MODELS OF BANK PORTFOLIO BEHAVIOUR WITH REFERENCE TO COMMERCIAL BANKS IN THE UNITED KINGDOM

by

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SYNOPSIS

The existing literature on the portfolio behaviour of commercial banks in general, and of UK commercial banks in particular, has been dominated by one particular set of models — the portfolio-theoretical models based on expected utility maximisation under the assumption of risk aversion. Most of these models are static models. In general, this set of models perform well in terms of the traditional "goodness of fit" measures and many authors have drawn important policy conclusions from them. However, the validity of these models have never been tested against other alternatives. This study attempts to remedy this situation by extending and building a number of alternative models. We consider three sets of models: models couched within the expected utility maximisation framework; models based on the Safety First principle and models based on the precautionary approach. Both static and dynamic versions of these models are examined. The data used are monthly balance sheet data of the London clearing banks for the period 1959-1983. This data period is much longer than that covered by the previous studies and encompasses a number of different control regimes. We make specific allowance for this. Our primary objective is not to arrive at one particular model of bank portfolio behaviour which "fits" the data well, but to test the different sets of models and to examine whether any of them can be accepted by the data. The various sets of models considered in this study are non-nested within each other. Thus, another major innovation of the thesis is to employ one recently developed non-nested test statistic to test the models. Non-nested testing in applied econometric work is still at an early stage and, to my knowledge, has not been used in testing bank portfolio models. Our main findings are that none of the static models considered can be accepted by the data but that two of the dynamic models based on the portfolio-theoretical approach cannot be rejected.

To the loving memory of my father

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CHAPTER ONE

INTRODUCTION

1.1 INTRODUCTORY

Banking was Italian, or to allow for its early (from about 1240) and steady development in Catalonia, Mediterranean, by birth. The word "bank" is derived from the Italian word "*banco*", a bench. By the fifteenth century the Italian banks offered their services of rapidly increasing diversity to an Italian and foreign clientele. Princes, merchants, and the general public alike turned to the banks to obtain credit, to pay bills for goods, to transfer funds, or to deposit money at interest. These Italian banks from the beginning assumed most of the essential functions which customers of modern banks are to expect from institutions of this kind [CMRSUC (1979)]. Today, banks occupy the most important position in the financial systems of most developed economies. As we can see from Table 1.1, banks' assets account for more than half of all financial assets held by financial intermediaries in the United Kingdom at the end of 1990. The banks also constitute the essential part of any national payment system and of monetary system as a whole which is vital for the smooth operation of modern economies.

Thus, an adequate explanation of bank behaviour is not only of great interest in that such institutions comprise a large sector of any modern economy, but also essential for a complete understanding of the mechanism by which the banking system affects the real goods markets and the level of economic activity. Banks, especially commercial banks, have traditionally been the focus of monetary policy. A detailed knowledge of commercial bank behaviour is therefore also vital in evaluating and enhancing the ability of monetary authorities to influence the behaviour of commercial banks and hence monetary aggregates. For example, if the monetary authorities wish to reduce the level of bank advances, given the private sector's demand for commercial bank loans, a

detailed knowledge about the banks' loan behaviour may improve the effectiveness of their policy measures. Finally, on a somewhat related plane, a proper understanding of the behaviour of commercial banks is essential in designing and implementing rules and legislations that would, on the one hand, impede as little as possible the efficient operation of the regulated units, but on the other, provide maximum protection to the depositors. In the light of recent wave of bank deregulation and bank failures, this is an issue of ever increasing importance.

Table 1.1
Holdings of Financial Assets by Financial Intermediaries in the UK
(end of 1990)

Intermediaries	£ million	%
Banks	1,031,228	53.68
Building Societies	217,027	11.30
Other Financial Institutions	672,972	35.02
Total	1,921,227	100.00

Source: Central Statistical Office, Financial Statistics, no. 351, July 1991, HMSO, London, pp. 64, 147 and 148.

The existing empirical literature on the portfolio behaviour of commercial banks in general, and of UK commercial banks in particular, has been donimated by one particular type of model — the portfolio-theoretical models based on expected utility maximisation under the assumption of risk aversion. The main attraction of this type of model is that they not only allow explicitly for risk aversion but also generate clearly defined demand equations and prior information concerning the signs and values of the parameters to be estimated. In general, this type of model performs well in terms of the traditional "goodness of fit" measures and many authors have often drawn important policy conclusions from them. However, the validity of these models have never been tested against other non-nested alternatives. This study represents an effort to remedy this situation by developing several alternative models.

One of the approaches examined in this study, the portfolio-theoretical approach, is well-known in the literature of bank portfolio behaviour. An integrated review of the existing empirical models based on this approach is provided in Chapter Two and a number of this type of models are extended and modified in Chapter Three. The Safety First principle is not entirely new in the portfolio literature but has not been applied to modelling the portfolio behaviour of commercial banks before, at least at the empirical level. We build a number of models of bank portfolio behaviour based on the Safety First principle in Chapter Four. Another approach, the precautionary or inventory-theoretical approach, has been applied to portfolio behaviour of banks at the theoretical level but has not been implemented at the empirical level. In Chapter Five, we extend and modify this approach in a number of important aspects and estimate a few models of bank portfolio behaviour derived from this approach. The primary objective of this study is, not to arrive at one particular model of bank portfolio behaviour which "fits" the data well, but rather, to test the different sets of models and to examine whether any of them can be accepted by the data. The various sets of models considered in Chapters Three to Five are non-nested within each other. Thus, another major innovation of the thesis is to employ one recently developed non-nested test statistic to test the models. Non-nested tests in applied econometric work are still at an early stage and, to my knowledge, have not been used in testing bank portfolio models. We do this in Chapter Six. The data used in this study are monthly data for the period 1959:01 - 1983:08. This data period is much longer than any of the previous studies and encompasses a number of different control regimes. We make specific allowance for the change of regimes.

The rest of the chapter is organised as follows. We discuss briefly in the next section the main features of commercial banks as compared to non-bank financial institutions (NBFIs). The nature of commercial banks is further considered in Section 1.3 through the examination of the aggregated balance sheet of the most important group of the UK commercial banks, the London clearing banks. In Section 1.4, we review the various theoretical models of bank behaviour in the literature but defer the discussion on

the empirical literature until Chapter Two. The scope and methodology of the thesis are set out in the final section.

1.2 FEATURES OF COMMERCIAL BANKS

An old and important industry banking may be, but there is no adequate definition of a bank at either the statutory or theoretical level. *The Banker's Book (Evidence) Act 1879, the Bills of Exchange Act 1882,* and *the Companies Act 1948* of the UK all describe a banker as a person or institution "carrying on the business of banking", without defining the word "banking" appropriately. Indeed, the word "banking" does not even appear as an entry in the *Dictionary of Banking and Finance* by Derrick G. Hanson (1985). It is not surprising, therefore, that in the case of United Dominions Trust Ltd v. Kirkwood (1966) Lord Denning, Master of Rolls, said, "A banker is easier to recognise than to define" [Hanson (1985)].

In *the Banking Act 1979* the problem of recognition, but not of definition, of a bank in the UK has been solved. The Act establishes the criteria by which the Bank of England may recognise an institution as a bank. The principal criteria for the recognition of a deposit-taking institution as a bank are

> *1. that the institution enjoys, and for a reasonable period of time enjoyed a high reputation and standing in the financial community;*
> *2. that the institution provides in the UK a wide range of banking services, or "highly specialised banking services";*
> *3. that at least two individuals effectively direct the business of the institution;*
> *4. that the institution has net assets of £5 million if providing a wide range of banking services and £250,000 if providing highly specialised banking services.*

Banking services include the following:

> *a. current or deposit account facilities, or the acceptance of funds in the*

wholesale money market;

b. finance in the form of overdraft or loan facilities, or the lending of funds in the wholesale money market;

c. foreign exchange services;

d. finance through the medium of bills of exchange and promisory notes, together with finance for foreign trade;

e. financial advice or investment management services and facilities for arranging the purchase and sale of securities.

An institution is not to be regarded as providing a wide range of banking services unless it provides all of the above services [Hanson (1985)].

In the United States a bank is defined by *the Bank Holding Company Act 1956* as an organisation that both accepts demand deposits and make commercial loans. Any financial intermediary that engages in only one of these two functions thus does not qualify as a bank [Candilis (1975)]. Although such a statement is again more of a criterion for recognition rather than a definition, it does emphasise the importance of demand deposit and commercial loans in the business of a commercial bank.

There exists a substantial literature on the issue of whether commercial banks are in any important sense different from other financial intermediaries. The debate was initiated by Gurley and Shaw (1955, 1960) when they stressed the similarities, as opposed to the differences, between banks and NBFIs. This "new view" of commercial banks was later reiterated in its most lucid version by Tobin (1963):

> (1) The distinction between commercial banks and other financial intermediaries has been too sharply drawn. The differences are of degree, not of kind. (2) In particular, the differences which do exist have little intrinsically to do with the monetary nature of bank liabilities. (3) The differences are more importantly related to the special reserve requirements and interest rate ceilings to which banks are subject. Any other financial industry subject to the same kind of regulations would behave in much the same way [p. 418].

This is in contrast to the traditional view that commercial banks are very different from

NBFIs. In their defence of the traditional view Guttentag and Lindsay (1968) demonstrate that "commercial banks have a greater capacity for varying the aggregate volume of credit than other financial intermediaries" [p. 991]. Specifically, they illustrate that shifts in the demand by commercial banks for reserves have significantly greater impact on total credit extended by all financial intermediaries than do similar shifts in the demand for reserves by NBFIs. Indeed, "it makes it clear that banks are a much more potent engine of credit creation, even if the expansion process is not exclusively a bank phenomenon" [p. 998]. And they argue that this can be interpreted as an operational uniqueness of commercial banks.

A practical distinction between a commercial bank and a NBFI rests upon the nature of their asset and liability portfolios. Operationally, NBFIs tend to cater for specific needs in the economic system and the choice of assets and liabilities in their portfolio is in general extremely restricted. On the one hand, for example, under *the Building Society Act 1962* these institutions may only lend by way of mortgage on property. On the other hand, banks can hold diversified portfolios of assets of varied liquidity where liquidity is defined as the ability to transform wealth holding into any form without loss of face value or delay. Funds secured by banks in the form of demand deposit, time deposit and ownership claims are invested in a wide variety of earning assets, the revenue from which constitutes the main source of bank income. These assets differ widely in terms of their expected rates of return, market and/or default risk characteristics, liquidity and so forth. It should be stressed though, that after the recent wave of financial market deregulation the distinctions between banks and NBFIs have becoming increasingly blurred. Banks are starting to intrude into the long-established territories of the NBFIs while the NBFIs, which have created the major impetus for innovation in the British financial system since the Second World War, have been encroaching into the domains of traditional banking. Commercial banks now provide mortgages for home buyers and most building societies offer cheque and cash withdrawal services for their shareholders as well as non-shareholder customers.

1.3 THE BALANCE SHEET OF A COMMERCIAL BANK

In order to understand the nature of a commercial bank we now investigate in more detail the asset and liability portfolios of a commercial bank. In practice, the balance sheet of a commercial bank usually contains many items and the number of items included may differ from one bank to another and for the same bank from one period to another, partly because of changes in the number of assets and liabilities available to commercial banks and partly due to changes in reporting or accounting practice. In general, however, the items in a balance sheet may be aggregated into some broad catagories. Table 1.2 presents the aggregated balance sheet of the London clearing banks.

Table 1.2
The Aggregate Balance Sheet of the London Clearing Banks
(end of 1982)

Liabilities	£m	Assets	£m
Sight and Time Deposits	78,840	Notes, Coins and Balances with the Bank of England	1,325
Certificates of Deposit	4,355	Money at Call and Short Notice	9,649
Other Liabilities	11,977	Treasury Bills	140
		Commercial Bills	2,361
		Certificates of Deposit	675
		Special & Supplementary Deposits	0
		British Government Stocks and Other Investment	5,886
		Loans and Advances	68,560
		Other Assets	6,576
Total	95,172	Total	95,172

Source: see Appendix.

From Table 1.2, it is clear that the main liabilities of the London clearing banks are sight deposits and time deposits. Sight deposits, which may or may not be interest-bearing, are withdrawable on demand and transferable by cheque. The term "sight deposit" originated in May 1975 when current accounts (demand deposits) needed to be amended to measure more precisely all those deposits which were repayable on demand [Bank of England (1975a)]. Thus sight deposits include demand deposits which do not bear any interest and those interest-bearing deposits which are immediately repayable such as callable wholesale deposits. Sight deposits are the most important means of payment in the economy. Time deposits, as the name implies, are deposits which are only repaid after certain period of time has elapsed. Before the introduction of Competition and Credit Control in September 1971 time deposits were interest-bearing and subject to notice of withdrawal, generally seven days, and not usually transferable by cheque. Since 1971, however, the London clearing banks have been prepared in addition to accept fixed-term deposits of more than seven days.

The next item on the liability side of the balance sheet is negotiable certificates of deposit (CD), which is closely related to the fixed-term time deposits but differ from fixed-term deposits in its negotiability. The certificates are generally donimated in either sterling or US dollars. As a result of the rapid growth in wholesale banking the sterling certificate of deposit was inaugurated in October 1968 [Bank of England (1968), p. 347]. A sterling certificate of deposit, as defined by the Bank of England, "is a document, issued by a U.K. office of a British or foreign bank, certifying that a sterling deposit has been made with that bank which is repayable to the bearer upon the surrender of the certificate at maturity" [Bank of England (1972b), p. 487]. Compared with time deposits or other ordinary fixed-term deposits where payment may be requested and be difficult to refuse, the certificates of deposit have the advantage from the bank's point of view that funds deposited in this form cannot be withdrawn before maturity. The certificates of deposit are also attractive to the holder. They not only earn the holder as good a rate of interest as a fixed-term deposit but also provide the holder with the additional advantage of selling the certificate at a discount in the secondary

market if the holder needs liquidity before the certificate matures. Prior to the introduction of Competition and Credit Control the London clearing banks were precluded from issuing certificates of deposit in their own names by the interest rate and other cartelistic arrangements then in force. Since then, however, the London clearing banks have made considerable use of such certificates as a means of attracting additional resources, especially in times of severe liquidity shortage.

Let us now turn to the other side of the balance sheet. The bank's assets consist of notes, coins and balances with the Bank of England, money at call and short notice, Treasury bills, commercial bills, certificates of deposit, Special and Supplementary Deposits, British government stocks, loans and advances and other assets. We discuss each item in turn.

Notes, coins and balances with the Bank of England are the most liquid assets held by the London clearing banks to meet deposit withdrawals. These items do not bear interest and the London clearing banks have traditionally been required by the Bank of England to hold a certain proportion of their total assets in this form. Just as an ordinary commercial bank can influence an individual's expenditure by requiring a certain minimum to be kept in a private bank account, it is possible for the Bank of England to restrict a bank's lending by requiring more or less to be held in the form of notes, coins or deposits with it. On the one hand, if the Bank of England requests that more be held in this form in the asset portfolio then less is available for the commercial bank to lend given the total deposits the bank can attract. On the other hand, if the Bank were to release some of the deposits a commercial bank has with it this would allow the commercial bank to make more loans or other forms of investment. It is therefore not surprising that this item has traditionally been used by the Bank of England as a tool of monetary policy, although as we shall see in the next section, the methods adopted by the Bank have changed significantly since the 1960s.

The second item in the commercial bank's asset portfolio is money at call and

short notice. As the name indicates, these are money loaned to the money market on a short-term basis by the commercial banks. They are so called because the money, most of which is lent to the discount houses, can be recalled immediately or at periods of notice of up to fourteen days, and thus they are regarded as the second most liquid assets of the commercial banks. The terms of the loans vary and in practice the money may not be called for long periods. The interest on call money is normally below the Treasury bill rate as the discount houses use the loaned money to deal in the Treasury bills and commercial bills to be discussed below. The commercial banks are willing to lend their liquid funds in this form to the money market because not only do the funds earn them interest but are secured by the assets of the discount houses and by the knowledge that the Bank of England is always prepared to act as the "lender of last resort". Thus, the primary function of this asset is to provide the commercial banks liquidity on demand.

Of course, the commercial banks themselves hold in their asset portfolio Treasury bills as well as commercial bills. Treasury bills are promisory notes issued by the Treasury to finance short-term government expenditure. As the bills do not carry interest, they are tendered for at a discount. The London clearing banks, by an understanding with the Bank of England and the discount houses, do not generally bid for Treasury bills at the weekly tender, at least not for themselves. Treasury bills bought on their own accounts by the clearing banks are usually held at maturity, though they are on occasions sold to the Bank of England if the latter is looking for maturities that are no longer held by the discount houses or if the commercial banks require liquidity. A commercial bill is in fact an IOU from one party to another which promises to pay a specific amount of money at a given date, usually three months ahead. As with the certificates of deposit commercial bills can be sold at a discount if one requires immediate liquidity. Thus, commercial bills are also regarded as part of a bank's liquid assets. Although commercial bills provide liquidity and a yield at maturity, they do not provide capital certainty since in order to obtain cash quickly one must usually sell them at a further discount. Therefore, the London clearing banks do not

engage in this activity to a great extent.

The London clearing banks also hold certificates of deposit in their asset portfolios. In fact, from the very beginning the London clearing banks themselves have held a considerable proportion of the certificates in issue, and the market in the certificates of deposit "has developed to a large extent as an extention of the ordinary inter-bank market in unsecured sterling funds" [Bank of England (1972b), p. 490]. In this respect, the certificates of deposit have served a similar purpose in helping distribute funds within the banking system, although since September 1971, the London clearing banks have used the instrument more actively as a means of bidding for funds from outside the banking system and the amount held by private holders other than the commercial banks has grown considerably.

British government stocks and other investment form another part of the bank's asset portfolio. These are fixed-interest securities issued by the government and traded on the Stock Exchange. They are often referred to as gilt-edged due to the fact that it is certain that interest will be paid and that they will be redeemed (where applicable) on the due date. Some of them are dated, some are undated and some are index-linked. Government securities are not, of course, a risk-free investment because of fluctuations in their market value. Government securities are playing a less significant role than before as a stablising element in the asset portfolio. Also, since September 1971, the authorities have not made a market in the gilts, so they are becoming increasingly capital uncertain as a result. The reason for this declining role attached to the gilt-edged securities is that "the controls imposed upon the banks to ensure stability of their asset portfolio is such that they do not need to resort to the usual measures to stablising them, i.e., to hold as much government stocks as do the other, relatively uncontrolled, institutions" [Wilson (1981), p. 81].

Undoubtedly the largest component of the London clearing banks' asset portfolio is the provision of loans and advances. This element of the asset portfolio accounted for

72% of total assets. This catagory includes a wide range of assets. Loans to the discount market are funds which are not immediately callable and are unsecured in contrast with money at call and short notice. Advances, which are quantitatively of major importance, comprise not only other advances but also advances in other currencies under various headings and not individually specified. Advances are of two main types, loans and overdrafts. With the loan the customer's account is credited with the amount whereas the overdraft is literally an overdrawing of a current account which is, debited accordingly. Advances are generally regarded as the most lucrative of the bank's assets. Before the introduction of Competition and Credit Control, rates on advances by the London clearing banks were tied to Bank rate under their cartel arrangements on interest rates. Each bank now fixes what it calls a base rate for advances. The base rate might and did differ between banks but was generally close to the minimum lending rate of the Bank of England before 1981. The actual rates charged for advances vary with the nature and status of the customer but the average rate was usually 2% higher than the base rate. The London clearing banks do have uniform rates for nationalised industry borrowing under Treasury guarantee and for certain other government-supported borrowing. Although loans are normally secured and repaid in regular instalments they are regarded as the most risky assets in the commercial bank's asset portfolio because of default risks.

1.4 THEORIES OF BANK PORTFOLIO BEHAVIOUR

A remarkable amount of theoretical and empirical work has been done in the last three decades on the behaviour, especially portfolio behaviour, of commercial banks. Research in this area has been stimulated by the need to analyse the effects of alternative regulatory schemes, as well as by the desire to improve knowledge about the *modus operandi* of the banking system as the most important link in the transmission process of monetary policy. There exists a wide variety of theoretical models on bank behaviour and the literature is still as unsettled and heterogeneous as ever [see Baltensperger (1980) and Santomero (1984)]. However, this heterogeneity should not be regarded as undesirable but rather as reflecting the difficult nature of the subject as well as the differing objectives pursued in various studies. We shall focus on the main approaches that have dominated the literature and there is, therefore, no pretence at being comprehensive or exhaustive. For convenience of presentation we shall review in this section only the various theoretical models of bank behaviour and defer the discussion on the empirical studies until Chapter Two.

1.4.1 The Traditional Approach

The traditional approach towards bank behaviour may be best described by the enlightening, though somewhat stereotyped, analysis of Robinson (1962). For Robinson, "the essential banking problem is to resolve the conflict between safety and profitability in the employment of bank funds" [p. 11]. He therefore sought to "describe methods of achieving the most profitable employment of commercial bank funds consistent with safety" [p. 4]. These methods essentially consist of setting and observing a hierarchy of priorities in the employment of funds. These priorities, in descending order, are: (1) primary or legally required reserves, (2) secondary reserves or protective investment, (3) customer credit demand, and (4) open-market investments for income. Commercial banks have the legal obligation to pay out their demand deposits without notice and on demand. So they must provide themselves with adequate primary reserves before any

other use of funds is considered. However, holding cash for all possible and remote contingencies requires a bank to forego earnings needlessly. For such a dual purpose protective investment is regarded as the second priority. "Once a bank has made it safe, it should devote itself to the business for which it is best fitted" [p. 16] — making loans. Only after a bank has provided the liquidity for safety and has satisfied in full the local customers' demand for loans, can it enter the investment market with any remaining funds. That is to say that open-market investment is entirely exogenous and derivative. Within such a hierarchical framework safety is the first priority and interest rates do not affect the bank's choice of portfolio. While this framework does provide some insights into bank motives for holding various assets, it does not indicate how a bank optimise when deciding whether or not to shift funds from one asset to another due to the total absence of marginal analysis which in turn arises from the exclusion of interest rates from influencing the choice variables.

Hodgman (1961, 1963) attempt to remedy some of the shortcomings of the traditional bankng theory by introducing interest rates and a deposit-loan relation into his analysis of behaviour of the banking firm. His most significant contribution is the development of the customer relation hypothesis in which the size of deposit is made dependent upon the size of loans under the assumption that some borrowers are also depositors. Thus, in Hodgman's view, "the individual banker cannot maximise his total return on earning assets by maximising his return per dollar lent or invested unless he takes into account the dependence of the size of his investment funds upon loans made" [Hodgman (1963), p. 14]. Deposit balance, contributing to the bank's capacity to acquire earning assets, is made endogenous through the deposit-loan relationship, which gives the bank a strong reason to lend to customers with large balances. Hodgman's work is useful in understanding such aspects of contemporary banking as prime rate conventions, compensating balance requirements and credit rationing [see also, Hodgman (1960, 1962)]. He also recognises the possibility of bank optimisation, but unfortunately he fails to follow this lead, since his primary objective is "to describe and analyse the habitual procedures, criteria of judgement, and attitudes of commercial bankers as they make

decisions concerning loan and investment policy" [Hodgman (1963), p. 1]. Marginal analysis, which is such a salient feature of modern economics, is still missing.

In both Robinson (1962) and Hodgman (1961, 1963) an objective is only loosely specified together with a set of legal and administrative constraints to obtain decision rules for the bank. Their theories of bank behaviour are descriptive rather than analytical. Chambers and Charnes (1961) improve upon this descriptive tradition by suggesting a linear programming framework [see also Charnes and Thore (1966)]. Chambers and Charnes view the bank's problem as one of constrained profit maximisation where the constraints are the "requirements laid down by bank examiners which are interpreted as defining limits within which the level of risk associated with the return on portfolio is an acceptable" one [p. 393], and the balance sheet constraint. Interest rates are introduced into the model through the objective function — the profit function. The main advantage of such a model is that marginal analysis can be derived directly from the solutions to the constrained maximisation problem. However, the soundness of this model is considerably weakened by the assumption that the bank knows "the levels that will prevail, at various dates in the future, of demand and time deposits, of rates of interest and of the bank's net worth" [p. 393]. Uncertainty is absent from the model.

"Probability is the foundation of banking. The solvency and profits of the banker depend upon the probability that he will not be called upon to meet at once more than a certain amount of his liability" [Edgeworth (1888), p. 113]. One of the sources of uncertainty in banking is the random deposit flows, whose importance for the determination of a bank's optimal portfolio is suggested by Edgeworth (1888) in the form of a simple game:

> I have imagined a new game of chance which is played in this manner: each player receives a disposable fund of 100 counters, part of which he may invest in securities not immediately realisable, bearing say 5 per cent per ten minutes; another portion of the 100 may be held at call, bearing interest at 2 per cent per ten minutes; the remainder is kept in the hands of the player as a *reserve* against certain liabilities. ... From time to time, say every two

> minutes, there are taken a certain number, say 22, digits at random from the pages of some mathematical or statistical tables. The sum of these digits constitutes the demand which the player has to meet. We need not consider the provision which is made to meet the *average* amount (99) of this demand. The special object of the reserve above mentioned is to provide against demands which exceed that average. If the player can meet this excess of demand with his funds in hand, well; but if not he must call in part, or all, of the sum placed at call, incurring a forfeit of 10 per cent on the amount called in. But if the demand is so great that he cannot even thus meet it, then he incurs an enormous forfeit, say 100l, or 1,000l [p. 120].

Strikingly, some seventy years elapsed before uncertainty was explicitly introduced into further models of bank portfolio behaviour by Porter (1961) and Orr and Mellor (1961).

1.4.2 The Precautionary Approach — Bank Behaviour under Risk Neutrality

Porter (1961) and Orr and Mellor (1961) are the first to apply the precautionary approach to the problem of bank portfolio behaviour under conditions of uncertainty and the approach has been taken up by others in more recent times [Morrison (1966), Poole (1968), Frost (1971), Baltensperger (1972a, 1972b, 1980), Pringle (1974), Hester and Pierce (1975), Knobel (1977) and Sprenkle (1987)]. This approach relies crucially upon two basic assumptions: (1) the bank is subject to random flows of deposits and knows (estimates) the probability distribution of deposit flows, and (2) the bank maximises expected profits, or minimises expected loss. Given the portfolio size the bank's problem is one of choosing the optimal beginning-of-period allocation of the funds among reserves and other assets to maximise expected profits. Some of the salient features of this approach can be illustrated by means of the following simple model.

Consider a bank which has a given amount of deposits, D, and a given amount of capital or net worth, W, and can choose between three assets, namely, cash reserves, A_1, securities, A_2, and loans, A_3. The bank is subject to a random withdrawal of deposits — cash requirement. Let x denote the random cash requirement during the planning period with an estimated (prior) probability density function, f(x). When x exceeds the beginning-of-period cash reserves, A_1, during the planning period, a reserve deficiency

occurs, which makes certain costly adjustments necessary for the bank, e.g., selling of securities, emergency borrowing from central banks. Assume that the bank operates in a perfectly competitive market and seeks to maximise expected profit for the planning period, the problem is then to choose the optimal beginning-of-period allocation of the given funds, D + W, among the three assets so as to minimise the expected losses due to the uncertain nature of the cash requirements during the planning period.

Suppose the net return on cash reserves is zero, the coupon return per unit (say a pound) worth of securities (at initial market price) is r_2, and the net return on loans (net of default risk) is r_3, with $r_2 < r_3$. Further suppose that the proportional unit cost of liquidating securities is c_2 and that of liquidating loans is c_3, with $c_2 < c_3$. It is thus optimal for the bank to first liquidate securities to meet cash requirements when the requirements exceed the beginning-of-period cash reserves. Only when the liquidation of securities is not sufficient to meet the cash requirements, i. e., when $x > A_1 + A_2$, will the bank start liquidating its loans. Therefore, we can write the bank's expected profit function for the planning period as

$$\mu_\pi = r_2A_2 + r_3A_3 - \int_{A_1}^{A_1+A_2} c_2(x - A_1)f(x)dx - \int_{A_1+A_2}^{\infty} [c_2A_2 + c_3(x - A_1 - A_2)]f(x)dx. \tag{1.1}$$

The bank's objective is to choose A_1, A_2 and A_3 to maximise (1.1) subject to the balance sheet constraint

$$D + W = A_1 + A_2 + A_3. \tag{1.2}$$

Forming the Lagrangian,

$$L = r_2A_2 + r_3A_3 - \int_{A_1}^{A_1+A_2} c_2(x - A_1)f(x)dx - \int_{A_1+A_2}^{\infty} [c_2A_2 + c_3(x - A_1 - A_2)]f(x)dx$$

$$+ \lambda (D + W - A_1 - A_2 - A_3) \qquad (1.3)$$

and differentiating it with respect to the three choice variables, A_1, A_2, A_3 and the Lagrangean multiplier, λ, yields the following first-order conditions:

$$\frac{\partial L}{\partial A_1} = c_2 \int_{A_1}^{A_1+ A_2} f(x)dx + c_3 \int_{A_1+ A_2}^{\infty} f(x)dx - \lambda = 0 \qquad (1.4)$$

$$\frac{\partial L}{\partial A_2} = r_1 + (c_2 - c_2) \int_{A_1+ A_2}^{\infty} f(x)dx - \lambda = 0 \qquad (1.5)$$

$$\frac{\partial L}{\partial A_3} = r_3 - \lambda = 0 \qquad (1.6)$$

$$\frac{\partial L}{\partial \lambda} = D + W - A_1 - A_2 - A_3 = 0. \qquad (1.7)$$

From (1.6), $\lambda = r_3$. Substituting for λ into (1.4) and (1.5) gives

$$r_3 = c_2 \int_{A_1}^{A_1+ A_2} f(x)dx + c_3 \int_{A_1+ A_2}^{\infty} f(x)dx \qquad (1.8)$$

$$r_3 - r_2 = (c_3 - c_2) \int_{A_1+ A_2}^{\infty} f(x)dx. \qquad (1.9)$$

The implications of these conditions are far from obvious at first sight. After some slight algebraic manipulation, however, they are ready to be interpreted. Rearranging (1.9) and substituting r_3 from (1.9) into (1.8) give

$$r_2 = c_2 \int_{A_1}^{\infty} f(x)dx. \qquad (1.10)$$

Note that

$$\int_{A_1+A_2}^{\infty} f(x)dx$$

is the probability that the net cash requirements will exceed the bank's total reserves (cash reserves and securities). Therefore,

$$c_2 \int_{A_1}^{A_1+A_2} f(x)dx \quad \text{and} \quad c_3 \int_{A_1+A_2}^{\infty} f(x)dx$$

are the expected marginal costs of liquidating securities and loans respectively. (1.9) thus requires that the bank allocate its funds between loans and total reserves such that the expected marginal profit of extending loans is equal to the expected marginal profit of holding reserves. Since it is assumed that cash reserves do not yield positive interest nor cause liquidation costs, (1.10) requires that the bank's total reserves determined by (1.9) are divided between cash reserves and securities such that the expected marginal profit of holding securities equal the expected marginal profit of holding cash reserves, which is zero. Equations (1.9) and (1.10) thus determine the bank's optimal portfolio selection. However, explicit solutions for A_1, A_2 and A_3 can only be obtained until a specific probability density function of cash requirements is specified for f(x).

In the above model, it is assumed, as in most of the existing precautionary models, that all asset yields are certain. This, however, may seem to be an unrealistic assumption as, say, securities are subject to capital gains or losses, and loans to default risks. By assuming a uniform distribution for the capital gains and losses Porter (1961) attempts to incorporate this important aspect of uncertainty into the precautionary model. As a result, the bank's decision on the optimal portfolio selection depends upon the coupon return as well as upon the expected capital gains or losses. Alternatively, to allow for capital uncertainty we can redefine c_2 as the liquidating cost net of capital gains or losses. It is also possible to introduce uncertainty into the rate of return on loans by means of replacing the net-of-default-risk rate with a more general gross rate

whose expected value is determined by the assumed probability distribution. It should be noted, however, that even in a simple three-asset model the introduction of asset yield uncertainty complicates the analysis considerably, as can be seen in Porter (1961). A general model of this kind would make the algebra rather intractable.

It is also assumed in the above model that the bank operates in a perfectly competitive market which implies that the rate of return on each asset is independent of the volume of the asset acquired. A good first approximation for the securities market this may be, but it is not a necessary assumption for the usefulness of the model. Loan market imperfections can be incorporated into the model quite easily, as are in Porter (1961) and Baltensperger (1980). If it is assumed that, r_3, rather than being a market-determimed parameter, is functionally related to the amount of loans the bank makes, that is,

$$r_3 = r_3(A_3)$$

with

$$r_3' = \frac{\partial r_3(A_3)}{\partial A_3} < 0,$$

i.e., the rate of return on loans is a declining function of the volume of loans, (1.9) then becomes

$$r_3 (A_3) + A_3 r_3'(A_3) - r_2 = (c_3 - c_2) \int_{A_1 + A_2}^{\infty} f(x)dx \qquad (1.11)$$

which is essentially the same as (1.9) except that the expected marginal rate of return on loans is no longer constant. The bank's decision on the allocation of total reserves between cash and securities is not, however, affected by the introduction of the loan market imperfections as (1.10) is independent of the rate of return on loans.

So far, the costs of adjusting to the optimal portfolio have been ignored. The level of reserves that the bank inherits at the beginning of each period is a result of the random cash requirements in the previous period and is not, in general, equal to the desired level of cash reserves. If beginning-of-period adjustments to the optimal position as determined by the above simple model were costless, the bank would continuously adjust its portfolio so that it starts each planning period at the optimal level independent of the inherited level of cash reserves from the previous period. In reality, however, adjustment costs do exist and an adjustment to the optimal position is profitable only if the resulting gains from moving the portfolio towards optimality are greater than the costs of adjustment. This aspect of adjustment costs has been incorporated into the above precautionary model by Miller and Orr (1966), Frost (1971), Baltensperger (1972a, 1980) and Knobel (1977).

Assume that the inherited level of reserves is A_1^0 and the adjustment costs are proportional to the volume of adjustments. Let q be the proportional adjustment cost. Then the Lagrangean (1.3) becomes

$$L = r_2A_2 + r_3A_3 - \int_{A_1}^{A_1+A_2} c_2(x - A_1)f(x)dx - \int_{A_1+A_2}^{\infty} [c_2A_2 + c_3(x - A_1 - A_2)]f(x)dx$$

$$- q\,(|\,A_1 - A_1^0\,|) + \lambda\,(D + W - A_1 - A_2 - A_3) \qquad (1.12)$$

where $|\,A_1 - A_1^0\,|$ indicates the absolute value of adjustments required. Maximisation of (1.12) with respect to A_1, A_2, and A_3 yields the following first-order conditions:

$$r_3 - r_2 = (c_3 - c_2) \int_{A_1+A_2}^{\infty} f(x)dx \qquad (1.13)$$

and

$$r_2 - c_2 \int_{A_1}^{A_1+ A_2} f(x)dx = \pm q. \qquad (1.14)$$

(1.13) is exactly the same as (1.9). This is so because of the assumption that adjustments only take place with the cash reserves. (1.14) shows that the presence of adjustment costs affect the bank's reserve asset composition in such a way that the marginal gains resulted from moving towards optimality is just equal to the marginal cost of making the adjustments. (1.14) also determines the range around A_1 within which the bank will let its reserves fluctuate freely without making any adjustments. If the inherited cash reserve A_1^0 is outside this range an adjustment to either the upper or the lower boundary will be made depending on whether A_1^0 has left the critical range on the upper or lower side. Though any adjustment towards the optimal level will still reduce the expected reserve deficiency cost, the marginal gain is so small that it is less than the adjustment costs, thus resulting in a net marginal loss.

It is assumed above that the adjustment costs are proportional to the volume of adjustments. If, however, adjustment costs consist of a lump sum element, Q, which is independent of the size of adjustment, an adjustment towards optimality is profitable only if the resulting gains are greater than all the adjustment costs — $(q \mid A_1 - A_2^0 \mid + Q)$. Though adjustment towards A_1 will be made, the range within which no active adjustment is profitable is further increased, see for example, Baltensperger (1972a, 1980). A useful diagrammatic exposition of the determination of adjustment boundaries in the presene of both a proportional and a lump sum adjustment costs is provided by Frost (1971).

The models reviewed so far are bank asset management models. While a lot of work has been done in this field, the liability side of the banking firm tends to be relatively neglected in the literature. In particular, the rather wide applicability of the precautionary approach has not been explored to model the bank's liability management. The reason for this neglect seems to lie in the argument that the bank does not have any choice other than simply to accept all the deposits offered to it by the public at the

ruling deposit rates; thus the volume and structure of deposits are beyond the control of the bank [Baltensperger (1980)]. Such an argument may be valid for the short-run portfolio behaviour of commercial banks, but it does not seem to be sufficient to justify the neglect. A bank has a variety of instruments such as interest rate paid on and services provided for a certain type of deposits, to influence the relative and absolute attractiveness of, and thus the public's demand for different types of deposits. This is reinforced by the fact that banks advertise continuously and vigorously for their deposit business. And indeed, in the last two decades liability management has become increasingly important for the survival of a banking firm. The Bank of England's Supplementary Special Deposit Scheme was introduced from December 1973 in response to the rapid growth of liability management by commercial banks and other financial intermediaries [see Bank of England (1982)]. Furthermore, even if the argument that banks' liabilities are somewhat determined by exogenous interest rates were true, it would still be sensible, methodologically, to ask what the bank's optimal volume and structure of liabilities are under these conditions. Finally and most importantly, a satisfactory theory of financial intermediation depends upon a better understanding of the liability side of a financial intermediary in general and of a banking firm in particular.

A serious and useful attempt has been made by Baltensperger (1973, 1980) to apply the precautionary approach to study the determination of the optimal liability structure of the banking firm. His models are analytically as simple as those described above and yet have important implications for the theory of financial intermediation and are helpful for a better understanding of the behaviour of the banking firm.

To keep the exposition as simple as possible, consider a bank with a total given beginning-of-period level of assets of a certain structure, A (= A_1 + A_2 + A_3), for example. The bank's total income, Y, from these assets for the planning period is, due to the uncertainty over end-of-period value (as demonstrated by the above model) and yields of assets, a random variable, with an estimated probability density function f(Y); and obviously f(Y) is dependent upon the volume and structure of the bank's asset

portfolio. As before, we assume that the bank has only two type of liabilities, namely, deposits, D, with paid interest rate, r_D, and capital (net worth), W, whose opportunity cost is r_W. If the bank's total end-of-period asset value (A + Y) is less than its total end-of-period debt $(1 + r_D)D$, the bank is then in a state of insolvency, which, as in the case of reserve deficiency, incurs costs for the bank. Thus, a state of insolvency will occur if

$$Y + A < (1 + r_D)\, D. \tag{1.15}$$

With the balance sheet identity, A = W + D, it is clear that the probability of this event occurring is, *ceteris paribus*, positively related to the size of D and negatively to W. Therefore, an increase in capital W reduces the probability of insolvency, and hence the costs of such an event on the one hand, while the use of capital, on the other hand, incurs an unit opportunity cost, r_W. These two type of costs must be balanced.

If, for simplicity, we assume a proportional cost p for the state of insolvency, the expected cost of insolvency for the bank is then

$$H = \int_{-\infty}^{Y} p[(1 + r_D)D - Y - A]f(Y)dY \tag{1.16}$$

where

$$Y = (1 + r_D)D - A.$$

The total expected profit of the bank for the planning period, given the total volume and structure of assets, is

$$\mu_\pi = E(Y) - \int_{-\infty}^{Y} p[(1 + r_D)D - A - Y]f(Y)dY - r_D D - r_W(A - D) \tag{1.17}$$

where

$$E(Y) = \int_{-\infty}^{\infty} Y\, f(Y)dY.$$

Differentiating μ_π with respect to D gives the first-order condition:

$$r_W - r_D = \int_{-\infty}^{Y} p(1 + r_D)f(Y)dY. \qquad (1.18)$$

Thus, optimality requires that the bank choose the size of capital such that the expected marginal gains — marginal reduction in the insolvency costs equal the marginal cost of employing capital in place of deposits. Of course, the optimal deposit is then derived through the balance sheet identiy A = D + W. It is clear from (1-18) that the optimal liability structure of the bank depends in this case upon the rates, r_D, r_W, p and the income probability density function f(Y).

The costs of producing and maintaining deposits have so far been ignored. This aspect can, however, be easily added to the above simple precautionary model through the introduction of a cost function. If we assume that the cost of producing and maintaining deposits is a function of the size of deposits, i.e., C(D), then insolvency will occur if

$$Y + A < (1 + r_D)D + C(D). \qquad (1.19)$$

The bank's expected total profit for the planning period now becomes

$$\mu_\pi = E(Y) - \int_{-\infty}^{Y} p[(1 + r_D)D + C(D) - A - Y]f(Y)dY - r_D D - C(D)$$

$$- r_W(A - D) \qquad (1.20)$$

where

$$Y = (1 + r_D)D + C(D) - A.$$

And the corresponding first-order condition is

$$r_W - [r_D + C'(D)] = \int_{-\infty}^{Y} p[(1 + r_D) + C'(D)]f(Y)dY \qquad (1.21)$$

where

$$C'(D) = \frac{\partial C(D)}{\partial D}.$$

This condition can be interpreted in the same way as for (1.18).

The above model can be extended to cover more than one debt instrument, e.g., demand deposits and time deposits, without any difficulty [see Baltensperger (1973, 1980), which also incorporate deposit insurance into the above model].

It is useful at this stage to distinguish between long-run and short-run models of bank liability management. The above model may be regarded as a long-run model as in practice a bank's decision regarding optimal capital and deposit structure is only taken in the long run, while a model of bank liability management with capital fixed and considering only the optimal structure of deposits should be regarded as a short-run model.

In the discussion of the first-order condition (1.18) above we mentioned that the optimal liability structure of the bank depends upon the income probability density function f(Y), which in turn depends upon the size and structure of the bank's optimal asset portfolio. This implies, of course, that the bank's optimal asset and liability structures are interdependent. However, all the models discussed so far are partial equilibrium models which do not allow the simultaneous determination of the bank's optimal asset liability structures and the portfolio size. Such an analysis, as

Baltensperger (1980) has shown, is also possible within the precautionary framework. To keep things simple, we will assume as before that the bank has only two type of liabilities, namely, deposits, D, and capital, W, but now with both D and W variable. The bank can only choose between three assets, A_1, A_2 and A_3. For simplicity, we shall only consider the following costs:

1) costs of funding ($r_D D + r_W W$);

2) liquidation costs G;

3) insolvency costs H;

where

$$G = c_1 \int_{A_1}^{A_1 + A_2} (x - A_1) f(x)dx + [c_1 A_1 + c_2(x - A_1 - A_2)] f(x)dx \qquad (1.22)$$

$$H = p \int_{-\infty}^{Y} [(1 + r_D)D + C(D, W, A_1, A_2, A_3) - A - Y] f(Y)dY \qquad (1.23)$$

with

$$Y = r_2 A_2 + r_3 A_3$$

$$A = A_1 + A_2 + A_3$$

$$Y = (1 + r_D)D + C(D, W, A_1, A_2, A_3) - A.$$

But the probability density function of cash requirement is now redefined as dependent on the size and structure of liabilities while the income probability density function is dependent upon the level and structure of assets.

The total expected profit of the bank for the planning period is then

$$\mu_\pi = r_2 A_2 + r_3 A_3 - r_D D - r_W W - G(D, W, A_1, A_2, A_3)$$

$$- H(D, W, A_1, A_2, A_3) - C(D, W, A_1, A_2, A_3). \qquad (1.24)$$

Forming the Lagrangean and differentiating it with respect to the choice variables give the following first-order conditions:

$$\frac{\partial L}{\partial A_1} = - G_{A_1} - H_{A_1} - C_{A_1} - \lambda = 0 \qquad (1.25)$$

$$\frac{\partial L}{\partial A_2} = r_2 - G_{A_2} - H_{A_2} - C_{A_2} - \lambda = 0 \qquad (1.26)$$

$$\frac{\partial L}{\partial A_3} = r_3 - G_{A_3} - H_{A_3} - C_{A_3} - \lambda = 0 \qquad (1.27)$$

$$\frac{\partial L}{\partial D} = - r_D - G_D - H_D - C_D + \lambda = 0 \qquad (1.28)$$

$$\frac{\partial L}{\partial W} = - r_W - G_W - H_W - C_W + \lambda = 0 \qquad (1.29)$$

$$\frac{\partial L}{\partial \lambda} = D + W - A_1 - A_2 - A_3 = 0. \qquad (1.30)$$

We have from (1.28) and (1.29)

$$r_D + G_D + H_D + C_D = r_W + G_W + H_W + C_W \qquad (1.31)$$

from (1.26) and (1.27)

$$- (G_{A_1} + H_{A_1} + C_{A_1}) = r_{A_2} - G_{A_2} - H_{A_2} - C_{A_2} \qquad (1.32)$$

and from (1.27) and (1.28)

$$r_{A_2} - G_{A_2} - H_{A_2} - C_{A_2} = r_{A_3} - G_{A_3} - H_{A_3} - C_{A_3} \qquad (1.33)$$

Equation (1.31) determines the bank's optimal liability structure and the portfolio size.

Optimality requires that the bank choose the levels of deposits and capital such that the expected marginal costs of acquiring deposit equal the expected marginal costs of acquiring capital. The bank's optimal asset structure is determined by equations (1.32) and (1.33). (1.32) requires that the expected marginal profit from holding cash reserves should equal the expected marginal profit from holding securities, and (1.33) requires that the expected marginal profit from holding securities should equal that from holding loans.

A bothersome feature of static profit maximisation in the absence of uncertainty is that optimal portfolios contain only the highest yielding asset. However, as argued earlier, the crux of the banking operation is uncertainty — uncertainty regarding cash requirements, capital gains or losses, loan defaults and market yields. The precautionary approach shows that a bank that maximises expected profit will in general hold a diversified portfolio provided that assets (liabilities) that have the highest (lowest) yields are at the same time characterised by highest (lowest) transactions costs. Models of this kind have been particularly useful for describing and explaining the bank's demand for liquid assets, especially for excess reserves [see Orr and Mellor (1961), Baltensperger (1974), Knobel (1977)]. Liquid assets are treated as buffe s that insulate a bank from rapid and costly sales or purchases of illiquid assets due to random cash requirements. Information considerations [see Baltensperger and Milde (1976), Stanhouse (1982, 1983, 1986)] as well as real resource costs [see Pesek (1970), Baltensperger (1972b), Saving (1977)] can also be incorporated into these precautionary models to determine both the optimal structures of assets and liabilities and the total size of the banking firm.

It should be stressed, however, that the managability and usefulness of this model depend critically upon the assumptions made about the forms of the probability density functions and the number of assets and liabilities. As the probability density functions are assumed to be dependent on the level and structures of the bank's assets and liabilities, the model may be algebraically intractable for some specific density functions and larger number of assets and liabilities.

A very often cited criticism of the precautionary approach is its assumption that banks are risk neutral. It is to these models whose foundation is the very assumption of risk aversion of the banking firm that we turn in the next sub-section.

1.4.3 The Portfolio-theoretical Approach — Bank Behaviour under Risk Aversion

Although uncertainty plays a very important role in the precautionary models, banks are nevertheless viewed as risk neutral. Banks care about risk only to the extent that it feeds back into, and thus is reflected in the expected profit, e.g., via the possibility of costly emergency adjustments. Another approach to the theory of the banking firm under uncertainty, the portfolio-theoretical approach, which stems from Tobin's (1958) paper on liquidity preference and Markowitz's (1952,1959) pioneering study of efficient portfolio selection, makes explicit allowance for risk aversion. The portfolio-theoretical approach assumes that the bank maximises expected utility whose arguments are usually the expected value and variance of return of the portfolio subject to the balance sheet constraint. The maximisation of expected utility for a risk averse bank will in general result in the selection of a diversified portfolio. Examples of models of this kind can be found in Kane and Malkiel (1965), Parkin (1970), Parkin, Gray and Barrett (1970), Klein (1971), Pyle (1971), Hart and Jaffee (1974) and Sealey (1980). In fact, most of the empirical work on bank portfolio behaviour is based on this approach. In this sub-section, however, we shall review only the theoretical models. Empirical models under risk aversion are examined in Chapters Two and Three.

The most elementary of the risk aversion models is the work by Pyle (1971), which, using a three-asset model where there is a riskless asset, as well as loans and deposits in the intermediary, was primarily intended to determine the necessary conditions for financial intermediation. Pyle shows that, in the case of stochastic independence between asset and liability yields, intermediation will occur only if there is a positive risk premium on loans and/or a negative premium on deposits, i.e., if there is a positive expected yield differential between assets and liabilities. Pyle also shows that these

conditions are still sufficient for financial intermediation even if there is a positive dependence between asset and liability yields. The results imply that variances and covariances of interest rates across the balance sheet are extremely important and that the optimal asset and liability choices are obviously interdependent except in the case of independence between all interest rates. Since Pyle's analysis has its roots in the Telser (1955) model of hedging the assumption of risk aversion is crucial to ensure a finite size of, and a place for the riskless asset in, the portfolio. Without risk aversion an arbitrator would, in the absence of transactions costs, instantly eliminate the interest rate differentials which form the basis of Pyle's analysis.

Pyel's model is not, however, developed specifically for commercial banks. The first model of portfolio selection applied to commercial banks is by Parkin, Gray, and Barrett (1970). In the model each bank is assumed to possess a Freund-type utility function of the form [Freund (1956)]

$$U = \alpha - \gamma e^{-\beta\pi} \qquad (1.34)$$

where U is utility, $\prod$ real profit, and α, β and γ parameters. The objective of the bank is assumed to be, subject to a balance sheet constraint, the maximisation of the expected value of the utility function in (1.34), which, with the normality assumption of real profit, is equivalent to the maximisation of

$$\Omega = \mu_{\pi} - \frac{\beta}{2}\sigma^2 \qquad (1.35)$$

where μ_{π} and σ_{π}^2 are the expected value and variance of real profit [see Chapter Two]. Parkin *et al* like Pyle (1971) assume that the rates are given exogenously and independent of the bank's decision on its portfolio. Maximisation of (1.35) results in the bank choosing the level of assets and liabilities on the efficient frontier, where additional return is only achievable at the expense of rising variance, such that the objective function's marginal rate of substitution between risk (variance of return) and return is equated with the market's opportunity set. As in Pyle (1971), the determinants of the

optimal solution are the characteristics of the probability distribution of return to each asset and liability, and the form of the bank's objective function.

Both Pyle (1971) and Parkin *et al* (1970) completely disregard liquidity and insolvency considerations, which arise from random cash requirements, default risks, etc., and real resource costs. By modifying the Tobin-Markowitz portfolio model Kane and Malkiel (1965) attempt to take account of the phenomenon of deposit variability through the incorporation into the risk and profit function of a shift parameter, which measures the quality of the relationship the bank is able to establish with its customer. Similar to the customer relationship arguments advanced by Hodgman (1963), Kane and Malkiel (1965) argue that "there exists a class of loan applications, L^* (L-star) loans, where the very failure to grant the loan itself increases aggregate risk. These L^* loan requests are distinguished primarily by the existence of a continuing (loan-deposit) relationship between the borrower and the bank" [p. 119]. An improved relationship will work towards reducing the variability of deposits while a worsening of the relationship will increase the variability of deposits. It is, therefore, natural for them to hypothesise that an improvement in the quality of the bank-depositor relationship both increases the bank's expected value and reduces the variance, of deposits, thus increases the bank's expected utility, and *vice versa.* While the consideration of deposit variability does add a new dimension to the portfolio selection it should be noted that random deposit flows can be better incorporated into the model via the liquidity deficiency cost function as in the precautionary models.

In an attempt to show that some basic results of the Tobin-Markowitz portfolio model also hold when applied to the banking firm Hart and Jaffee (1974) incorporate into their model some of the characteristics of the banking firm, such as no short sales and reserve requirements. Presumably, the most important results of their study are the existence of a separation theorem and the comparative static properties of the model similar to those obtained from the basic Tobin-Markowitz portfolio model [see Tobin (1958), Sharpe (1964), Lintner (1965) and Hicks (1967)]. However, their analysis

depends critically on the absence of a positive and fixed net worth component of liabilities and on the assumption of perfect competition. The importance of the separation theorem is that it indicates that the scale and structure of the bank's portfolio decisions can be separated. While the scale of the bank's portfolio will depend upon the utility function the relative proportion of assets and liabilities held in an efficient portfolio are uniquely determined and are independent of the preferences of the banking firm.

Klein (1971) is among the first to question the applicability of the portfolio-theoretical approach to the banking firm. Klein bases his criticism on the existence of imperfectly competitive markets and shows that some basic theorems of portfolio theory such as the separation theorem are not applicable under imperfect market structure. The exclusion of liquidity insolvency considerations and real resource costs from the portfolio-theoretical models are also questioned by, for example, Pesek (1970), since, as discussed in the previous sub-section, diversification of bank's assets and liabilities may result from cost differentials as well as from the basic portfolio considerations of risk aversion.

To remedy some of these shortcomings Sealey (1980) tries to "develop a model of intermediary behaviour that intergrates the risk consideration of the portfolio-theoretical approach with the market conditions, and deposit rate-setting behavioural mode of the firm-theoretical approach" [p. 1140]. This is achieved through the incorporation of deposit rate-setting behaviour, liquidity and production costs of the bank into the basic Tobin-Markowitz portfolio model. As the model takes into account a number of important aspects of the banking firm within a Tobin-Markowitz portfolio-theoretical framework we shall outline Sealey's model here.

Random deposit supply is one of the sources of uncertainty of banking and occupies a prominent place in the precautionary models discussed before. In Sealey's model random deposit supply is included via an implicit supply function of the form

$$\Phi(D^S, r_D, \psi) = 0 \qquad (1.36)$$

where D^S and r_D are the quantity supplied of and interest rate paid on a homogenous type of deposit, respectively, while ψ is the random element of deposits which has a known subjective or estimated probability distribution. For rate-setting behaviour the deposit supply function in (1.36) can be respecified as

$$D^S = D(r_D, \psi) \qquad \text{with} \quad \frac{\partial D^S}{\partial r_D} > 0 \quad \text{and} \quad \frac{\partial D^S}{\partial \psi} > 0. \qquad (1.37)$$

For simplicity, the bank's balance sheet is assumed to be of the following form

$$L = D^P + Z \qquad (1.38)$$

where L = loans, D^P = the quantity of deposits purchased, and Z = a composite liquid asset variable which measures the difference between money market borrowing and lending and hence can be either positive or negative or even zero. Since deposits are stochastic liquidity adjustment will affect the bank's expected profit.

Suppose the liquidation costs are as follows

$$\varphi(Z) = \begin{cases} p_1 Z & \text{if } Z < 0 \\ 0 & \text{if } Z = 0 \\ p_2 Z & \text{if } Z > 0 \end{cases} \qquad (1.39)$$

where p_1 and p_2 are certain lending and borrowing rates respectively. To avoid introducing discontinuity into the objective function it is assumed that $p_1 = p_2 = p$ although this assumption is relaxed in a later section of Sealey's paper. Uncertainty is also assumed to enter the model through stochastic loan rate, r_L, which has a known subjective probability distribution.

To introduce real resource costs Sealey (1980) specifies the following cost functions

of servicing deposits and loans: $C(D^P)$ and $C(L)$ with

$$C'(D^P) = \frac{\partial C(D^P)}{\partial D^P} > 0, \qquad C'(L) = \frac{\partial C(L)}{\partial L} > 0,$$

$$C''(D^P) = \frac{\partial^2 C(D^P)}{\partial D^{P2}} > 0, \quad \text{and} \quad C''(L) = \frac{\partial^2 C(L)}{\partial L^2} > 0.$$

The bank is assumed to maximise the expected utility of profit. The utility function is denoted by $U(\Pi)$ with

$$U'(\Pi) = \frac{\partial U(\Pi)}{\partial \Pi} > 0 \quad \text{and} \quad U''(\Pi) = \frac{\partial U(\Pi)}{\partial^2 \Pi} \begin{matrix} > \\ = \\ < \end{matrix} 0$$

depending on the bank's risk preferences. Using (1.37), (1.38), (1.39) and the additional assumption that $D^P = D^S$ under market clearing, the bank's profit function can be written as

$$\Pi = r_L L - r_D D(r_D, \psi) - p[L - D(r_D, \psi)] - C(D(r_D, \psi)) - C(L). \qquad (1.40)$$

Thus the bank's expected utility function can now be specified as

$$E[U(\Pi)] = E\{U[r_L L - r_D D(r_D, \psi) - p[L - D(r_D, \psi)] - C(D(r_D, \psi)) - C(L)]\}. \qquad (1.41)$$

The bank will choose L, the loan size, and r_D, the interest rate paid on deposits, such that (1.41) is maximised. The first-order conditions are

$$\frac{\partial E[U(\pi)]}{\partial L} = E\{U'(\pi)[r_L - p - C'(L)]\} = 0 \qquad (1.42)$$

$$\frac{\partial E[U(\pi)]}{\partial r_D} = E\{U'(\pi[p(\frac{\partial D}{\partial r_D}) - r_D(\frac{\partial D}{\partial r_D}) - D(r_D, \psi) - C'(D(r_D, \psi))\frac{\partial D}{\partial r_D}]\} = 0. \quad (1.43)$$

Sufficient conditions for an optimum are (a) that the utility function implies risk aversion and that profit is concave, or (b) that the bank is risk neutral and that profit is strictly concave. Solutions for L and r_D can be obtained by solving (1.42) and (1.43) and the optimal level of the liquid asset, Z, can then be determined via the balance sheet equality. However, as in any Tobin-Markowitz portfolio model the solutions will critically depend upon the specification of the utility function. In addition, the solutions will depend on the assumptions concerning the functional form through which uncertainty is introduced and the form of the joint distribution of loan rate, r_L, and the random element of deposits, ψ.

Sealey (1980) shows that many of Pyle's (1971) conclusions reached within the confines of a quantity-setting model cannot be generalised to the model with deposit rate-setting behaviour. Moreover, he shows that neither Hyman's (1972) conclusion that increases in risk aversion reduce the bank's scale of operation nor Pringle's (1973) result that optimal loans are lower under risk aversion is consistent with a deposit rate-setting model. In fact, in Sealey's model it is possible for the bank to reduce risk through greater scale of operation than would be consistent with expected profit maximisation, in which case, the bank should have lower profit and should employ more resources than expected profit maximisation would predict.

The strength of the portfolio-theoretical approach lies in the fact that it provides an explicit allowance for risk aversion. It is very often argued that banks are likey to be risk averse due to the uncertain nature of banking. Such risk aversion can arise either because a bank's objective function is convex in return or because influential depositors, whose deposits are the major source of the bank's funds, or banking authorities induce it to act as a risk averter. However, this approach suffers also from several important

deficiencies when applied to the banking firm.

First of all, the results are very sensitive to the specifications of the utility function and the distributions of the rates of return. The common practice in the literature is to assume a quadratic or exponential utility function whose only arguments are expected value and variance of return, or some indices of them. However, unless the utility functions are quadratic or profits normally distributed, as Feldstein (1969) points out, it is improper to ignore higher moments [see Chapter Two for more detailed discussion].

Secondly, the portfolio-theoretical approach implies that, given the portfolio size, the bank's optimal selection depends only upon relative expected rates of return on available assets and their associated risks, i.e., given the balance sheet constraint, the demand for a particular asset depends not only on its own expected rate of return but also on the expected rates of return on all other available assets. Banks, however, choose from a large set of assets with characteristics which cannot be uniquely mapped into the mean-variance space. For example, the terms on many type of bank loans, as Hester (1967) argues, are determined by negotiations between a bank and its customers in which the size of the loan, the maturity, type of repayment, and the size of compensating balance are considered. Another example of these characteristics is provided by Hodgman's (1963) customer relationship argument that the profitability of a loan depends also upon the value of a borrower to the bank as a future depositor and user of other bank services [see also Wood (1975)]. This kind of complexity, which is presumably typical in banking and, especially in market of commercial bank lending cannot be handled easily by the portfolio-theoretical approach, especially in models with large number of assets and liabilities.

Finally, stochastic cash requirements, and hence the problem of liquidity management, which are a salient feature of the banking firm, are also difficult to incorporate in the context of this approach. Indeed, the problem of liquidity management has seldom been treated by students of the portfolio-theoretical approach.

1.4.4 Bank Behaviour under Imperfect Market Structure

Most of the models discussed so far make the assumption that the rate of return on each asset is exogenous to the bank. This amounts to assuming that banks operate in perfectly competitive financial markets and act as price-takers. The implication of this assumption for a bank is that the demand functions for its deposits and loans, as well as the supply functions of bonds to it, all display infinite elasticity with respect to the own rate of return. Some authors argue, however, that such an assumption, a good first approximation to the bond market it may be, is inadequate as far as the loan and deposit markets are concerned [Monti (1971, 1972), Pringle (1974), Towey (1974), Sealey and Lindley (1977)]. There exists a group of models in which the presence of monopoly power play a prominent role in the determination of the banks' optimal decision. There are two basic assumptions common in this kind of models: (1) the bank faces a scale as well as a portfolio decision, where scale is identified with the volume of deposits, and (2) the bond market is perfectly competitive while the loan and deposit markets are characterised by imperfection.

Monti (1971, 1972) consider the behaviour of a bank that operates in such an imperfect market under alternative specifications of the bank's objective function. He assumes that demand functions for loans and deposits are linear functions of their own rates and of the exogenous bond rate, implying that imperfect portfolio substitution exists for both depositors and loan recipients. For a profit maximising bank the optimal asset selection rule is to equate the marginal revenue from loans and from bonds. One of the main results of Monti (1971, 1972) and some other authors [see for example, Klein (1971)] is that the optimal loan rate, and hence the optimal loan level, depends only upon the bond rate and the parameters of the loan demand function. Neither the deposit rate nor the parameters of the deposit demand function enter the loan rate function. This implies, of course, that the bank's optimal loan (asset) decision is totally independent of its deposit (liability) decision. The major weakness of Monti's models, however, lies in the assumption, as he realises himself, that both the volumes and prices

are perfectly certain, i.e., random changes in either bank deposits or the rate of return on bonds as well as default risks are completely disregarded. A consequence of this assumption is the absence of the liquidity problem which is such an essential feature of the banking firm.

Klein (1971) improves on this by incorporating uncertainty into the monopolistic model. As Klein's analysis is regarded as the most representative of the monopoly models of bank behaviour in the literature, it seems useful to restate his model.

The bank is assumed to have two primary sources of funds, namely, equity, W, and two type of deposits, D_1 (demand deposits) and D_2 (time deposits), and can choose among three assets: reserve assets, government bonds and loans. To introduce imperfections into the loan market, the bank is assumed to face a demand curve for its loans which is a function of the contract rate of interest and a vector of exogenous variables. Since the non-interest terms of loans are exogenous, the expected rate of return on loans, r_L, is simply a function of the loans extended by the bank, that is,

$$r_L = h(X_L) \quad \text{with} \quad h'(X_L) = \frac{\partial h(X_L)}{\partial X_L} < 0 \tag{1.44}$$

where X_L is the proportion of total funds allocated to loans. Similarly, imperfections are introduced into the deposit market through the following equations,

$$D_1 = D_1(r_1) \quad \text{with} \quad D_1'(r_1) = \frac{\partial D_1(r_1)}{\partial r_1} > 0 \tag{1.45}$$

$$D_2 = D_2(r_2) \quad \text{with} \quad D_2'(r_2) = \frac{\partial D_2(r_2)}{\partial r_2} > 0 \tag{1.46}$$

where r_1 and r_2 are the interest rates paid on demand and time deposits respectively. To incorporate liquidity consideration the net outflow of deposits as a fraction of total funds, Z, is assumed to be a random variable which has a density function f(Z). In

particular, f(Z) = 1/ (c - b) where c and b are the largest and lowest conceivable deposit loss, i.e., Z is uniformly distributed with c and b. The bank is presumed to incur a proportional penalty cost, n, if its cash holdings as a fraction of total funds, X , is insufficient to meet its net deposit outflow. Thus, the expected deficiency cost on cash management is

$$n \int_{X_r}^{c} (Z - X_r) f(Z) dZ = n \left[\frac{(X_r - c)^2}{2(c - b)}\right]. \tag{1.47}$$

The objective of the bank is to maximise the expected return on equity, r_W, which, using equations (1.44) — (1.47), can be written as

$$r_W = \left[1 + \frac{D_1(r_1) + D_2(r_2)}{W}\right]\left[X_L h(X_L) + X_g r_g - \frac{n(X_r - c)^2}{2(c - b)}\right]$$

$$- \frac{1}{W}\left[r_1 D_1(r_1) + r_2 D_2(r_2)\right] \tag{1.48}$$

subject to the balance sheet,

$$X_L + X_g + X_r = 1 \tag{1.49}$$

where X_g and r_g are the proportion of total funds allocated to bonds and the expected exogenous return on bond respectively.

Forming the Lagrangean and differentiating it with respect to the decision variables yield the following first-order conditions:

$$\frac{\partial L}{\partial r_1} = \frac{D_1'(r_1)}{W} E_a - \frac{1}{W}\left[r_1 D_1'(r_1) + D_1(r_1)\right] = 0 \tag{1.50}$$

$$\frac{\partial L}{\partial r_2} = \frac{D_2'(r_2)}{W} E_a - \frac{1}{W}\left[r_2 D_2'(r_2) + D_2(r_2)\right] = 0 \tag{1.51}$$

$$\frac{\partial L}{\partial X_L} = \left[1 + \frac{D_1(r_1) + D_2(r_2)}{W}\right]\left[X_L h'(X_L) + h(X_L)\right] - \lambda = 0 \qquad (1.52)$$

$$\frac{\partial L}{\partial X_g} = \left[1 + \frac{D_1(r_1) + D_2(r_2)}{W}\right] r_g - \lambda = 0 \qquad (1.53)$$

$$\frac{\partial L}{\partial X_r} = \left[1 + \frac{D_1(r_1) + D_2(r_2)}{W}\right]\left[-\frac{n(X_r - c)}{(c - b)}\right] - \lambda = 0 \qquad (1.54)$$

$$\frac{\partial L}{\partial \lambda} = X_L + X_g + X_r - 1 = 0 \qquad (1.55)$$

where

$$E_a = X_L h(X_L) + X_g r_g - \frac{n(X_r - c)^2}{2(c - b)} \qquad (1.56)$$

and λ is the Lagrangean multiplier.

From equations (1.52) — (1.54) we have

$$X_L h'(X_L) + h(X_L) = r_g = -\frac{n(X_r - c)^2}{2(c - b)} \qquad (1.57)$$

which determines the optimal composition of the portfolio. The expression on the left-hand side of (1.57) is simply the marginal rate of return on loans while the expression $n(X_r - c)^2/2(c - b)$ is the marginal reduction in the expected deficiency cost which can be viewed as the marginal rate of return on reserve assets. Optimality thus requires that the marginal rates of return from loans and reserve assets be equated with the exogenous expected rate of return on bonds. From (1.50) and (1.51) we obtain

$$r_1 = E_a - \frac{D_1(r_1)}{D_1'(r_1)} \qquad (1.58)$$

$$r_2 = E_a - \frac{D_2(r_2)}{D_2'(r_2)} \qquad (1.59)$$

or

$$E_a = r_1[1 + \frac{1}{\epsilon_1}] \qquad (1.58')$$

$$E_a = r_2[1 + \frac{1}{\epsilon_2}] \qquad (1.59')$$

where ϵ_1 and ϵ_2 are the deposit supply functions' elasticities. (1.58) and (1.59) determine the optimal structure of liabilities and the optimal size of the bank's portfolio, which require that the bank equate the marginal expected revenues from assets, E_a, with the marginal costs of obtaining demand and time deposit respectively.

From the optimal conditions for the bank's asset selection it is clear that, once again, neither the costs of deposits (r_1 and r_2) nor the parameters of the deposit supply functions affect the bank's optimal asset selection. On the other hand, however, the asset yield, E_a, which is dependent on the optimal asset portfolio does affect the bank's decision on the optimal size of the portfolio via the interest rates the bank is willing to pay for deposits. These results are in agreement with those derived by Monti (1971) under perfect certainty. Klein's finding that there is independence of loan policy and deposit rates, however, depends critically upon the special assumptions made in the model, and does not hold in general. Pringle (1973) shows, for example, that if there exists at least one controllable source of funds with rate of interest exogenously determined and hence constant rather than being a function of the quantity chosen and if the net constant cost of this source to the bank exceeds the bond rate the optimal loan decision will now depend on the source of funds with constant cost. The crucial and somewhat questionable assumption (as in most of the precautionary models) is that the deposit flow distribution f(Z) is fixed, independent of the composition of deposits and homogenous of degree one in total assets. If the parameters of this distribution are affected by the demand and time deposit mix, the independence conclusion, as Klein

(1971) is aware of himself, no longer holds. Based on Klein's model Miller (1975) considers the effect of reserve requirements for demand and time deposits on the independence result. He shows that if different reserve requirements are applied to demand and time deposits the asset decision is no longer independent of the deposit supply functions because the marginal expected rate of return on reserve assets — the marginal reduction in the expected deficiency cost, is now dependent on the deposit supply functions.

By making the level of liquidity deficiency dependent upon the volume of loan demand which is random, Prisman, Slovin and Sushka (1986) also show that the optimal deposit and loan rates are positive functions of the deficiency penalty costs and hence the liability and asset decisions of the bank are interdependent.

It should also be noted that Klein (1971) assumes that the expected loan rate depends upon the relative proportion of loans in the portfolio instead of the actual volume of loans. This assumption seems most strange since in Klein's model the total size of the portfolio is not fixed but endogenously determined. This assumption implies that the bank can sell loans at unchanged conditions if only its portfolio structure remains unchanged regardless of the absolute level of loans. Such behaviour obviously contradicts the assumption of market imperfection for loans.

A common feature of the imperfect market models is the absence of uncertainty and real resource costs. The cost of producing and maintaining deposits and loans is completely ignored. As mentioned earlier the presence of monopoly power in the deposit and loan markets plays a crucial role in this type of models and ensures a finite size of the banking firm. Although in the real world banks are seen to display a certain degree of monoply in some asset and liability markets, it should be stressed that models which rely solely on such an assumption are methodologically inadequate. Indeed, the monopoly models will completely break down when the bank is forced to operate in perfectly competitive markets. Moreover, the monopolistic models are difficult,

if not impossible, to be generalised into models with large number of assets and liabilities.

1.4.5 Summary

The traditional approach towards bank behaviour is descriptive rather than analytical, and most of its features have been incorporated into models of the precautionary approach. The imperfect market structure approach emphasises the importance of imperfections of the market structure in which a bank operates. But as we have indicated earlier, market imperfections can also be dealt with in both the precautionary and portfolio-theoretical frameworks. Although uncertainty plays a very important role in both the precautionary and portfolio-theoretical approaches, the emphasis of the two approaches differ: while the portfolio-theoretical approach emphasises the importance of uncertainty over future rates of return, uncertainty over future deposit withdrawals, hence uncertainty over future liquidation costs form the core of the precautionary models. Banks in the precautionary models are assumed to be risk neutral and expected profit maximising, caring about risk only to the extent that it feeds back into, and thus reflected in expected profit via the possibility of costly emergency adjustments. In the portfolio-theoretical models banks not only care about expected profit but also its variability, thus banks are assumed to be risk averse and they maximise expected utility.

A common weakness of all the approaches reviewed above is that each of them emphasises one particular aspect of the bank's operations and cannot be easily, if at all possible, extended to take into account of other aspects which are equally important to the bank. Even in models where such extentions are possible they are usually restricted to three or at most four assets and/or liabilities, and are extremely difficult to be generalised into n asset/liability models. However, as have seen in Section 1.3, commercial banks usually have a large number of assets and liabilities in their portfolios. Models which are incapable of dealing with large number of assets and liabilities are likely to be of little use at the empirically level.

1.5 SCOPE AND METHODOLOGY

Any model of the bank must, due to the complexity of the institution, be relatively abstract. The nature and degree of abstraction is determined by the author's perception of what is particularly important about the bank. Unfortunately, arguments over such matters often seem to approach the metaphysical with the result that little of value emerges. It is nevertheless important at the outset to set forth the general way in which a bank will be modelled in this thesis. As we have seen in the proceeding section, there exist in the theoretical literature a large number of models of bank portfolio behaviour, each of which emphasises a different aspect of the bank's operations. The scope of this study is to extend, develop and test a number of alternative short-run portfolio models of commercial banks, with special reference to the London clearing banks. Primarily, we are interested in determining, given the total funds available, which variables influence portfolio composition of a commercial bank and in analysing the manner in which it adjusts to changes in such variables. Our main purpose is not to derive one particular model which fits well, but to test the different models against the evidence and to see which model, if any, is consistent with the data. This is particularly important since, as we shall see in Chapter Two, the empirical literature of bank portfolio behaviour is dominated by one particular type of models — the portfolio-theoretical models and many important policy conclusions are often been drawn from these models. However, although these models usually provide good fits they have not been tested against other non-nested alternatives. It is possible and indeed it is the conclusion of this study that most of these models have to be rejected when tested against other non-nested alternative models.

It is assumed throughout this thesis that the portfolio behaviour of commercial banks is one of constrained optimisation. A specification of constrained optimising behaviour involves a description of the *actions* a unit can undertake, the *constraints* on those actions imposed by the environment (resources, technology, market conditions, law and regulations), and most importantly, the *objective function* that evaluates the actions.

When uncertainty is present, as it is often than not, the specification of optimising behaviour also involves a description of the *sources of uncertainty.* In this section we discuss these four aspects for the commercial banks.

1.5.1 The Bank's Objective Function

The classical theory of the firm rests upon the assumption of profit maximisation. Loosely interpreted, such an assumption if applied to a bank would suggest that the bank should be found holding a portfolio consisting of only a single asset which has the highest yield. Since in reality banks are found to hold diversified portfolios comprising of many assets, the usual presumption is that they do not maximise profits. As discussed in the last section, a number of different analytical approaches have been developed to explain the holding of diversified portfolios. The precautionary models assume that banks maximise expected profits whose arguments include expected rates of return and expected liquidation costs while the portfolio-theoretical models suppose that banks maximise expected utility which are functions of expected rates of return on individual assets and their risks (usually approximated by the variance of the return on the portfolio). As no *a priori* information is available regarding which of these two objective functions is more appropriate for describing the portfolio behaviour of commercial banks, both of these objective functions and some others will be considered in this study.

1.5.2 The Set of Bank Activities

Banks, like most commercial enterprises, make many decisions every day. The decisions involve personnel, salaries, lending terms, asset diversification, public relations, trust department policy, underwriting, and so on. Of these decisions, few are well documented or observable by outside investigators; published summary data about bank behaviour reflect a large number of individual decisions. Theories of a commercial bank's behaviour that are to be subjected to empirical verification must imply the

existence of relations among reported variables. It is mainly for this reason that the scope of the theory in this thesis will be confined to specifications that yield hypotheses about variables recorded by bank regulatory authorities and for which data are available for use by the general public.

Banks are principally engaged in providing the services of intermediation by directing flows of funds from surplus units to deficit units. It is therefore convenient to view all bank variables as flows and to assume that the stock of an asset is uniquely associated with a flow of services that the bank controls through investing and lending decisions. A deposit liability is associated with a flow of services and is basically controlled by the customers (depositors). The controls by banks and customers are not strict, however. For instance, commercial loans are sometimes unexpectedly renewed, and depositors sometimes temporarily relinquish control of their deposits when they commit their funds to a bank for a fixed-term. Nevertheless, as a first approximation, it seems useful to view banks as attempting to select assets in order to maximise their objective function subject to externally determined deposit and regulatory constraints.

It is recognised that this formulation is highly simplified and omitts a number of important aspects of bank behaviour. Thus, compensating balance practice, lines of credit and revolving credit arrangements do not naturally fit into this framework. Similarly, the very appealing notion of a long-term customer relationship [Hodgman (1963] and the intricate bilateral determination of lending terms [Hester (1962)] are not readily incorporated in this formulation. However, the simplifications are necessary if the analysis is to be tractable and the derived hypotheses are to be empirically verifiable.

While flow variables may be measured continuously or discretely, in practice, all financial flows are recorded over discrete time intervals, and often they are not measured directly so that their magnitude must be inferred from net changes in stock variables between two observation dates. Such measures are appropriate only if a theory suggests that inflows and outflows during the period have symmetric effects on and/or are

symmetrically affected by other variables in the system. In this study the decision period is defined as that span of time over which the bank sets and does not intend to alter its asset portfolio. Such a definition is obviously unrealistic for every bank is always planning and replanning its asset portfolio. However, it may be argued that portfolios are not planned with the intention of making frequent changes and it seems appropriate to assume that the basic portfolio decisions with respect to the fundamental components are made with reference to a period of time. The decision period is usually not known to the general public and it is likely that the length of the decision period differ from one bank to another. In order to facilitate estimation of the theoretical models it will be necessary to assume that the decision period is the same for all banks and it coincides with the observation period. As the models we test are short-run models we shall employ monthly data in our estimation. It is believed that "banks' decision period is considerably shorter than a quarter so that the specification error induced by using temporally aggregated data with dynamic models is reduced [Bewley (1981), p. 192]. The use of monthly data also ensures that we have adequate degrees of freedom in models with large number of parameters. The data period covered by this study is 1959:01 - 1983:08. This period is much longer than any of the previous studies and encompasses a number of different bank control regimes. This is also a period when consistent balance sheet data for the London clearing banks are publicly available.

1.5.3 Constraints

In choosing its asset portfolios a bank is subject to both institutional and technical constraints. Technological constraints are analogous to those encountered in the theory of the production function. Like any other firms, a commercial bank's capacity to operate will be determined to a great extent by the available technology. However, since our emphasis is on the short-run portfolio behaviour of commercial banks it is assumed that the technological constraints will not affect a bank's portfolio choice and technological constraints will not be taken into account in our study. Institutional restrictions include

accounting, legal and regulatory constraints. Accounting constraints refer to balance sheet identities such as the condition that the sum of a bank's assets must equal the sum of its liabilities. Such identities will be explicitly imposed in this thesis. Other institutional constraints, i.e. legal and regulatory constraints, may differ from one group of banks to another, and may also change from one time period to another. In the following paragraphs only those legal constraints imposed upon the London clearing banks during the relevant period will be discussed.

The prime institutional restriction on bank portfolio behaviour is that a bank be prepared to honour requests by demand depositors for cash and cheque withdrawals up to the amount of their total deposits without notice and by time depositors at very short notice. Regulatory constraints related to this institutional restriction are the cash and liquid asset ratios and later the reserve asset ratio imposed by the Bank of England upon the London clearing banks (and others). During the period covered by this study the London clearing banks have also been subjected to credit ceilings, Special Deposit Scheme, and Special and Supplementary Deposit Scheme. Some of these constraints are further discussed in Chapters Two and Three and we shall therefore discuss here only briefly the relevance of these constrains to our modelling of bank portfolio behaviour [for general discussions on banking regulation and supervision during the period see, for example, Bank of England (1969, 1975b, 1982, 1983), Johnson (1972), Barge and Wise (1977), Artis (1978)].

Since 1951 and prior to the introduction of Competition and Credit Control the London clearing banks were required to hold a minimum of 30% of gross deposits in liquid assets (reduced to 28% in 1963), at least 8% of which must be held in cash. The liquid assets of the banks for this purpose were defined as comprising, in addition to cash, money at call and short notice, Treasury bills and commercial bills. These ratios were replaced in September 1971 by the minimum reserve asset ratio. The reserve asset ratio required that the banks hold at least 12.5% of their eligible liabilities in reserve assets [for details see Bank of England (1971a, 1971b)]. An *a priori* decision

as to whether these constraints were binding or not on the bank's portfolio choice will of course depend upon the knowledge of the parameters of the bank's objective function. Unfortunately, such a knowledge is usually not available and this creates a problem for model building. In the absence of any knowledge regarding the parameters of the bank's objective function, our guiding principle in treating these constraints will have to be a practical one.

A cursory inspection of the balance sheets of the London clearing banks for the relevant period in question reveals that the banks had always held 8% of their total deposits in the form of cash while they had always maintained their holdings of liquid assets above the required minimum. It has been argued that, since the clearing banks had always maintained their holdings of liquid assets above the legal minimum, the incorporation of the observed liquid asset ratio will not contain any useful and extraneous information in the estimation of the analytic models. The liquid asset ratio will affect the bank's portfolio behaviour only if it is binding and effective. Since it has not been observed to be effective the exclusion of such a constraint from the formal analysis is at least empirically justified [Parkin, Gray and Barrett (1970), Courakis (1974, 1975, 1980)]. However, others argue that the fact that the observed holdings of liquid assets above the legal minimum alone does not entail one to discard the liquid asset ratio constraint from the formal analysis of commercial bank portfolio behaviour. Indeed, it has been suggested that the London clearing banks were influenced by the legal liquid asset ratio in that, when the legal minimum was reduced by two percentage points in October 1963, average holdings of liquid assets in the subsequent period by the London clearing banks went down *pari passu* [Ford (1983), see also Bewley (1981)]. In order to test whether the liquid asset ratios have affected the short-run portfolio behaviour of the London clearing banks we shall develop and estimate both models with and without the liquid asset ratio constraints. However, partly because data on some assets included in the reserve asset set are not available and partly because the definitions of reserve assets and eligible liabilities changed over time [see Bank of England (1972, 1975a, 1985)], it will not be possible to take into account of the minimum reserve asset ratio in our models



for the post-1971 period.

During the period prior to 1971 the London clearing banks were from time to time subject to direct guidance and restrictions on advances imposed by the Bank of England. These guidance and restraints varied from mild requests to banks to restrain their lending to strict quantitative ceilings on growth of bank lending. The treatment of these restrictions has always been a controversial and difficult issue in the empirical studies of UK commercial bank portfolio behaviour and we shall defer our discussion on this issue until Chapter Two.

Apart from the various ratios and credit ceilings the London clearing banks had been further subjected to a Special Deposit scheme before December 1973 and a Supplementary Special Deposit scheme thereafter. The Special Deposit scheme, operated between July 1958 and December 1973, was intended to provide a mechanism for reducing banks' lending by means of varying the liquidity ratio of the banks. The clearing banks were instructed from time to time to increase their gross deposits at the Bank of England by a certain percentage. These special deposits, bearing interest at approximately the Treasury bill rate, did not count as liquid assets. As the percentage of deposits to be called for were determined by the Bank of England, the clearing banks had no control over the amount of these deposits and had always complied with the Bank's instructions. It is important to note, however, that the calls for special deposits were often announced beforehand so that the clearing banks always knew the percentages of deposits to be increased in the next period. The Supplementary Special Deposits (SSD) scheme, known as the "corset", was a system of direct controls on the sterling operations of the commercial banks. The scheme was introduced in response to the rapid growth of "liability management", which made it difficult for the monetary authorities to restrict the growth of broad monetary aggregates in the face of strong demand for bank credit. It imposed penalties on individual institutions whose interest-bearing eligible liabilities (IBELs), essentially their interest-bearing sterling deposits, grew faster than a prescribed rate. The greater the excess over the prescribed

growth the more severe the penalties became [for details of the scheme see Bank of England (1982)]. We shall test the effects on bank's portfolio behaviour of the Special Deposit and Supplementary Special Deposit schemes by introducing dummy variables.

1.5.4 Risk and Uncertainty

The environment in which a bank operates is one characterised by risk and uncertainty. Any positive or normative theory of bank behaviour must take into account the presence of risk and uncertainty. There are three major areas of risk and uncertainty facing a commercial bank. First, there is the investment or default risk connected with the assets a bank holds: as a creditor the bank faces the risk that its debtors are unable or unwilling to meet their obligations at the agreed time. This type of risk is particularly important in the case of commercial loans. Second, there is the liquidity or solvency risk in connection with the bank's liabilities: as a debtor the bank faces the possibility that its depositors may withdraw their deposits at an unspecified time or that they are unwilling to extend or renew their credit to the bank. This type of risk is most obvious and significant in the case of demand deposits, where the depositor can withdraw all of his funds at any time and without advance notice. These two aspects of risk and uncertainty are clearly not unrelated. If the bank's assets were always marketable at par and loans callable on demand and without possibility of default, the bank could never become illiquid or insolvent no matter how unpredictable the behaviour of its depositors is. Last, the bank faces uncertainty over the rates of return on its assets. Rates of return on assets are usually determined exogenously and are therefore subject to fluctuation. For example, the bank cannot know exactly the market value of the non-matured securities in its portfolio at any moment of the future. Even when the interest rate on an asset is determined by the bank, as in the case of some commercial loans, the future rate of return will differ because of loan defaults or later repayments.

There are those who postulate a major operational distinction between risk and uncertainty, particularly with respect to investment decisions [see, for example, Knight

(1921) and Shackle (1952, 1955, 1961)]. The standard definition is that risky situations are those for which an objective probability function is available, while uncertainty is applied to unique situations where the frequency distribution cannot be objectively specified. In general, repetitive decision problems involving risk are theoretically easier to handle than those involving uncertainty. They lend themselves to reformulation as deterministic problems with the expected value of the variable under consideration being substituted for the certain value of the certainty model. This is most useful in cases such as those involving insurable occurances where a large number of independent events are insured simultaneously.

The case for employing expected values in uncertain situations is more controversial. Here, there are no series of observations to provide a statistical basis for choosing parameter values. The situation at hand is likely to be unique. However, uncertainty does not mean complete ignorance. Even though there may not be a well-defined, objectively determined probability distribution available, a decision-maker is likely to have some knowledge about the problem he is concerned with. He is likely to have some kind of expectations and these expectations may themselves be translatable into the kind of subjective probability defined by Savage (1950, 1951, 1954). The major difference between objective frequency distribution and subjective one is that the former can be determined by methods external to the thought processes of the individual decision-maker involved while the latter cannot.

This poses certain problems in constructing a model of bank behaviour: many of the uncertainty problems that a bank faces lend themselves to statistical evaluation but the analysis of the others may depend upon subjective probability or degree of belief. Yet, it would seem that the distinction, if one is to be made, between objective and subjective probability, or between risk and uncertainty, is not clear-cut. It can be argued that even under uncertainty, the decision-maker may well have information upon which to base a subjective probability distribution. Assuming that a decision must be made, there is, *a priori*, no reason why a subjective probability distribution should be utilised

differently from an objectively determined distribution. Thus, there seems to be no strong argument for making an operational distinction between objective and subjective probability. The question is rather whether probability, be it objective or subjective, should be applied to decision making under uncertainty at all.

Professor Shackle, perhaps one of the staunchest and most vehement critics on the use of probability as a uncertainty variable, dismisses probability (be it objective or subjective) as a measure of uncertainty or doubt about any outcome and the relevance of an expectation principle. His opposition to the use of probability as a measure of uncertainty rests on two major arguments. The first is that decisions are usually non-repeatable, or in his own words, non-seriable, and indeed they are unique:

> The theory of probability, in the form which has been given to it by mathematicians and actuaries, is adapted to discovering the tendencies of a *given* system under *indefinitely repeated* trials or experiments. In any set of such trial, each trial is, for the purpose of discovering such a tendency, given equal weight with all the others. No individual trial is considered to have any importance in itself for its own sake, and any tendency which may be inductively discovered, or predicted *a priori*, for the system, tells us nothing about any *single* individual trial which we may propose to make in future. It follows that in forming expectations, actuarial general principles and particular facts will only help us when the following conditions are satisfied:
> 1. We are sure that the system, whose future behaviour we wish to know, will remain a *given* system and not undergo changes during the interval of future time in question.
> 2. We are interested only in the total result of a 'large number' of trials, all of which count equally or virtually so in building up this total.
> 3. *We feel sure that we shall, in fact, have the opportunity* to carry out a sufficient number of trials, and not, for example, find that our plans depended, in fact, on the successful issue of a few early trials whose failure has deprived us of continuing [Shackle (1952), pp. 5-6].

Shackle's second argument against the application of probability calculus to decisions under uncertainty is that probability, defined as relative frequency, is itself a distributional variable, which implies that all the possible outcomes that characterise a prospect must be mutually exclusive and that the probabilities sum to unity. That the sum of unity is distributed across the competing outcomes has further implications: if the individual decision-maker believes, or is made to believe, that another, previously unthought of, outcome is possible, at least one of the probabilities of the original outcomes in the prospect must be adjusted, indeed, reduced. Without such a realignment the newly

conceived outcome could not be alloted any chance of occurence. However, the fact that an additional outcome is perceived to be possible might in no way affect the degree of belief that the individual decision-maker attaches to the occurence of the remaining outcomes [Ford (1987)], so that subjective probability, interpreted as degree of belief, may still be applicable to decision making under uncertainty. For these reasons we shall not make any distinctions in the rest of this thesis between the risk and uncertainty aspects of the commercial bank's operations.

CHAPTER TWO

BANK PORTFOLIO BEHAVIOUR UNDER RISK AVERSION

2.1 INTRODUCTION

It is often argued that banks are risk averse. The portfolio-theoretical approach, which is commonly reduced to the mean-variance approach, to the theory of bank portfolio behaviour under uncertainty, which stems from Tobin's (1958) paper on liquidity preference and Markowitz's (1952, 1959) pioneering study of efficient portfolio selection, makes explicit allowance for risk aversion [see Markowitz (1987) for a comprehensive account of the general mean-variance portfolio analysis]. Underlying this approach is the idea that the determinants of alternative portfolios can be reduced to measures of expected return and risk of the portfolio, where the former is the mean of the probability distribution of return and the latter is usually approximated by the variance of that distribution. In fact, most of the existing empirical work on commercial bank portfolio behaviour employ such an approach [see for example, Parkin (1970), Parkin, Gray and Barrett (1970), Courakis (1974, 1975, 1980, 1988), Sharpe (1974), White (1975) and Bewley (1981)]. In this chapter we attempt to provide an integrated discussion of the portfolio-theoretical approach as applied to bank portfolio behaviour with special reference to the UK commercial banks.

The rest of this chapter is organised as follows. Section 2.2 discusses briefly the definition of risk aversion. The traditional mean-variance analysis, a special case of the portfolio-theoretical approach, to portfolio selection under uncertainty is outlined in Section 2.3. The Parkin-Gray-Barrett model, an important attempt to apply the expected utility approach to model bank portfolio behaviour, is described in Section 2.4, followed by a critique of the model in Section 2.5. Extentions of the Parkin-Gray–Barrett model are discussed in Section 2.6. Section 2.7 concludes.

2.2 RISK AVERSION

Before we proceed to discuss bank portfolio behaviour under risk aversion it seems useful to note briefly what is meant by risk aversion. A person is risk averse relative to a lottery (p, W_1, W_2) if the utility of its expected value is greater than the expected value of its utility, i.e., if

$$U\,[p\,W_1 + (1 - p)\,W_2] > p\,U(W_1) + (1 - p)\,U(W_2) \qquad (2.1)$$

where W_1 and W_2 are different wealth levels, and p and $1 - p$ are their corresponding probabilities of occurrence. A risk averter prefers a *certain* outcome to an *uncertain* one with the same expected value. If (2.1) holds for all $0< p <1$ and all W_1 and W_2 within the domain of the utility function, the utility function is strictly concave over its domain(1). Thus, if $U''(W) = d^2U/dW^2 < 0$, the person is a risk averter. The sign of the second derivative of the utility function provides an indication of the individual's attitude towards risk, but it cannot be employed to indicate the level of risk aversion since its magnitude is not invariant under a linear transformation. We thus seek a measure which is based on $U''(W)$ but constructed in a way so as to remain invariant under positive linear transformation of the utility function. Pratt (1964) and Arrow (1971) define such measures, known as absolute risk aversion and relative risk aversion, in terms of the ratio of the second and first derivatives of the utility function.

A measure of absolute risk aversion (ARA) is provided by the ratio of the second and first derivatives of the utility function,

$$ARA = -\frac{U''(W)}{U'(W)} = -\frac{d\ln U'(W)}{dW}. \qquad (2.2)$$

This measure is positive, negative, or zero as the person averts, prefers, or is neutral towards risk. Also, it has the desirable property of being invariant under a linear transformation. Let $T = \alpha + \beta U$ with $\beta > 0$, then

$$ARA = - \frac{T''(W)}{T'(W)} = - \frac{\beta U''(W)}{\beta U'(W)} = - \frac{U''(W)}{U'(W)}.$$

Relative risk aversion (RRA) is defined as

$$RRA = - \frac{WU''(W)}{U'(W)} \qquad (2.3)$$

which is also invariant to a linear transformation of the utility function.

Two classes of utility functions are often quoted to describe risk aversion: the quadratic and the negative exponential utility functions. The quadratic utility function is of the form,

$$U = W - \alpha W^2 \qquad (2.4)$$

with $\alpha > 0$ and $0 < W < \alpha/2$. Since $U' = 1 - 2\alpha W > 0$ and $U'' = - 2\alpha < 0$, it describes risk-aversion behaviour. The negative exponential function,

$$U = - e^{-\beta W} \qquad \text{with } \beta > 0 \qquad (2.5)$$

also describes risk aversion, since

$$ARA = - \frac{U''(W)}{U'(W)} = - \frac{-\beta^2 e^{-\beta W}}{\beta e^{-\beta W}} = \beta > 0. \qquad (2.6)$$

In fact, the negative exponential utility function exhibits constant absolute risk aversion as (2.6) is independent of W. This differs from the quadratic utility function which displays increasing absolute risk aversion with wealth. From (2.4) we have absolute risk aversion for the quadratic utility function as

$$\gamma = \frac{2\alpha}{1 - 2\alpha W} \tag{2.7}$$

so

$$\frac{\partial \gamma}{\partial W} = \frac{4\alpha^2}{(1 - 2\alpha W)} > 0 \tag{2.8}$$

i.e., absolute risk aversion increases with wealth. Relative risk aversion for the quadratic utility function is obtained by multiplying (2.7) by W, and we see immediately that relative risk aversion also increases with wealth. This is a counterintuitive result which limits the use of the quadratic utility function.

2.3 THE MEAN-VARIANCE APPROACH TO PORTFOLIO SELECTION

Suppose there are n types of assets available to an investor. The ith asset has a rate of return r_i, which is a stochastic variable. We are considering an investor who has a given amount of wealth, W, deciding how much of each of the n assets to hold in his portfolio *for a definite period of time.* The orthodox approach to this problem is the expected utility approach based on the application of probability theory, which assumes, similar to the theory of consumer behaviour under uncertainty, that the individual investor has an actual utility function U which depends exclusively upon the actual, realised end-of-period return, $\prod$, on the chosen portfolio, and that he chooses *at the beginning of the decision period* that set of placings of his initial wealth across the n assets that produces the probability distribution of return which maximises the expected value of his utility function.

Provided that (a) the investor's utility function is quadratic, or (b) the actual net return of the portfolio, $\prod$, is normally distributed, the portfolio choices of an expected utility maximising investor can be analysed in terms of the two parameters, mean, μ_π and variance, σ^2, of his subjective probability distributions of the returns from alternative possible portfolios[2][see Borch (1969), Feldstein (1969) and Tobin (1969)]. That is, we can assume that his expected utility is dependent upon only the first two moments of the probability distribution of the returns,

$$E(U) = U(\mu_\pi, \sigma^2_\pi). \qquad (2.9)$$

As we have seen in the previous section both the quadratic and negative exponential utility functions describe risk aversion behaviour, but their risk aversion properties differ. The quadratic utility function exhibits increasing absolute and relative risk aversion as wealth increases. This property of the quadratic utility function has been widely attacked in the literature [see, for example, Feldstein (1969)]. In addition, as we can appreciate from (2.4), it imposes an upper limit on wealth as marginal utility satiates at $W = \alpha/2$.

However, the alternative assumption that the real return from the portfolio is normally distributed is not only unrealistic since it rules out all asymmetry and skewness in the probability distribution of returns but, as we shall see later, imposes severe restrictions upon the investor's asset choice. The normality assumption of returns has also been criticised by Fama (1965) on the empirical ground that sample variances of returns on securities commonly imply infinite population variances.

Actual return from a given portfolio *at the end of the decision period*, $\prod$, can be written as

$$\prod = a' \; r \tag{2.10}$$

and assuming that the investor operates in a perfectly competitive market and thus can hold whatever level of each asset he chooses we can define μ_{π} and σ^2_{π} as

$$\mu_{\pi} = a' \; e \tag{2.11}$$

$$\sigma^2_{\pi} = a' \; \Omega \; a \tag{2.12}$$

where

$a = (a_1 \; a_2 \; a_3 \; \ldots \; a_n)'$ is an n × 1 vector of asset holdings,

$r = (r_1 \; r_2 \; r_3 \; \ldots \; r_n)'$ is an n × 1 vector of actual rates of return,

$e = (e_1 \; e_2 \; e_3 \; \ldots \; e_n)'$ is an n × 1 vector of expected rates of return

and

$$\Omega = \begin{bmatrix} \sigma_{11}^2 & \sigma_{12} & \sigma_{13} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22}^2 & \sigma_{13} & \cdots & \sigma_{2n} \\ \sigma_{31} & \sigma_{32} & \sigma_{33}^2 & \cdots & \sigma_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \sigma_{n3} & \cdots & \sigma_{nn}^2 \end{bmatrix}$$

is the variance-covariance matrix of the n asset returns.

$\prod$ in (2.10) is seen to be a sum of random variables. A sufficient condition for $\prod$ to be normally distributed is that all r's are normally distributed since any linear functions of normally distributed variables are themselves normally distributed. If, however, the r's are not all normally distributed, then each time the composition of asset holdings changes, the distribution of $\prod$ changes as well. In general, when not all r's are normally distributed, only a limited set of asset combinations will enable $\prod$ to be normally distributed. Therefore, the normality assumption places severe restrictions on the investor's asset choice.

Whichever assumption we make the investor's decision is now one of constrained maximisation, i.e., he chooses the a vector to maximise his expected utility in (2.9) subject to the wealth constraint,

$$W = i'a \qquad (2.13)$$

where i is an n × 1 summation vector. Let λ be the Lagrangean multiplier, then the constrained maximisation process yields the following first-order conditions[3]:

$$\frac{\partial U}{\partial \mu_\pi^2} e + 2 \frac{\partial U}{\partial \sigma_\pi^2} \Omega a - \lambda i = 0$$

$$W - i'a = 0 \qquad (2.14)$$

which in matrix notation is

$$\begin{bmatrix} 2\frac{\partial U}{\partial \sigma_\pi^2}\Omega & -i \\ -i' & 0 \end{bmatrix} \begin{bmatrix} a \\ \lambda \end{bmatrix} = \begin{bmatrix} -\frac{\partial U}{\partial \mu_\pi} e \\ -W \end{bmatrix}. \tag{2.15}$$

Let H denote the coefficient matrix on the left-hand side of (2.14), then there exists a unique, non-trivial solution for this system of equations provided that H has an inverse. H will have an inverse if (1) the submatrix in the upper left-hand corner of H, $2(\partial U/\partial \sigma_\pi^2)\Omega$, is non-singular, and (2) $2i'(\partial U/\partial \sigma_\pi^2)\Omega i$ is non-zero [see, for example, Goldberger (1964), p. 27]. The submatrix consists of two parts, $\partial U/\partial \sigma_\pi^2$, the marginal utility of the variance of return, and Ω, the variance-covariance matrix of the rates of return of individual assets. Risk aversion implies that the marginal utility of the variance of return is negative definite, the two conditions will then be satisfied if the variance-covariance matrix, Ω, is non-singular, since the second condition is simply a quadratic form in Ω. *The necessary condition for Ω to be non-singular is that all r_i's, the individual rates of returns, are not linearly dependent in the mathematical sense.*

Granted that all r_i's are not linearly dependent Ω is then symmetric and positive definite. Denoting the inverse of H by G and partitioning G conformably, we have the solution system of demand equations for the n assets,

$$\begin{bmatrix} a \\ \lambda \end{bmatrix} = \begin{bmatrix} G_1 & G_2 \\ G_3 & G_4 \end{bmatrix} \begin{bmatrix} -\frac{\partial U}{\partial \mu_\pi} e \\ -W \end{bmatrix}. \tag{2.16}$$

Thus

$$a = G_1 \left(-\frac{\partial U}{\partial \mu_\pi}\right) e - G_2 W. \tag{2.17}$$

Therefore, the investor's demand for assets will depend upon the expected rates of return as well as his initial wealth. It should be noted that, when the normality of $\prod$ is used as the maintained assumption and all r's are not normally distributed, there is no guarantee that the investor's optimal asset holdings in (2.17) will be such that $\prod$ is

normally distributed.

From the definition of an inverse matrix we have

$$\begin{bmatrix} \frac{\partial U}{\partial \sigma_{\pi}^2}\Omega & i \\ i' & 0 \end{bmatrix} \begin{bmatrix} G_1 & G_2 \\ G_3 & G_4 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0' & 1 \end{bmatrix} \tag{2.18}$$

where I is an identity matrix of order n × n, 0 is an n × 1 null vector and 1 is a unit scalar. Expansion of (2.18) gives

$$i'G_1 = 0' \tag{2.19}$$

$$i'G_2 = 1. \tag{2.20}$$

Thus, given that G_1 is the coefficient matrix of the expected rates of return and G_2 the coefficient vector of wealth in the solution system of equations (2.17), (2.19) and (2.20) are, as in consumer demand theory, the Cournot aggregation and the Engel aggregation conditions. The Cournot condition follows trivially from the existence of the wealth constraint while the Engel condition emerges from the fact that if only the investor's level of wealth changes, the total change in that wealth has to be apportioned across all the n assets. In fact, as will be seen below, the matrix G_1 is symmetric, so that *ceteris paribus* changes in the expected asset yields will have symmetric effects across the asset holdings.

Applying the standard formula for block inversion to (2.14) we have [see, for example, Goldberger (1964)]

$$G_1 = \left(\frac{\partial U}{\partial \sigma_{\pi}^2}\Omega\right)^{-1} \{I - i[i'\left(\frac{\partial U}{\partial \sigma_{\pi}^2}\Omega\right)^{-1}i]i'\left(\frac{\partial U}{\partial \sigma_{\pi}^2}\Omega\right)^{-1}\} = G_1' \tag{2.21}$$

since Ω^{-1}, being the inverse of a symmetric non-singular variance-covariance matrix, is also symmetric [see Goldberger (1964), p. 25]; therefore G_1 is indeed symmetric. Thus

$$G_1 \; i = 0. \qquad (2.22)$$

Given risk aversion, the investor's demand equations for assets in (2.17) will allow for substitution as well as complementarity, depending on the signs of the covariance between rates of returns [for a mathematical derivation of this result see Royama and Hamada (1967)]. It should be stressed that in the above model of expected utility maximisation, where expected utility depends upon risk and return alone, the investor can only distinguish between assets in terms of their risk and return characteristics. Two assets with the same risk and return characteristics are, from the point of view of the expected utility maximising investor, the same asset. If there do exist two assets which have the same return and risk characteristics in the choice set, two rows of Ω, the variance-covariance matrix, are then identical. And this implies that a unique solution can not be obtained for the system of equations in (2.15).

2.4 THE PARKIN-GRAY-BARRETT MODEL OF UK COMMERCIAL BANK PORTFOLIO SELECTION

The Parkin-Gray-Barrett model (1970) of commercial bank portfolio behaviour under risk aversion is based on the expected utility maximisation framework discussed in the last section(4). As this model tends to be referred to extensively in the literature, presumably for its generalisation and its empirical application, the model and its empirical results are restated in this section with some notational alterations.

Each bank is assumed to possess a Freund-type utility function of the form [Freund (1956)]

$$U = \alpha - \gamma e^{-\beta\pi} \qquad (2.23)$$

where

U = utility per decision period

$\prod$ = real profit per decision period

α, β and γ are parameters (β, $\gamma > 0$).

As we have noted in Section 2.2 the marginal utility of return of this utility function is always positive, i.e., $U' = dU/d\prod > 0$, but decreasing as return increases, i.e., $U'' = d^2U/d\prod^2 < 0$, while absolute risk aversion is constant. It is assumed that real profit $\prod$ is normally distributed with mean μ_π and constant variance σ_π^2, i.e.,

$$\prod \sim N(\mu_\pi, \sigma_\pi^2). \qquad (2.24)$$

The objective of the bank is presumed to be , subject to the balance sheet constraint, the maximisation of the expected value of the utility function in (2.23), which

is

$$E(U) = \alpha - \gamma E(e^{-\beta\pi}). \tag{2.25}$$

We can rewrite (2.25) by noting that $E(e^{-\beta\pi})$ is the moment generating function (MGF) of the random variable $\prod$, $M_{\pi}(-\beta)$, and that given the normality assumption (2.24),

$$M_{\pi}(-\beta) = E(e^{-\beta\pi}) = e^{-\beta[\mu_{\pi} - (1/2)\beta\sigma_{\pi}^2]}. \tag{2.26}$$

This follows from the fact that the MGF for any random variable x is written as $M_x(t) = E(e^{tx})$ where t is an undefined constant [see, for example, Searle (1971), pp. 41-43]. It is immediately obvious that the maximisation of (2.25) is therefore equivalent to maximising

$$Q = \mu_{\pi} - \frac{\beta}{2}\sigma_{\pi}^2. \tag{2.27}$$

To complete the definition, actual profit is specified as

$$\prod = r' A \tag{2.28}$$

which is decomposed as

$$\prod = (e + u)'(\hat{A} + v)$$

$$= e'\hat{A} + e'v + u'\hat{A} + u'v \tag{2.29}$$

where r and e are now n × 1 vectors of actual and expected yields and borrowing rates, A and $\hat{A}$ are n × 1 vectors of actual and expected values of assets and liabilities (with labilities entered negatively) respectively; and u and v are n × 1 vectors of random

errors in r and A accordingly. By assuming that u and v are independently distributed with mean vector equal to zero and constant covariance matrices, Parkin, Gray and Barrett (1970) are able to obtain

$$\mu_{\pi} = E(\prod) = e' \hat{A} \tag{2.30}$$

and

$$\sigma_{\pi}^2 = e' V e + \hat{A}' \Omega \hat{A} + \Psi \tag{2.31}$$

where

$$V = E(v v') \tag{2.32}$$

$$\Omega = E(u u') \tag{2.33}$$

$$\Psi = E(u' v v' u). \tag{2.34}$$

Note that this definition of σ_{π}^2 differs from that in (2.12) because the actual levels of assets and liabilities at the end of the decision period are also assumed to be unknown at the moment of decision.

For UK commercial banks the elements of A are specified as

$$A' = (CM \quad TB \quad GB \quad CB \quad AD \quad SD \quad CA \quad BI \quad -DD \quad -TS) \tag{2.35}$$

where

CM = Money at Call and Short Notice (Call Money),

TB = Treasury Bills,

CB = Commercial Bills,

GB = Government Bonds,

AD = Advances,

SD = Special Deposits with the Bank of England,

CA = Notes, Coins and Balances with the Bank of England (Cash),

BI = Balancing Items,

DD = Demand Deposits,

TD = Time Deposits.

As the bank is not free to determine the volume of each and every asset/liability in its portfolio in every decision period, it is necessary to partition A into choice items, A_1, and non-choice items, A_2, and accordingly, r into r_1 and r_2. Parkin *et al* assume that CM (call money), TB (Treasury bills), CM (commercial bills) and GB (Government bonds) are the only choice variables which the bank can choose, and others are non-choice variables. The detailed definitions of A and r are given as

$$
\begin{aligned}
\hat{A}' &= (\mathrm{CM}\ \ \mathrm{TB}\ \ \mathrm{GB}\ \ \mathrm{CB}\ \ \mathrm{AD}\ \ \mathrm{SD}\ \ \mathrm{CA}\ \ \mathrm{BI}\ \ -\mathrm{DD}\ \ -\mathrm{TD}) = (\hat{A}_1' : \hat{A}_2') \\
v' &= (v_1\ \ 0\ \ 0\ \ 0\ :\ v_5\ \ v_6\ \ v_7\ \ v_8\ \ v_9\ \ v_{10}) = (v_1' : v_2') \\
e' &= (e_1\ \ e_2\ \ e_3\ \ e_4\ :\ e_5\ \ e_6\ \ 0\ \ 0\ \ 0\ \ e_{10}) = (e_1' : e_2') \\
u' &= (u_1\ \ u_2\ \ u_3\ \ u_4\ :\ u_5\ \ u_6\ \ 0\ \ 0\ \ 0\ \ u_{10}) = (u_1' : u_{2'}) \qquad (2.36)
\end{aligned}
$$

During the period covered by the study of Parkin *et al*, the London clearing banks were required to hold a minimum of 30% (reduced to 28% on 16 October 1963) of their total deposits in liquid assets, at least 8% of which must be held in cash, where liquid assets comprised, in addition to cash, money at call and short notice, Treasury bills, and commercial bills. Also, these banks, together with the Scottish banks, were requested from time to time to increase Special Deposits with the Bank of England. Although the call for Special Deposits was expressed as a percentage of a bank's total deposits the percentage varied from one time to another. In algebraical terms, these

requirements impose the following restrictions on the bank's portfolio choice:

$$CA \geqslant 0.08\ (DD + TD) \tag{2.37}$$

$$CM + TB + CB + CA \geqslant 0.3\ (DD + TD) \tag{2.38}$$

$$SD = \delta\ (DD + TD)\,. \tag{2.39}$$

Parkin *et al* argue that the liquid assets ratio had never been binding so that the liquid constraint, (2.38), can be dropped. Since the percentage and timing of call for Special Deposits were determined by the authorities Special Deposits were treated as given. Parkin *et al* further argued that the cash ratio was always binding and effective and (2.35) was to be treated as an identity. They added that since cash was uniquely linked to the total deposits, which were assumed to be exogenously given, so was cash.

The problem now is a standard constrained maximisation, i.e., given $\hat{A}_2$, e, V and Ω, choose $\hat{A}_1$ to maximise

$$Q = \mu_\pi - (\beta/2)\sigma_\pi^2$$

subject to

$$i_1' \hat{A}_1 + i_2' \hat{A}_2 = 0 \tag{2.40}$$

where i_1 and i_2 are appropriate summation vectors. (2.40) is thus the balance sheet constraint, namely, the sum of (expected) assets must equal the sum of (expected) liabilities at any time. Denoting the Lagrangean multiplier by λ, the maximisation procedure yields the following solution vector,

$$\begin{bmatrix} \hat{A}_1 \\ \lambda \end{bmatrix} = \begin{bmatrix} \beta\,\Omega_{11} & -i_1 \\ -i_1' & 0 \end{bmatrix}^{-1} \begin{bmatrix} e_1 - \beta\,\Omega_{12}\,\hat{A}_2 \\ i_2'\,\hat{A}_2 \end{bmatrix} \tag{2.41}$$

provided, as discussed in the last section, that $\beta\,\Omega_{11}$ is nonsingular. Parkin *et al* asserted that this condition is satisfied since Ω_{11}, a submatrix of the covariance matrix Ω, partitioned conformably with the detailed definitions of A and r in (2.34), is positive definite. Thus,

$$\hat{A}_1 = (1/\beta)\,G\,e_1 - G\,\Omega_{12}\,\hat{A}_2 - H\,i_2'\,\hat{A}_2 \tag{2.42}$$

where

$$G = \Omega_{11}^{-1} - \frac{\Omega_{11}^{-1} i_1\, i_1'\, \Omega_{11}^{-1}}{i_1'\,\Omega_{11}^{-1} i_1} \tag{2.43}$$

and

$$H = \frac{\Omega_{11}^{-1} i_1}{i_1'\,\Omega_{11}^{-1} i_1}. \tag{2.44}$$

As we have seen in section 2.1, G is symmetric since Ω_{11} is a covariance matrix, and has zero row and column sums, while the vector H has a sum of -1[5]. Thus the Cournot and Engel conditions hold.

Taking the second and third terms of (2.40) together yields the system of portfolio demand equations for an individual bank,

$$\hat{A}_1 = (1/\beta)\,G\,e_1 + H^*\,\hat{A}_2^* \tag{2.45}$$

where A_2^* differs from A_2 in that elements of A_2 which have the same coefficients are consolidated into a single component, namely,

$$A_2^* = (AD \quad SD \quad -TD \quad -DD^*) \tag{2.46}$$

where

$$DD^* = CA + BI - DD \qquad (2.47)$$

and H* is of course the sum of the coefficients of the second and third terms of (2.42).

To obtain estimatable aggregate portfolio demand equations Parkin *et al* make the following assumptions:

(1) that all banks hold the same expectations about interest rates and they all have the same subjective covariance matrix of forecasting errors;

(2) that e_1 is given without error by $\bar{r}_1$ which is defined as the simple average of thirteen weekly rates centred on quarter end;

(3) that forecasting errors on A_2^* are zero in aggregate across banks, i.e.,

$$\sum_{i=1}^{N} A_{2i}^* = \Sigma_1^* A_{2i}^* \qquad (2.48)$$

where N is the number of banks under consideration.

Therefore, by summing over the N banks the aggregate demand equations are arrived at as

$$\hat{A}_1 = B^* G \bar{r}_1 + H^* A_2^* \qquad (2.49)$$

where

$$\hat{A}_1 = \Sigma_1^{\pi} A_{1i},$$

$A_2^* = \Sigma_1^m A_{2i}^*$,

$B^* = 1/\beta$.

However, (2.49) is the optimal behaviour of banks, which is not observable. In order to estimate the demand equations Parkin *et al* make the additional assumption that actual behaviour of banks deviate from the optimal behaviour stochastically. Specifically,

$$A_1 = B^* G \bar{r}_1 + H^* A_2^* + \epsilon \qquad (2.50)$$

where ϵ is a vector of random errors normally distributed with mean vector zero and constant variance $\sigma^2 I$.

To allow for seasonality and a data dummy (2.49) is modified as

$$A_1 = B^* G \bar{r}_1 + H^* A_2^* + J D + J_5 D_5 + \epsilon \qquad (2.51)$$

where J is a 4 × 4 matrix of coefficients and D is a 4 × 1 vector of seasonal dummies and D_5 and J_5 are the data dummy and its coefficient vector respectively. Since the maximisation procedure specifies no constant in the demand equation, the four seasonal dummies must be constrained to a zero sum.

Using a restricted least square trace minimisation procedure (RLSTM) Parkin *et al* then estimated (2.51) with the following five sets of restrictions imposed:

(1) Symmetry of G;

(2) i' G = 0;

(3) i' H = -1;

(4) i' J = $0'$;

(5) $J\ i = 0$.

The data used in the estimation are quarterly data for the London clearing banks from the fourth quarter of 1953 to the fourth quarter of 1967. Only the t statistics and R^2's are reported for the four equations and no attempt is made by Parkin *et al* to test the theoretical restrictions. The reason for this seems to lie in the argument that singularity of the residual covariance matrix precludes "an F test on the significance of the difference in the error sum of squares in regressions which impose restrictions compared with unrestricted regressions" [Parkin (1970), p. 487]. Instead, the authors resort to examining whether the individual estimates are significant by using the t test. They find that "of the ten independent interest-rate coefficients, six are significantly different from zero and four are not" [p. 241]. Also, from examining the cross relations they revealed which assets are complements and which substitutes. The complements are (1) commercial bills and Treasury bills and (2) call money and Government bonds. The substitutes are (1) Treasury bills and Government bonds, (2) Treasury bills and call money and (3) commercial bills and Government bonds.

Yet, informative though these t tests may be, one should not forget that from the standpoint of evaluation of the validity of portfolio theory as applied to commercial banks, little importance attaches to the extent to which individual coefficients are statistically significant. The question is rather whether the theoretical restrictions hold in general. In fact, as we shall see in the next section, the Parkin-Gray-Barrett model has serious inconsistencies in its theoretical derivation as well as its estimation. Thus, their conclusions cannot be interpreted to imply that bank asset choice behaviour may be adequately interpreted as conforming to an expected utility maximisation.

2.5 A CRITIQUE OF THE PARKIN-GRAY-BARRETT MODEL

The appeal of the Parkin-Gray-Barrett model is that uncertainty in the interest rate structure is accounted for explicitly rather than implicitly. In addition, it yields clearly defined demand equations and prior information concerning the pattern of the signs and values of the parameters to be estimated. Specifically, as we have seen in the last section, the model implies the sort of over-identifying restrictions similar to those derived in system-wide linear expenditure models of consumer demand, namely, symmetry, Cournot aggregation and Engel aggregation.

However, these positive features are to be balanced against the strong assumptions about the distribution of real profits, inconsistencies in the derivation of the estimatable equations, the arbitrary assumptions regarding the choice set of assets and the treatment of the non-choice items. In this section we attempt to provide a synthetical critique of the Parkin-Gray-Barrett model as outlined in the last section. Although these criticisms are based upon the original Parkin-Gray-Barrett model, in most cases they apply equally to those variant models to be mentioned later.

2.5.1 Distribution of Real Profits

In the Parkin-Gray-Barrett model an implicit assumption is made that the distributions of u and v are such that $\prod$ is normally distributed, or (2.27) will not hold. As Berndt, McCurdy and Rose (1980) point out, Parkin *et al* did not discuss the necessary and sufficient conditions on the distributions of u and v for $\prod$ to be normally distributed. To illustrate this point (2.29) is reproduced here,

$$\prod = e' \hat{A} + e' v + u' \hat{A} + u' v. \qquad (2.29)$$

Here $\prod$ is seen to be the sum of four products and a sufficient condition for $\prod$ to be normally distributed is that the four products are individually, normally, independently

distributed. We recall that e and $\hat{A}$ are vectors of expected rates of return of individual assets and expected holdings of these assets. Thus, the first term on the right-hand side of (2.29) is a constant, which does not present any distributional problem. The second and third terms are each the sum of products of a random variable and a constant. Therefore, for $e'v$ and $u'\hat{A}$ to be normally distributed independent of each other, u and v must be normally and independently distributed. If this is so, however, the forth term becomes a product of two normally and independently distributed vectors. Two problems arise. First, if u and v are normally distributed vectors independent of each other, the product $u'v$ can under no conditions be normally distributed [for the properties of products of random variables see, for example, Bacon (1980)]; Second, if $u'v$ is normally distributed, it is hardly justifiable to assume that $u'v$ is distributed independent of $e'v$ and $u'\hat{A}$.

In fact, as first pointed out by Sharpe (1973), the strong symmetry property of the cross interest rate coefficients depends crucially upon the very assumption that u and v are independently distributed. When u and v are not statistically independent the symmetry result no longer holds. In addition, the expected returns of non-choice assets/liabilities enter the demand equations for choice assets.

Assuming that u and v are not statistically independent with

$$E\ (u'\ v) = K$$

$$E\ (v\ u') = E\left[\begin{bmatrix} v_1u_1 & v_1u_2 & v_1u_3 & \cdots & v_1u_n \\ v_2u_1 & v_2u_2 & v_2u_3 & \cdots & v_2u_n \\ v_3u_1 & v_3u_2 & v_3u_3 & \cdots & v_3u_n \\ \vdots & \vdots & \cdot & \cdot & \\ v_nu_1 & v_nu_2 & v_nu_3 & \cdot & v_nu_n \end{bmatrix}\right] = \Sigma_{vu}$$

and maintaining that $E\ (u) = E\ (v) = 0$, we have from (2.29)

$$\mu_\pi = E\ (\prod) = e'\ \hat{A} + K \qquad (2.30')$$

$$\sigma_{\pi}^2 = E\,[(\Pi - \mu_{\pi})] = E\,[(e'\,v + u'\,A + u'\,v - K)^2]$$

$$= e'\,V\,e + 2e'\,\Sigma_{vu}\,A + A'\,\Omega\,A + 2e'\,V_u + 2\Omega_v'\,A + \Psi - K^2 \qquad (2.31')$$

where

$$V_u = E\,(v\,v'\,u)$$

$$\Omega_v' = E\,(v'\,u\,u')$$

and V, Ω and Ψ are as defined in (2.32) - (2.34). The important thing to note here is that Σ_{vu} is non-symmetric, i.e., $\Sigma_{vu} \neq \Sigma_{vu}'$, since in general $E\,(v_1 u_2) \neq E\,(u_1 v_2)$. Maximising (2.27), but with μ_{π} and σ_{π}^2 redefined in (2.30') and (2.31'), subject to the balance sheet constraint (2.40) yields the following demand equations for the choice assets:

$$A_1 = (1/\beta)\,G\,(I - \beta\,\Sigma_{v_1u_1}')\,e_1 - G\,\Sigma_{v_2u_1}'\,e_2 - G\,\Omega_{12}\,A_2$$

$$- H\,i'\,A_2 - G\,\Omega_{v_1} \qquad (2.42')$$

where G and H are as defined in (2.43) and (2.44) and Ω_{v_1} is the conformable sub-vector of Ω_v [see Chapter Three for details].

From (2.42') we note the following:

(1) Although G is still symmetric, the product of G and the non-symmetric matrix $(I - \beta\,\Sigma_{v1u2}')$ is non-symmetric. This implies, of course, that if u and v are not independently distributed, the symmetry restrictions of cross interest rate coefficients are no longer valid. Therefore, an attempt to impose the symmetry restrictions in estimating the asset demand equations would introduce specification error. It seems that a more

appropriate procedure would be to obtain unconstrained interest rate coefficients and then to see whether they are consistent with the hypothesis of symmetrical interest rate responses. Let

$$G_0 = G\ (I - \beta\ \Sigma'_{v1u1})$$

then

$$i'\ G_0 = i'\ G\ (I - \beta\ \Sigma'_{v1u1}) = 0$$

since

$$i'\ G = 0.$$

Thus, the coefficients of any expected rate of return in the choice set sum to zero across all equations, hence the zero column-sum property, as in the Parkin–Gray-Barrett model. However, the zero row-sum property (i.e., the summation of within-equation rate of return coefficients to zero) is not automatically preserved, since

$$G_0\ i = G\ (I - \beta\ \Sigma'_{v1u1})\ i = G\ i - \beta\ G\ \Sigma'_{v1u1}\ i \neq 0$$

unless one assumes, as Perrin (1980) does, that $i'\ v_1 = 0$, i.e., errors in forecasting choice asset levels sum to zero. If $i'\ v_1 = 0$, then given the definition of Σ_{v1u1}, we have

$$\Sigma'_{v1u1}\ i = E\ (v_1\ u_1^1)'\ i = E\ (u_1\ v_1^1)\ i = E\ (u_1\ i'\ v_1) = 0.$$

(2) Unlike (2.42), the expected rates of return of the non-choice assets/liabilities also enter the demand equations for choice assets in (2.42'). This result follows directly from the statistical dependency between the error terms of rates of return and

asset/liability levels.

(3) The term G Ω_v involves only the variance-covariance matrices. When u and v are not independently distributed, the maximisation process will specify an intercept in the asset demand equations provided that the variance-covariance matrices can be assumed constant over time.

2.5.2 Aggregation and the Composite Error Term

We now turn to examining the inconsistencies in the assumptions of the error terms and aggregation in the Parkin-Gray-Barrett model. In (2.50) ϵ is introduced into the aggregate demand equations independently of the explanatory variables and therefore independent of their error terms. However, as Berndt *et al* (1980) point out, this is hardly consistent with the previous assumptions made about the error terms. From (2.29), (2.36) and (2.45) we can write

$$A_1 - v_1 = (1/\beta)\ G\ (r_1 - u_1) + H^*\ (A_2^* - v_2)$$

or

$$A_1 = (1/\beta)\ G\ r_1 + H^*\ A_2^* - [(1/\beta)\ G\ u_1 + H^*\ v_2 - v_1] \qquad (2.52)$$

It is apparent that the composite disturbances vector $[(1/\beta)\ G\ u_1 + H^*\ v_2 - v_1]$ is correlated with the two vectors of explanatory variables r_1 and A_2^*. Therefore, any distribution in the demand equation of an individual bank can not be assumed to be independent of the explanatory variables. An alternative as noted by Berndt et al would be to provide an explicit expectation formulation such that

$$\bar{r}_1 = e_1 + \omega \qquad (2.53)$$

where ω is a vector of the measurement error term in the expectations, then

$$A_1 = (1/\beta)\ G\ (\tilde{r}_1 - \omega) + H^*\ (A_2^* - \nu_2) + \nu_1$$

$$= (1/\beta)\ G\ \tilde{r}_1 + H^*\ A_2^* - [(1/\beta)\ G\ \omega + H^*\ \nu_2 - \nu_1]. \qquad (2.54)$$

Again, $\tilde{r}_1$ and ω are obviously correlated. In fact, such an expectations formula is what Parkin *et al* employed. (2.54) is the actual demand equation of a single bank. To obtain aggregate demand equations, (2.54) is summed up over the N banks, thus

$$\sum_{i=1}^{N} A_{1i} = \sum_{i=1}^{N} (1/\beta_i)\ G\ \tilde{r}_1 + \sum_{i=1}^{N} H^*\ A_{2i}^* - \sum_{i=1}^{N} [(1/\beta_i)\ G\ \omega + H^*\ \nu_{2i} - \nu_{1i}]$$

or

$$A_1 = B^*\ G\ \tilde{r}_1 + H^*\ A_2^* - \sum_{i=1}^{N} [(1/\beta)\ G\ \omega + H^*\ \nu_{2i} - \nu_{1i}]. \qquad (2.55)$$

To get round the correlation problem Parkin *et al* made two additional assumptions: $\omega = 0$ and $\Sigma_1^N\ \nu_{2i} = 0$, so that (2.55) reduces to (2.50) with $\Sigma_1^N\ \nu_{1i} = \epsilon$. However, these two assumptions are very arbitrary indeed. If, for example, e_1 can be assumed to be given by $\tilde{r}_1$, why can not A_2^* be given by $\tilde{A}_2^*$ where $\tilde{A}_2^*$ is defined analogously to $\tilde{r}_1$? Substitution of $\tilde{A}_2^*$ for A_2^* into (2.50) soon reveals that such an assumption would violate the balance sheet constraint. Therefore, the assumption that $\Sigma_1^N\ \nu_{2i} = 0$ serves a dual purpose: to eliminate the correlation problem and to avoid the violation of the balance sheet constraint.

In (2.36) v_1 is assumed to have zeros everywhere except in the call money position. The reason advanced by Parkin *et al* for this peculiar assumption is that "some asset(s) must take up the effects of departures of the actual values of the exogenous items from their expected levels, and ... it is always call loans which serve this purpose" [p. 235]. There is no explanation given as to why it is always call loans which serve this purpose. Moreover, such an assumption is inconsistent with the full specification of the error vector ϵ in (2.50). By definition, $\mathbf{A}_1 = \mathbf{A}_1 + \Sigma_1^N v_{1i}$. Given that $\Sigma_1^N v_{1i} = \epsilon$ and the vector $\Sigma_1^N v_{1i}$ has zeros everywhere except in the call loan position, all but the call loan demand equation in (2.50) will be non-stochastic. In fact, $\Sigma_1^N v_{2i} = 0$ together with the assumption that v_1 has zeros everywhere except in the call loan equation implies that all demand equations in (2.50) are non-stochastic.

We recall that actual (realised) return at the end of a decision period from a particular portfolio A is given by (2.28). In the traditional portfolio theory discussed in Section 2.3, the vector A is assumed to be certain, namely, the investor can hold for the whole decision period whatever level of assets he chooses at the beginning of the decision period; and his chosen (optimal) asset level is the same as the actual (realised) asset level at the end of that decision period. The investor's actual (realised) return differs from his expected return only because the actual (realised) yields at the end of the decision period are not known with certainty at the moment of his decision. In the Parkin-Gray-Barrett model, however, the bank's total asset and liability levels at the end of the decision period, A, are also unknown at the moment of decision because of "uncertainty ... about the levels of deposit liabilities" [p. 233]. Therefore, the bank's expected profit as given by (2.30) differs from its actual (realised) profit not only because actual yields are stochastic but also because actual (realised) asset and liability levels are unknown at the beginning of the decision period. However, the bank can choose its desired level (called expected by Parkin *et al*) of choice assets, $\hat{A}_1$, with certainty since at the moment of decision it knows exactly the level of the non-choice items, $\hat{A}_2$.

Granted the Parkin *et al* dichotomy of the bank's balance sheet between choice and non-choice items, the balance sheet constraint requires that

$$i_1' \hat{A}_1 + i_2' \hat{A}_2 = 0 \qquad (2.40)$$

i.e., the sum of desired choice assets must equal the sum of non-choice items at the moment of decision. Given the static nature of the Parkin–Gray–Barrett model, which assumes the absence of transactions costs, the desired level of choice assets, $\hat{A}_1$, will differ from their actual level, A_1, only because the level of non-choice items at the beginning of the decision period, $\hat{A}_2$, may differ from their actual level, A_2, at the end of the decision period. However, the balance sheet constraint must also hold at the end of the decision period so that

$$i_1' A_1 + i_2' A_2 = 0. \qquad (2.56)$$

From (2.29) we can rewrite (2.56) as

$$i_1' (\hat{A}_1 + v_1) + i_2' (\hat{A}_2 + v_2) \qquad (2.57)$$

which together with (2.40) implies that

$$i_1' v_1 + i_2' v_2 = 0. \qquad (2.58)$$

Interpreting v_1 and v_2 as changes in the choice and non-choice items during the decision period (2.58) requires that the sum of changes in the choice items equal the sum of changes in the non-choice ones. Aggregating across all the N banks under consideration we obtain

$$i_1' \sum_{i=1}^{N} v_{1i} + i_2' \sum_{i=1}^{N} v_{2i} = 0 \qquad (2.59)$$

which, with $\Sigma_1^N \nu_{2i} = 0$, reduces to

$$\sum_{i=1}^{N} \nu_{1i} = 0. \qquad (5.60)$$

Therefore, all demand equations in (2.50) are indeed non-stochastic. It should be noted that, in interpreting ν_1 and ν_2 as changes in the choice and non-choice items during the decision period, we are implicitly assuming that the bank takes a decision at the beginning of the period and his decisions regarding the optimal portfolio is independent of the changes in the non-choice items during the period. This assumption is not inconsistent with the static nature of the model, which is optimal only in the absence of transactions costs.

All these problems arise from the static nature of the Parkin-Gray-Barrett model. The system of portfolio demand equations for an individual bank in (2.45) describes the optimal (desired) level of choice assets at the beginning of decision period, which is not only unobservable but also different from the actual level at the end of the decision period, because of changes in the non-choice items during the decision period. What is required here is the specification of a mechanism of the bank's adjustment process from the beginning-of-period optimal position to a new one when changes in the non-choice items take place, a mechanism which will then translate the desired level of choice assets into its actual level. In order to accomplish this a dynamic model will have to be specified. We develop such a model in Chapter Three.

2.5.3 Choice Set and the Treatment of Non-choice Items

Parkin *et al* excluded advances from the bank's choice set of assets because: (1) "for about one-half of the period covered ... advances were subject to ceilings imposed by the Bank of England"; (2) "even when advances are not limited by the authorities, the price of a loan is administered and therefore, if advances are a choice variable, some other non-price control must be used to achieve the desired level of advances" [p. 235].

However, neither of these two arguments is very convincing.

It is true that during about one-half of the period covered by the study of Parkin *et al* (from the fourth quarter of 1953 to the fourth quarter of 1967) bank advances were subject to controls by the Bank of England. However, these controls rarely took the form of explicit quantitative ceilings. The fact that banks were subject to lending controls during only one-half of the period concerned should in itself reject the idea of treating advances as exogenous, since for the other one-half of the period banks were *not* subject to controls by the authorities, and if credit controls were the only constraint on banks' choice of advances, banks would then be able to choose their desired level of advances. Moreover, even if banks were subject to credit controls for the entire period, this fact alone does not, *a priori*, necessarily form the basis for treating advances as an exogenous variable because banks' desired level of advances which result from the expected utility maximising procedure may or may not exceed the imposed ceilings. Only if the banks' desired level of advances is higher than the imposed ceiling and the ceiling is legally binding will the ceiling be effective. Therefore, only if banks were subject to lending ceilings all the time and if the lending ceilings were always binding and effective, could we treat advances as truely a non-choice variable. When such ceilings did exist during part of the period concerned the methodologically more desirable procedure is to treat advances as a non-choice variable but to introduce a dummy variable to capture the effects of lending ceilings, and then test whether such ceilings are indeed effective.

As regards the argument of the administered price of a loan, it is a well-known fact that, although the London and Scottish clearing banks linked their lending rate to the then Bank rate during the relevant period, they had always charged a range of rates for loans much wider than the conventional formula of two percent plus the Bank rate, and as Courakis (1974) points out, "were always in a position to alter the *average* rate on advances by altering the share alloted to different rates" [p. 187]. More importantly, even if the price of a loan were administered, this argument is relevant to the exogenous

treatment of advances only if the advances market was always in excess supply, i.e., the amount of advances supplied by the banks always exceeded that demanded by the public at the administered rate. By invoking the administered price argument, Parkin *et al*, therefore, assumed *a priori* that banks were always faced with a deficient demand for their loans during the period. Such an assumption, however, is inconsistent with the fact that during the relevant period the Bank of England had to impose from time to time ceilings on commercial bank lending. As long as the advances market is not always at a position of excess supply banks can, with or without using non-price instruments, choose their desired level of advances, and accordingly, advances should be treated as a non-choice variable.

Now let us turn to the treatment of those so-called non-choice items in the Parkin-Gray-Barrett model. In (2.46) A_2^* is defined as

$$A_2^* = (AD \quad SD \quad -TD \quad DD^*)$$

where

$$DD^* = CA + BI - DD.$$

We recall that CA is notes, coins and balances with the Bank of England, BI is balancing items and DD is demand deposits. No justification is provided by Parkin *et al* in defining the composite variable DD*. The same aggregation is carried out by Courakis (1974) because "within the confines of our objective functions assets carrying a zero rate of return may be aggregated" [p. 189].

As we noted in Section 2.3, in models of expected utility maximisation, where expected utility depends upon risk and return, the individual bank can only distinguish between assets in terms of their risk and return characteristics. Two assets with the same risk and return characteristics are therefore the same asset as defined by the

expected utility maximisation paradigm. Since CA and BI are both assets carrying zero return and by definition having no risk, then they are the same asset from the point of view of the individual expected utility maximising bank. Similarly, as DD does not carry a rate of interest it has the same risk and return characteristics as CA and BI and is, therefore, the same asset. This line of reasoning highlights the inadequacy of the standard portfolio theory. What is required here is the introduction of some means of distinguishing between assets other than on grounds of differences in risk and return characteristics such as liquidity.

However, aggregations of this sort present some problems. For example, given the definition of DD* and that CA = 0.08 (DD + TD) in (2.37) there is a clear algebraic relationship between DD* and TD, and therefore, these two explanatory variables in (2.50) are not independent. This gives rise to two problems. First, as pointed out by Bailey, Driscoll and Ford (1980), if two explanatory variables are not independent of each other in the mathematical sense, the interpretation of H^* in (2.50) is difficult to maintain. The usual interpretation given to G and H^* is that they are matrices of partial derivatives between the dependent and independent variables. Such an interpretation, in order to be correct, requires that all the non-choice variables are independent of each other. Second, if DD* and TD are not independent of each other, although OLS estimates of the parameters will still be best unbiased, small variations in the data, e.g., dropping or adding a few observations, may produce substantial variations in the OLS coefficient estimates. Therefore, the conclusions of Parkin *et al* can be called into question.

In addition, the relation CA = 0.08 (DD + TD) implies that the restriction

$$i' H^* = -1$$

is *a priori* incorrect since a *ceteris paribus* increase in TD and/or DD will increase CA by 0.08 (ΔDD + ΔTD) and the other choice variables by only 0.92 (ΔDD + ΔTD)

where Δ denotes an increase. Thus the appropriate restriction is

$$i' H^* = -0.92. \qquad (2.56)$$

In fact, there is a more fundamental conceptual problem with the treatment of non-choice variables in the Parkin-Gray-Barrett model. In the reduced form equations of (2.49) the non-choice items, advances, Special Deposits, demand deposits and time deposits are included as separate variables. It is not clear whether there is a correspondence between these reduced form equations and the form of the model that one would expect to follow from the mean-variance approach employed by Parkin *et al.* In the mean-variance portfolio theory outlined in Section 2.3, the presence of the wealth variable, W, simply determines how much is availabe to the individual investor to invest in his portfolio, i.e., the portfolio size. One would anticipate, therefore, that the non-choice items in total affect the bank's portfolio choice only in that they determine what funds are left to be held in the choice assets. That is to say that within the confines of the mean-variance portfolio theory the composition of the non-choice items in the portfolio will not have any effect on the bank's demand for choice assets; indeed all non-choice items should be consolidated into one variable whose role in the demand equations is similar to that of a wealth variable.

Of course, as Bailey, Driscoll and Ford (1980) point out, there are many reasons why the bank's portfolio choices might be affected by the composition of non-choice items. For example, the ratio of demand deposits to time deposits might be an important determinant of portfolio choice as these two types of deposits differ in their liquidity characteristics. Demand deposits are provided at call and can be withdrawn without notice, and are thus more volatile than time deposits. An increase in demand deposits relative to time deposits might therefore lead the bank to make its portfolio more liquid. However, reasons such as this are extraneous to the mean-variance portfolio theory, which does not cater for the liquidity of assets and uncertainty of deposit flows, and attempts to allow for such effects are likely to violate the logical structure of the

model. Indeed, the inability of the mean-variance portfolio approach to incorporate such features as liquidity and random deposit flows, which are so relevant to modern commercial banking, highlights the inappropriateness of applying the standard mean-variance portfolio analysis to commercial bank portfolio behaviour.

2.6 EXTENTIONS OF THE PARKIN-GRAY-BARRETT MODEL

Since the publication of the seminal paper by Parkin, Gray and Barrett (1970) on the portfolio behaviour of UK commercial banks, attempts have been made by Courakis (1974, 1980), White (1975), Berndt, McCurdy and Rose (1980), and Bewley (1981), to extend this work and, at the same time, to employ more efficient and consistent estimation techniques as well as more rigorous tests of model specification than those used by Parkin *et al.*

As we indicated earlier, Parkin *et al* used a restricted least square trace minimisation (RLSTM) procedure to estimate the system of equations in (2.51). This procedure, as Parkin *et al* themselves realised, is deficient in this case because of the intrinsic singularity of the system (2.51), which arises from the portfolio balance sheet constraint and the assumption that $i'\hat{A}_{2t} = i'A_{2t}$. Singularity poses two problems. Firstly, singularity implies that only 3 of the 4 equation disturbances are linearly independent, hence only 3 of the 4 equations can be estimated independently. Secondly, singularity of the disturbance covariance matrix implies that the matrix is non-diagonal; failure to account for this non-diagonality implies that the RLSTM estimator yields inefficient estimates of the parameters and the disturbance covariance matrix [Berndt, McCurdy and Rose (1980)]. In Parkin *et al* neither the linear dependency nor the non-diagonality is taken into account when the RLSTM procedure is used, thus, their "success" at capturing significant interest rate response estimates should be called into question.

To allow for singularity and non-diagonality Courakis (1974), using the data from Parkin *et al*, employs a full information maximum likelihood (FIML) estimator to estimate the Parkin-Gray-Barrett model by deleting one equation. Later, Berndt *et al* (1980) estimate the system of equations in (2.51) with the call loan equation deleted by using a maximum likelihood (ML) estimator. As shown by Barten (1969) and Berndt and Savin (1975) the ML estimators are numerically invariant to the choice of which equation is deleted and indeed in the fully restricted case the estimates for the deleted

equation can be derived from the restrictions. After the deletion of one equation from the n-equation system, the n-1 element disturbance vector ϵ_{n-1} is independently, and in the Parkin-Gray-Barrett model, multivariantly, normally distributed with mean vector zero and constant non-singular covariance matrix; therefore, the ML estimates are consistent, asymptotically efficient and asymptotically normally distributed [Berndt and Savin (1975)].

Courakis's (1974) FIML estimates of the Parkin-Gray-Barrett model differ markedly from those obtained by Parkin *et al.* Of the ten interest rate coefficients only three are statistically significant, and they all appear in the call loan equation. In fact, all the four own interest rate coefficients are of the wrong negative signs.

Courakis (1974) also distinguishes between what he calls myopic and non-myopic models of bank portfolio selection. The former are models based on the assumption that decisions regarding the distribution of a given total between choice items are made in complete disregard of the composition of the rest of the portfolio, while the latter are models resting on the presumption that the allocations among choice items are made with explicit consideration of the whole of the portfolio. Experimenting with both the quadratic and the negative exponential utility functions Courakis derives asset demand equations for each of the above models, and finds that asset demand equations derived from the negative exponential utility function are special cases of those resulting from a quadratic utility function. In addition, he argues that the demand equations derived from a quadratic utility function on the non-myopic assumption that the composition of the non-choice items enter the decision-taking unit's objective function, "may be treated as the general case consistent with the Tobin-Markowitz tradition of choice under uncertainty all other objective functions considered ... yielding outcomes that imply additional constraints" [p. 193].

Unfortunately, his empirical results suggest that "neither the more general case nor any of its more restricted variants can be accepted as providing an adequate explanation of bank portfolio behaviour in the UK over the relevant period" [p. 193]. Moreover, the

constraints as were imposed by Parkin *et al*, such as symmetry, homogeneity, are found by Courakis as not consistent with the data.

It should be noted that the demand equations as estimated by Courakis (1974) are not entirely consistent with those one would derive from a mean-variance approach. The inconsistencies include the presence of an intercept and a trend variable as well as the separate entries of the non-choice items in each of the demand equations.

The ML estimates of Berndt *et al* (1980) also differ significantly from those of Parkin *et al.* For example, using the individual t test, only two of the ten interest rate coefficients are significant, compared with six in Parkin *et al.* Also, two sign changes occurred when the ML is employed. Using the ML estimates Berndt et al then tested the validity of the theoretical restrictions derived from the expected utility maximisation process and they concluded that the theoretical restrictions of no intercept, homogeneity and symmetry are not consistent with the data.

Of course, the above empirical results are based on the assumption that the disturbances are temporarily independent. To test whether first-order vector autocorrelation is present, Berndt *et al* re-estimated the system of equations in (2.51) with ϵ following a stationary first-order autoregression (AR1) process

$$\epsilon_t = R\, \epsilon_{t-1} + \omega_t \qquad (2.57)$$

where ω_t is an independently identically distributed random vector and R is a 4×4 matrix of unknown autoregressive coefficients. The R matrix can either be specified as full or diagonal. Again, the call loan equation is dropped from the regression and the results reject the null hypothesis of no first-order vector autocorrelation regardless of whether the alternative hypothesis is that R is diagonal or that R is full. Berndt *et al* are unable to reject the null hypothesis of "no intercept" under both of the specifications of R (full or diagonal). Using these as the maintained hypothesis they then tested the

homogeneity and symmetry constraints. Maintaining that R is diagonal, the homogeneity and symmetry restrictions are rejected when tested jointly as was under the assumption of no first-order vector autocorrelation. However, when R is assumed full, the test results are more ambiguous. Berndt *et al* thus concluded that "extent of the rejection of the theoretical restrictions varies considerably depending on the generality of the autoregression specification" [p. 871].

One of the possible explanations suggested by Courakis (1974) as to why the traditional mean-variance portfolio analysis has failed to provide an empirically consistent and statistically acceptable explanation of UK commercial bank asset choice behaviour, is that the choice set as perceived by these banks may differ from that by Parkin *et al* and Courakis (1974). As we commented earlier, the arguments provided by Parkin *et al* for treating advances as a non-choice variable are neither empirically convincing nor methodologically sound. In an attempt to rectify this situation White (1975) decomposes advances into a non-choice component and other advances, and estimates two sets of asset demand equations with other advances being a choice variable in one set and exogenous in another. The data employed by White are aggregated quarterly data covering the London clearing banks, the Scottish and Northern Irish banks for the period from the first quarter of 1963 to the fourth quarter of 1970.

By way of comparing the weighted average standard error of estimates (WASEE, which is calculated by summing the gross residual sum of squares of the equations common to both sets, dividing by the total available degrees of freedom and taking the square root) for alternative systems the goodness of fit is superior when other advances are regarded as a choice variable, and on the whole, the estimated parameter signs seem to be more acceptable. A second indication that other advances should be included in the bank's choice set is the significant influence exerted by the interest rate of advances in the other equations, notably those for call money and British Government bonds. White also estimates dynamic versions of the models and similar results are obtained.

Recognising the informal nature of this evidence, White points out that "nearly all the results suggest that other advances should be considered as a choice set asset" [p. 489]. What is more interesting is the fact that in the static models the symmetry restrictions are again rejected by the data no matter which of the alternative specifications of choice set is assumed. However, this should not be interpreted as implying that the mean-variance model itself as applied to commercial banks is rejected by the data. The reasons are as follows. Firstly, as we have argued and demonstrated in the last section, the symmetry property of the response to changes in the expected rates of return on assets depend upon the assumption that the error terms in the levels of asset/liability and in the rates of return are independently distributed. Thus, the rejection of the symmetry restriction by the data may simply imply that the statistical independence assumption is inconsistent with the data. Secondly and more importantly, the symmetry and homogeneity properties relate to the *expected*, not *actual*, rates of return on assets. Since the expected rates of return are not observable they have to be estimated or proxied from actual rates of return on assets. It is therefore possible that the rejection of the symmetry restrictions by the data might imply the rejection of the particular specification of the expected rates of return used in the estimation of the model rather than the rejection of the symmetry result of the model as pertaining to the expected rates of return. Finally, the symmetry characteristics of the response to changes in the expected rates of return on assets, which derive from the symmetry property of the variance-covariance matrix, correspond to the portfolio behaviour at *each* particular point in time. In estimating the asset demand equations as derived from the expected utility framework it is explicitly assumed that the variance-covariance matrix is constant over time. However, the perceived variance-covariance matrix of rates of return on assets may change from one period to another, and may not be constant over the sample period. Therefore, it can be argued that the rejection of the symmetry property on the basis of regressions pertaining to the portfolio behaviour of commercial banks *over time* need not imply rejection of the theoretical model but rather that the auxiliary hypothesis that the variance-covariance matrix is constant over time is inconsistent with the data.

The asset demand equations derived from the mean-variance analysis relate to the behaviour of an individual bank. However, perhaps due partly to the unavailability of individual bank balance sheet data and partly to the desire of the reseacher to reach macroeconomic policy conclusions only aggregate data relating to groups of institutions, such as the London clearing banks, have been utilised to study UK commercial bank portfolio behaviour. In order to estimate models of bank portfolio behaviour using aggregate data the microeconomic model derived from the mean-variance analysis has to be translated into a macroeconomic one. All the empirical studies discussed so far have proceeded on the assumption that all banks have the same attitude towards risk, hold identical expectations about interest rates and perceive the same variance-covariance matrices of returns, so that behaviour of the group of banks at the aggregate can be described by the behaviour of an individual bank multiplied by the number of banks making up the aggregate. Such aggregation over a group of banks is usually justified on the grounds that, for example, "the balance sheets of London clearing banks may well differ in details and in some substantial points from those of Scottish clearing banks, but all London clearing banks will have very similar balance sheets, and so will all Scottish clearing banks" [Revell (1973), p. 135].

Through examining the balance sheets of the eleven London clearing banks during the period from January 1959 to July 1967 Courakis (1980) argues that not only do these banks vary in size and in experience in terms of deposit variability and growth, but also "there are considerable and persistent differences in the proportions of total portfolio alloted to different assets by each of the eleven institutions" [p. 319]. He contends that such disparities between these seemingly homogeneous banks may well be accounted for by, among others, differences in their preferences between risk and return, in their expectations about interest rates and in their subjective variance-covariance matrices of returns, and it would be "unwise to readily aggregate over these institutions when examining their behaviour" [p. 320]. Accordingly, he estimates asset demand equations for each of the eleven London clearing banks for the period indicated above.

Another innovation of Courakis (1980) is the inclusion of the expected intraperiod changes in non-choice items as explanatory variables in the choice asset demand equations. Given the static single-period nature of the model, however, the addition of such variables seems most strange, and as we discussed in the previous section, from the point of view of model consistency, very *ad hoc* indeed.

In an attempt to resolve some of the problems concerning the classification of advances Bewley (1981), using monthly data of the London Clearing Banks for the period 1963 - 1971, extends earlier studies of bank portfolio behaviour by introducing dynamics of the Brainard-Tobin (1968) type into the Parkin–Gray–Barrett model. As we mentioned earlier, this type of dynamic model was also estimated by White (1975) for UK commercial banks. Bewley's model, however, differs from that of White in one important aspect: the model is allowed to switch between two regimes; "one regime specifies an exogenous role for advances when Bank of England controls were in force, and the other regime assumes that advances behaved endogenously when these controls were inoperative" [Bewley (1981), p. 191]. It is assumed that in both regimes the banks had the same long-run equilibrium relationship of the form derived in (2.45), but the banks' adjustment process was different for the two regimes. Technically, this amounted to adding an additional independent variable, namely, the difference between the desired level of advances and the level of advances in the last period, to the banks' adjustment equations when the Bank of England controls were in force. Therefore, instead of treating advances either as exogenous, as in Parkin *et al* (1970) and Courakis (1974, 1980), or as endogenous as in White (1975), and estimating the asset demand equations for the whole period in question, Bewley truncates the whole sample period into two subperiods, namely, regime 1 when Bank of England controls were in operation and regime 2 when such controls were lifted, and estimtes the asset demand equations for each subperiod separately. However, one might argue that such a situation can be handled simply by treating advances as endogenous for the whole sample period and estimating the asset demand equations with a dummy variable included to account for the effects of control over advances.

Another, and perhaps more interesting, extention by Bewley (1981) is the testing and subsequent acceptance of a hypothesis concerning the liquid assets ratio. The studies by Courakis (1974, 1980) and White (1975) on UK commercial bank portfolio behaviour have, following Parkin *et al* (1970), treated the liquid assets ratio as not binding on the grounds that the clearing banks had always maintained their liquid asset holdings above the required ratio. Recognising that although the observed liquid asset ratio of the clearing banks does not provide any useful information, Bewley suggests that, "through the period in question, the clearing banks operated with a desired exogenous liquid assets ratio above the legal minimum with the observed deviations from it being accounted for by the adjustment process" [p. 192]. Presumably, such an exogenous desired liquid asset ratio is "independent of the level of interest rates and other exogenous assets" [p. 203]. And this implies, if the hypothesis is accepted, that the banks' optimal *total* holdings of liquid assets will be independent of the interest rates on all assets and the distribution of the *total* among the individual liquid assets will depend upon only the interest rates on the liquid assets. However, Bewley argues that "the restrictions are such that the interest rates and exogenous assets affect the distribution of the assets within that set" and claims that "many of these effects are significant"[p. 203]. It is not clear at all why this should be the case.

2.7 CONCLUDING REMARKS

In this chapter we have concentrated our discussion on the Parkin–Gray-Barrett model of UK commercial bank portfolio behaviour under the assumption of risk aversion. Subsequent attempts to extend the original Parkin-Gray-Barrett model have involved employing more efficient and consistent estimation techniques and more rigorous tests of the theoretical restrictions by Courakis (1974) and Berndt *et al* (1980), experimenting with alternative treatments of advances by White (1975) and Bewley (1981), using individual bank balance sheet data for estimation by Courakis (1980), as well as specifying dynamic versions of the model by White (1975) and Bewley (1981).

However, within the confines of the static mean-variance framework as employed by Parkin *et al* (1970) several interesting and important areas, which are relevant to the analysis of bank portfolio behaviour, have been neglected by most of the above mentioned studies. First of all, no attention has been paid to modelling the bank's expectations formation process regarding future interest rates and/or to test whether the variance-covariance matrix of returns is indeed constant over time as almost universally assumed by the existing studies. Secondly, although models have been estimated with alternative treatments of advances, no attempt has been made to test formally the endogeneity or exogeneity of advances. Finally, all the studies mentioned above confined their empirical work to the period prior to the Competition and Credit Control era. Since data is now available it would be interesting to estimate models using post-1971 data and then to see whether there are any structural changes in UK commercial bank portfolio behaviour.

Before proceeding to Chapter Four we should recapitulate on the main points emerged from the discussions in this chapter:

(1) The normality assumption of real profit employed in the bank's objective function imposes severe restrictions on the distributions of asset/liability levels

and their returns. Indeed, given the form of the decomposition of real profit by Parkin *et al* real profit cannot be normally distributed;

(2) The symmetry result of interest rate coefficients in the asset demand equations depends critically upon the assumption of statistical independence between asset/liability levels and their returns. It is possible that asset/liability levels and their returns are not independently distributed and therefore, the symmetry result may not hold;

(3) Advances, at least other advances, should be regarded as a choice variable;

(4) The separate treatment of non-choice items not only present some econometric problems due to the functional dependence between certain items, but is also inconsistent with the mean-variance approach. It is suggested that the non-choice items should be consolidated into a single wealth variable, as in the traditional mean-variance analysis.

It is to these issues we turn in the next chapter.

Notes:

1. A function f(x) is strictly concave over the interval (a,b) if

$$f[\lambda x_1 + (1 - \lambda)x_2] > \lambda f(x_1) + (1 - \lambda)f(x_2)$$

for all $0 < \lambda < 1$ and $x_1 > a$, $x_2 < b$. (2.1) is seen to be identical to this definition with $\lambda = p$, $x_1 = W_1$ and $x_2 = W_2$.

2. Samuelson (1970) shows that in only a limited number of cases will the central limit law be applicable so that an approximation to normal distribution of returns becomes tenable. See Samuelson, Paul A (1970), "The Foundamental Approximation Theorem of Portfolio Analysis in Terms of Means, Variances and Higher Moments", Review of Economic Studies, Vol. 37, pp. 537-542. See also Chapter Four.

3. For convenience we have dropped the expectations operator before the utility function and denote the expected utility function simply by U thereafter.

4. There are two papers employing the same theoretical structure published at roughly the same time, namely, Parkin, J. M. (1970), "Discount House Portfolio and Debt Selection", Review of Economic Studies, vol. 37, pp. 469-497, and Parkin, J. M., Gray, M. R. and Barrett, R. J. (1970), "The Portfolio Behaviour of Commercial Banks", in Hilton, K. and Heathfield, D. F. (ed), The Econometric Study of The United Kingdom, Macmillam and Co. Ltd., Edinburgh, 1970. We refer to the latter.

5. In Parkin, Gray and Barrett (1970) liabilities are entered negatively, so the Engel Condition becomes $i'H = -1$.

CHAPTER THREE

ALTERNATIVE MODELS OF BANK PORTFOLIO BEHAVIOUR UNDER RISK AVERSION

3.1 INTRODUCTION

As we have seen in the last chapter, when modelling commercial bank portfolio behaviour most authors still employ the traditional mean-variance approach based on expected utility maximisation. Banks are assumed to form expectations with regard to the probability of realising different profits from a certain portfolio and then to choose, subject to the balance sheet constraint, between portfolios on the basis of the mean and variance of these probability distributions, having regard to their preference as between risk and return. Each of the asset demand equations resulting from such an approach consequently includes the own expected rate of return and the expected rates of return on all competing assets, their variances and covariances: it is these variances and covariances which provide the parameters of the equations on the assumption that they remain constant over time. To comply with the balance sheet constraint, the parameters are subject to cross-equation adding-up restrictions. The advantage of this approach is that it yields clearly specified demand equations and prior information concerning the pattern of the signs and values of the parameters to be estimated. Specifically, the approach results in the sort of over-identifying restrictions that are familiar in system-wide linear expenditure models of consumer demand, namely Cournot conditions, Engel conditions, homogeneity and symmetry.

In this chapter we specify and test a number of static as well as dynamic models of the portfolio behaviour of UK commercial banks. The underlying static relationship is derived using the mean-variance approach explored in Chapter Two. Dynamics are introduced with the general stock adjustment process first propounded by Brainard and

Tobin (1968). This approach is well known but the models under consideration differ from previous studies on the portfolio behaviour of UK commercial banks by Parkin *et al* (1970), Courakis (1974, 1980), White (1975) and Bewley (1981) in a number of important aspects. The first is the treatment of non-choice items in the bank's portfolio. In all the above cited studies non-choice items appear as separate explanatory variables in the bank's demand equations for choice items. As we have seen in the last chapter, this approach not only encounters estimation problems and makes the interpretation of the parameters difficult because of the linear relationship between some of the non-choice items, but is also inconsistent with the spirit of the underlying mean–variance framework. In this chapter we develop models which treat the non-choice items as a single wealth variable. The consolidation of the non-choice items into a single wealth variable avoids the estimation and interpretation problems and, more importantly, is in line with the traditional mean-variance approach. In the existing empirical literature on bank portfolio behaviour the symmetry restrictions on the parameters of expected rates of return have often been rejected by the data. The symmetry result depends crucially on the assumption that the rates of return on assets/liabilities are independent of the asset/liability levels. In this chapter we also derive models under the assumption of statistical dependence between the rates of return and asset/liability levels. The resulting models not only allow for asymmetric effects of expected rates of return but also specify intercepts in the demand equations for choice assets. The final innovation of this chapter is the incorporation into the bank's asset demand equations of the liquid asset ratio constraint which has often been disregarded in the existing empirical literature.

The rest of the chapter is organised as follows. Section 3.2 introduces the bank's objective function, followed by the specification of alternative static asset demand equations in Section 3.3. In Section 3.4 the bank's portfolio adjustment process is discussed and its dynamic asset demand equations are derived. The problems of aggregation and econometric specifications are discussed in Section 3.5. Section 3.6 presents the empirical results for the London clearing banks. Some concluding remarks are provided in the final section.

3.2 THE BANK'S OBJECTIVE FUNCTION

Following previous studies on the portfolio behaviour of UK commercial banks we assume that each bank maximises an objective function depending only on the mean and variance of its subjective probability distribution of returns on its portfolio. This is to say, the bank is seen as forming expectations as to the possible profits of a certain portfolio, with these expectations taking the form of a probability attached to each conceivable outcome. The mean yield derived from the resultant probability distribution is taken as a measure of the likely yield and the variance as a measure of the risk that actual yield will deviate from the likely yield. In particular, we assume that the bank's objective function is of the form

$$Q = \mu_\pi - \frac{\beta}{2} \sigma_\pi^2 \qquad (\beta > 0) \qquad (3.1)$$

where μ_π is the expected value of return from a given portfolio and σ_π^2 is the associated variance of return.

As we have seen in Chapter Two, such an objective function can be derived from a negative exponential utility function with the auxiliary assumption that actual return from the portfolio is normally distributed. It is also noted in Chapter Two that the normality assumption of actual return imposes severe restrictions on the distributions of individual asset returns and the distributions of asset levels. However, the use of the objective function in (3.1) can be justified along the following lines.

Suppose that the bank's utility function of return from a certain portfolio is positively sloped and concave downward and that its objective is the maximisation of expected utility. That is, the bank maximises the expected value of

$$U = U(\Pi) \qquad (3.2)$$

with $U'(\Pi) = \partial U/\partial \Pi > 0$ and $U''(\Pi) = \partial^2 U/\partial \Pi^2 < 0$

where $\prod$ is actual return from the given portfolio. Further, we assume that the utility function is at least twice differentiable. Then, expanding (3.2) around the expected value of return, μ_π, by the Taylor series and ignoring all terms beyond the second order we can rewrite the bank's utility function as

$$U \approx U(\mu_\pi) + U'(\mu_\pi)(\prod - \mu_\pi) + (1/2)\, U''(\mu_\pi)(\prod - \mu_\pi)^2 \qquad (3.3)$$

where $U(\mu_\pi)$ is the utility function evaluated at the expanding point, μ_π, and $U'(\mu_\pi)$ and $U''(\mu_\pi)$ are the first- and second-order derivatives of the utility function evaluated at the same point. Taking expectations we have

$$E(U) = U(\mu_\pi) + (1/2)\, U''(\mu_\pi)\, \sigma_\pi^2 \qquad (3.4)$$

since $E(\prod - \mu_\pi)^2 = \sigma_\pi^2$ and the second term on the right-hand side of (3.3) vanishes upon taking expectations. Setting

$$U(\mu_\pi) = \mu_\pi \qquad (3.5)$$

and

$$U''(\mu_\pi) = -\beta \qquad (3.6)$$

we obtain (3.1). Therefore, the objective function in (3.1) may be regarded as an empirical approximation to the true objective function. In addition, given the definition of risk aversion of Chapter Three and the fact that $\beta > 0$, (3.1) describes absolute risk aversion with β being the constant risk aversion parameter.

The model to be developed in the next two sections may be viewed as an approximation to the solution of an optimal control problem in which the bank maximises (3.1) when there are explicit and implicit adjustment costs associated with

changes in the asset and liability levels of the portfolio. The bank's optimal adjustment strategy is determined by minimising total adjustment costs subject to the constraint that the balance sheet identity is always binding. Following the methodology of Sharpe (1973) we approach the problem in two stages. First, we derive from the maximisation of (3.1) the bank's static equilibrium asset demand equations. These static demand equations represent the bank's desired levels of asset holdings given its expectations about future yields on individual assets. Second, we specify and analyse the bank's portfolio adjustment process to the static equilibrium solution in the presence of adjustment costs.

3.3 ALTERNATIVE SPECIFICATIONS OF THE BANK'S STATIC ASSET DEMAND EQUATIONS

The bank's static asset demand equations are derived by maximising (3.1) subject to the balance sheet constraint. However, in this section we are also concerned with alternative specifications of the bank's static asset demand equations resulting from different assumptions about the statistical distributions of asset/liability levels and asset yields and from alternative treatment of the non-choice items in the balance sheet. These two issues have already been touched upon in Chapter Two and our discussion concerning them can be quite brief here. Unlike the individual investor, the bank is not free to choose the level of each and every item in its portfolio. Some items, such as deposits, are mainly supply-determined, while others like the Special and Supplementary Deposits are determined by monetary authorities. The balance sheet items are therefore conventionally divided into choice and non-choice sets. Choice set items are those among which a bank can choose according to its maximising procedure when allocating its available financial resources while non-choice items are those whose levels are presumed to be given exogenously or determined independently of the bank's maximising procedure. Naturally, then, the asset demand equations we are to derive are for the choice-set assets only.

We consider two alternative specifications of the balance sheet. These two specifications differ in only one important aspect, namely, the treatment of the non-choice items. In most of the existing studies on the portfolio behaviour of UK commercial banks, the different forms of non-choice items are regarded as separate variables in the bank's portfolio decision. We shall refer to such a specification as Balance Sheet I. Although there are many reasons why the bank's portfolio choices might be affected by the composition of non-choice items, the separate treatment of non-choice items, as we have argued in Chapter Two, may be inconsistent with the spirit of the traditional mean-variance analysis [Bailey, Driscoll and Ford (1981)]. We therefore also consider an alternative specification in which the non-choice items are consolidated into a single

wealth variable termed "total disposable fund", which is defined as total liabilities minus total non-choice assets, on the assumption that the presence of non-choice items *in toto* simply determines how much is available to the bank to allocate among the choice assets. We refer to such a specification as Balance Sheet II.

Let A denote the n × 1 vector of actual levels of assets and liabilities in the portfolio and r the n × 1 vector of their rates of return. Two alternative forms of the bank's actual profit from a given portfolio can be specified from the above two balance sheets. Treating liabilities as negative assets, we may write actual profit corresponding to Balance Sheet I, $\prod_I$, as

$$\begin{aligned}\prod_I &= r' A = (e + u)' (\hat{A} + v) \\ &= e' \hat{A} + e'v + u' \hat{A} + u'v. \end{aligned} \tag{3.7}$$

where e and $\hat{A}$ are the vectors of expected values of r and A respectively; and u and v are the n × 1 vectors of random errors in r and A. Corresponding to Balance Sheet II actual profit, $\prod_{II}$, can be written as

$$\begin{aligned}\prod_{II} &= r_1' A_1 = (e_1 + u_1)' (\hat{A}_1 + v_1) \\ &= e_1' \hat{A}_1 + e_1'v_1 + u_1' \hat{A}_1 + u_1'v_1. \end{aligned} \tag{3.8}$$

where the subscript 1 refers to the choice set. The rationale for such a definition of actual profit is that, since the levels of non-choice items are independent of the bank's optimising procedure, only the choice assets and their corresponding returns will be relevant to the maximisation of the bank's objective function in (3.1).

The objective function in (3.1) depends only on the expected value of profit and its variance. However, the precise forms of both the expected value and the variance will in turn depend upon the assumptions one makes about the statistical relationship between u and v, the forecasting error vectors in actual returns and asset/liability levels. Most of

the existing studies employing the mean-variance framework on the portfolio behaviour of UK commercial banks have made the assumption that u and v are independently distributed, each with mean vector zero and constant variance-covariance matrix, under which assumption the expected values of actual profits corresponding to Balance Sheets I and II are respectively,

$$\mu_{\pi_I} = E(\prod_I) = e' \hat{A} \tag{3.9}$$

as in (2.30), and

$$\mu_{\pi_{II}} = E(\prod_{II}) = e_1' \hat{A}_1. \tag{3.10}$$

Accordingly, their variances are

$$\sigma^2_{\pi_I} = E[(\prod_1 - \mu_{\pi_I})^2] = e' V e + \hat{A}' \Omega \hat{A} + \Psi \tag{3.11}$$

where V, Ω and Ψ are as defined in (2.32) – (2.34), and

$$\sigma^2_{\pi_{II}} = E[(\prod_2 - \mu_{\pi_{II}})^2] = e_1' V^* e_1 + \hat{A}_1' \Omega^* \hat{A}_1 + \Psi^* \tag{3.12}$$

where

$$V^* = E(v_1 v_1') \tag{3.13}$$

$$\Omega^* = E(u_1 u_1') \tag{3.14}$$

$$\Psi^* = E(u_1' v_1 v_1' u_1). \tag{3.15}$$

As we discussed earlier, however, the forecasting error vectors, u and v, may not be independently distributed. It is likely that a bank would react to an unexpected change in the rate of return on one of its assets by adjusting the levels of its assets.

In what follows we consider a special form of the general statistical dependency between u and v as discussed in Section 2.5.1. In particular, we assume that forecasting errors in asset levels and their returns are statistically dependent within each set but independent across sets, i.e., u_1 is distributed independently of v_2 but not of v_1 and u_2 is distributed independently of v_1 but not of v_2. Such an assumption is justified since the bank's portfolio is partitioned into a choice set and a non-choice set, and by definition non-choice assets cannot be influence by changes in the rates of return on choice assets and *vice versa.*

Maintaining that $E(u) = E(v) = 0$ and defining

$$E(v\,u') = E\left[\begin{bmatrix} v_1u_1 & v_1u_2 & \cdots & v_1u_m & 0 & 0 & \cdots & 0 \\ v_2u_1 & v_2u_2 & \cdots & v_2u_m & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ v_mu_1 & v_m5u & \cdots & v_mu_m & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & v_{m+1}u_{m+1} & v_{m+1}u_{m+2} & \cdots & v_{m+1}u_n \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & v_nu_{m+1} & v_nu_{m+2} & \cdots & v_nu_n \end{bmatrix}\right]$$

$$= \begin{bmatrix} \Sigma_{v_1u_1} & \mathbf{0} \\ \mathbf{0} & \Sigma v_2u_2 \end{bmatrix} = \Sigma^*_{vu} \qquad (3.16)$$

we have corresponding to Balance Sheet I

$$\mu_{\pi_I} = e'\,\hat{A} + K \qquad (3.17)$$

which is the same as (2.30'), and

$$\sigma^2_{\pi_I} = E\left[(\prod_1 - \mu_{\pi_I})^2\right] = E\left[(e'\,v + u'\,\hat{A} + u'\,v - K)^2\right]$$

$$= e'\,V\,e + 2e'\,\Sigma^*_{vu}\,\hat{A} + \hat{A}'\,\Omega\,\hat{A} + 2e'\,V_u + 2\Omega'_v\,\hat{A} + \Psi - K^2 \qquad (3.18)$$

where

$$V_u = E\ (v\ v'\ u)$$

$$\Omega_v' = E\ (v'\ u\ u')$$

and V, Ω and Ψ are as defined in (2.32) - (2.34). Similarly, corresponding to Balance Sheet II we obtain,

$$\mu_{\pi II}^{*} = e_1'\ A_1 + K^{*} \tag{3.19}$$

$$\sigma^2_{\pi II}{}^{*} = e_1'\ V^{*}\ e_1 + 2e_1'\ \Sigma_{v_1 u_1} A_1 + A_1'\ \Omega^{*}\ A_1 + 2e_1'\ V^{*}_u + 2\Omega^{*}_v{}'\ A_1 + \Psi^{*} - K^{*2} \tag{3.20}$$

where

$$K^{*} = E\ (u_1'\ v_1)$$

$$V^{*}_u = E\ (v_1\ v_1'\ u_1)$$

$$\Omega^{*}_v{}' = E\ (v_1'\ u_1\ u_1') \tag{3.21}$$

and V^{*}, Ω^{*} and Ψ^{*} are as defined in (3.13) - (3.15).

The bank's static asset demand equations are obtained by maximising (3.1) subject to the balance sheet constraint. For Balance Sheet I, the constraint requires that at any moment total liabilities must equal total assets. That is,

$$i_1'\ A_1 + i_2'\ A_2 = 0. \tag{3.22}$$

The constraint for Balance Sheet II is simply that total choice assets must equal total

disposable funds, i.e.,

$$i' \hat{A}_1 = \hat{w}. \tag{3.23}$$

where

$$\hat{w} = i_2' \hat{A}_2.$$

Depending on the assumptions regarding the statistical relationship between asset/liability levels and their returns four sets of static asset demand equations are derived:

1. Static Asset Demand Equations Under the Assumption of Statistical Independence

Model S1A: Non-choice items as separate variables: maximisation of (3.1) with μ_π and σ_π^2 defined in (3.9) and (3.11), subject to the balance sheet constraint in (3.22), gives

$$\hat{A}_1 = (1/\beta)\ G\ e_1 - H^*\ \hat{A}_2 \tag{3.24}$$

where

$$H^* = G\ \Omega_{12} + H\ i_2'$$

with G and H as defined in (2.43) and (2.44).

Model S1B: Non-choice items as a composite wealth variable: maximisation of (3.1) with μ_π and σ_π^2 defined in (3.10) and (3.12), subject to the balance sheet constraint in (3.23), yields

$$\hat{A}_1 = (1/\beta)\ G\ e_1 + H\ \hat{w}. \tag{3.25}$$

2. Static Asset Demand Equations Under the Assumption of Statistical Dependence

Model S1C: Non-choice items as separate variables: maximisation of (3.1) with μ_π and σ_π^2 defined in (3.17) and (3.18), subject to the balance sheet constraint in (3.22), produces

$$\hat{A}_1 = (1/\beta)\ G^*\ e_1 - H^*\ \hat{A}_2 - G\ \Omega_{\nu_1} \qquad (3.26)$$

where

$$G^* = G\ (I - \beta\ \Sigma'_{\nu_1 u_1})$$

with $\Sigma'_{\nu_1 u_1}$ and Ω_ν as defined in (3.19).

Model S1D: Non-choice items as a composite wealth variable: maximisation of (3.1) with μ_π and σ_π^2 defined in (3.19) and (3.20), subject to the balance sheet constraint in (3.23), yields

$$\hat{A}_1 = (1/\beta)\ G^*\ e_1 + H\ \hat{w} - G\ \Omega^*_\nu \qquad (3.27)$$

where Ω^*_ν is defined in (3.21).

The above static asset demand equations possess several interesting properties:

(1) The expected returns on all choice assets appear in each asset equation. Therefore, the quantity demanded for each choice asset depends on its own expected return as well as on the expected returns on all other assets in the choice set;

(2) Any column of G sums to zero. The implication for models S1A and S1B of this result, which follows directly from the balance sheet constraint, is that the impact of a change in the expected return on any choice asset must sum to zero across all equations. As we have shown in Chapter Two the

matrix G is also symmetric and therefore, any row of G also sums to zero, implying that if the expected returns on all choice assets increase by the same amount the desired asset/liability levels will not be affected. However, as is also shown in Chapter Two the matrix G^* is not symmetric, although any column or row of G^* still sums to zero. This implies for models S1C and S1D that, while the impact of a change in the expected return on any one choice asset must sum to zero across equations and an equal amount increase in the expected returns on all the choice assets will not alter the bank's desired asset levels, the expected return responses will not be symmetric;

(3) The elements of the column vector H sum to unity so that the balance sheet identity holds. For models S1A and S2C, this result suggests that if only the levels of non-choice items change the changes will have to be apportioned across all the choice items. Similarly, for models S1B and SD, it implies that a *ceteris paribus* change in total disposable funds is apportioned among all the choice items;

(4) If one assumes that the covariances between asset/liability levels and their returns are constant over time models S1C and S1D will specify a constant in each of the static asset demand equations and the sum of the constants across the equations will be zero.

Remembering that $\hat{w} = i' A_2$ we can rewrite the four models more compactly as

$$\hat{A}_1 = \alpha + G e_1 + H A_2 \qquad (3.28)$$

where α is an m × 1 vector; G is an m × m matrix; H is an m × (n-m) matrix for Models 1A and 1C, where n-m is the number of non-choice items, and is an m × 1 vector for Models 1B and 1D; and similarly, A_2 is a (n-m) × 1 vector for Models 1A and 1C, and is a scalar equal to $\hat{w}$ for Models 1B and 1D. Specifically, the following restrictions apply to the four models:

Model S1A:

(1) $\alpha = 0$; (3.29a)

(2) $i' G = (1/\beta)\ i' G = 0'$; (3.29b)

(3) $G\ i = (1/\beta)\ G\ i = 0$; (3.29c)

(4) $i' H = i' H^* = -i'$; (3.29d)

Model S1B:

(1) $\alpha = 0$; (3.30a)

(2) $i' G = (1/\beta)\ i' G = 0'$; (3.30b)

(3) $G\ i = (1/\beta)\ G\ i = 0$; (3.30c)

(4) $i' H = i' H = -1$; (3.30d)

Model S1C:

(1) $i' \alpha = i' G \Omega_V = 0$; (3.31a)

(2) $i' G = (1/\beta)\ i' G^* = 0'$; (3.31b)

(3 $G\ i = (1/\beta)\ G^*\ i = 0$; (3.31c)

(4) $i' H = i' H^* = -i'$; (3.31d)

Model S1D:

(1) $i' \alpha = i' G \Omega_V^* = 0$; (3.32a)

(2) $i' G = (1/\beta)\ i' G^* = 0'$; (3.32b)

(3) $G\ i = (1/\beta)\ G^*\ i = 0$; (3.32c)

(4) $i' H = i' H = -1$. (3.32d)

We note that Model S1A is nested within Model S1C while Model S1B is nested within Model S1D with $\alpha = 0$. Further, given that $\hat{w} = i_2 A_2$ we observe that Model S1B and Model S1D are in turn nested within Model S1A and Model S1C respectively with $h_{i1} = h_{i2} = \ldots = h_{im} = h_i$ for all i =1, 2, ... n.

3.4 THE BANK'S ADJUSTMENT PROCESS

The asset demand equations derived in the last section are the static equilibrium solutions. The levels of assets that the bank inherits at the beginning of each decision period are a result of the decisions taken by the bank and of the deposit flows in the last period. In the absence of transactions costs the bank will continuously adjust its portfolio towards the optimal static position. In the real world, however, not only asset transactions are costly, but assets also vary in liquidity characteristics, hence making continuous portfolio adjustment very difficult, if not impossible. Thus, in the presence of transactions costs instantaneous adjustment of actual to the static equilibrium value of an asset will not be optimal.

If the bank does adjust its actual portfolio towards the optimal position, it will incur certain adjustment costs, either explicit or implicit. In the case of buying and selling securities such costs might include commissions as well as capital losses. The more quickly a bank adjusts its portfolio, the greater one would expect the transactions costs to be. For example, bank advances are usually committed for a certain period of time. If a bank wishes to reduce its stock of advances before the agreed repayment is due it will incur additional costs in terms of damaging the future customer relationship and/or explicit financial penalties for breaking an agreement. As there are positive transactions costs involved in either increasing or decreasing the holding of an asset, we may, to follow the suggestion of Feige (1967), approximate such costs by the quadratic function,

$$C_{1i} = \gamma_i \, (a_{1it} - a_{1it-1})^2 \qquad (3.33)$$

where γ_i is a proportional unit transactions cost parameter, a_{1it} and a_{1it-1} are the actual level of choice asset i at time t and t-1 respectively. It should be emphasised that the quadratic cost function is used here primarily on the basis of expediency. Although a quadratic cost function may be regarded as a good first approximation to buying and

selling commissions of securities, it is doubtful whether all portfolio adjustment costs are symmetric. To take advances again as the example, the costs involved in granting an additional unit of loans to a customer will certainly be different from the costs resulting from recalling a unit of loans from a customer. However, since our prime concern here is to provide a sound economic justification for employing a general stock adjustment process to explain dynamic bank portfolio behaviour, the deficiencies of the quadratic cost function should not worry us unduly.

However, at any moment there are also explicit and implicit costs involved in deviating from the static equilibrium portfolio. Such costs encompass the opportunity costs of forgone earnings and the explicit and/or implicit costs of possessing either a too risky or a too conservative portfolio. For instance, if the holding of liquid assets is in excess of the desired static level there is an opportunity cost in terms of forgone earnings from investing in less liquid but more profitable assets. Or, if the level of advances exceeds the static equilibrium level, the bank might have to borrow at very high rates to meet expected deposit withdrawals. In choosing its adjustment strategy the bank will have to balance transactions costs against the costs of being out of static equilibrium. We assume that the cost of being out of equilibrium for asset i is of the form

$$C_{2i} = \sum_{j=1}^{m} \theta_{ij} \, (a^{*}_{1it} - a_{1it})(a^{*}_{1jt} - a_{1jt}). \qquad (3.34)$$

where a^{*}_{1it} and a^{*}_{1jt} are the desired levels of choice assets i and j at time t, and θ_{ij} is a cost parameter. This cost function is also of the quadratic type but it differs from the simple quadratic cost function in (3.33) in one important aspect: it assumes that the cost of disequilibrium in any one asset depends also on the distribution of the disequilibrium gaps in the remaining assets. The rationale for employing such a cost function can be best illustrated by the illuminating example given in Sharpe (1973).

Suppose the portfolio consists of four assets with assets 1 and 2 being the liquid

assets and assets 3 and 4 being the highly illiquid. Consider two possible cases in Table 3.1. Case 1 portrays a situation of what might be termed "liquidity-adequate" portfolio disequilibrium, where the shortage of liquid asset 1 is compensated by the excess of liquid asset 2. However, Case 2 represents a situation of very serious liquidity shortage since both liquid assets 1 and 2 fall below their optimal static equilibrium levels. It seems appropriate, then, to assume that Case 1 involves a lower cost of portfolio disequilibrium than Case 2, indicating that the cost of being out of equilibrium for asset i, C_{2i}, is dependent upon the distribution of gaps between actual and static equilibrium levels of the other assets in the portfolio.

Table 3.1
Portfolio Disequilibria

	$a^*_{1t} - a_{1t}$	$a^*_{2t} - a_{2t}$	$a^*_{3t} - a_{3t}$	$a^*_{4t} - a_{4t}$
Case 1	20	-20	-20	20
Case 2	20	20	-20	-20

One way of interpreting (3.34) is to assume that the costs of disequilibrium in asset i follow the simple quadratic form

$$C_{2i} = \theta_i \, (a^*_{1it} - a_{1it})^2 \tag{3.35}$$

but that the opportunity cost coefficient, θ_i, consists of a constant, c_i, and a variable component which is a function of the disequilibrium gaps in the remaining assets:

$$\theta_i = c_i + \frac{\sum_{i \neq j} \theta_{ij} \, (a^*_{1it} - a_{1jt})}{a^*_{1it} - a_{1it}}. \tag{3.36}$$

In fact, (3.35) is the exact functional form assumed by Sharpe (1973) to represent the cost of being out of equilibrium for asset i.

Equations (3.33) and (3.34) are the transactions costs and costs of being out of static equilibrium for one particular asset i. However, because of the balance sheet constraint, an increase (decrease) in asset i must be offset by a reduction (increase) of an equal amount in the remaining choice assets. Thus, to obtain the total costs of any portfolio adjustment we must sum these costs across the m choice assets. Letting C be the total costs of adjustment and using matrix notation we have

$$C = (A_{1t} - A_{1t-1})' \Gamma (A_{1t} - A_{1t-1}) + (\hat{A}_{1t} - A_{1t})' \Theta (\hat{A}_{1t} - A_{1t}) \tag{3.37}$$

where

$$\Gamma = \begin{bmatrix} \gamma_1 & 0 & 0 & \cdots & 0 \\ 0 & \gamma_2 & 0 & \cdots & 0 \\ 0 & 0 & \gamma_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \gamma_m \end{bmatrix}$$

is a m-dimensioned diagonal matrix of transactions cost coefficients and

$$\Theta = \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} & \cdots & \theta_{1m} \\ \theta_{21} & \theta_{22} & \theta_{23} & \cdots & \theta_{2m} \\ \theta_{31} & \theta_{32} & \theta_{33} & \cdots & \theta_{3m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \theta_{m1} & \theta_{m2} & \theta_{m3} & \cdots & \theta_{mm} \end{bmatrix}$$

is a m-dimensioned square matrix of disequilibrium cost coefficients. It is assumed that Θ has full rank. From (3.35) and (3.36) we can immediately appreciate that Θ is symmetric with $\theta_{ii} = c_i$ for i = 1, 2, ..., m.

Since the bank's balance sheet identity holds at any moment the following constraint must also hold,

$$i_1' A_{1t} = i_1' \hat{A}_{1t} \tag{3.38}$$

namely, the sum of actual choice asset holdings must equal the sum of desired asset holdings. The bank will choose its portfolio adjustment strategy so as to minimise the total adjustment costs subject to the balance sheet constraint. Forming the Lagrangean

$$L = (A_{1t} - A_{1t-1})' \Gamma (A_{1t} - A_{1t-1}) + (\hat{A}_{1t} - A_{1t})' \Theta (\hat{A}_{1t} - A_{1t})$$

$$+ 2\lambda (i_1' A_{1t} - i_1' \hat{A}_{1t}) \qquad (3.39)$$

and differentiating it with respect to the choice asset vector A_{1t} and λ yield the following first-order conditions,

$$\frac{\partial L}{\partial A_{1t}} = (\Gamma + \Theta) A_{1t} - \Gamma A_{1t-1} - \Theta \hat{A}_{1t} + \lambda i_1 = 0 \qquad (3.40)$$

$$\frac{\partial L}{\partial \lambda} = i_1' A_{1t} + i_1' \hat{A}_{1t} = 0 \qquad (3.41)$$

which in matrix notation is

$$\begin{bmatrix} \Gamma + \Theta & i_1 \\ i_1' & 0 \end{bmatrix} \begin{bmatrix} A_{1t} \\ \lambda \end{bmatrix} = \begin{bmatrix} \Theta \hat{A}_{1t} + \Gamma A_{1t-1} \\ i_1' \hat{A}_{1t} \end{bmatrix} \qquad (3.42)$$

Since both Γ and Θ are matrices of full rank and hence $(\Gamma + \Theta)$ is non-singular there exists a unique solution for (3.42). Letting the inverse of the coefficient matrix on the right-hand side of (3.42) be M and partitioning M conformably we obtain the solutions for A_{1t} and λ as

$$A_{1t} = (M_{11} \Theta + M_{12} i_1') \hat{A}_{1t} + M_{11} \Gamma A_{1t-1} \qquad (3.43)$$

$$\lambda = (M_{21} \Theta + M_{22} i_1') \hat{A}_{1t} + M_{21} \Gamma A_{1t-1} \qquad (3.44)$$

where

$$M_{11} = (\Gamma + \Theta)^{-1} - \frac{(\Gamma + \Theta)^{-1}\, i_1\, i_1'\, (\Gamma + \Theta)^{-1}}{i_1'\, (\Gamma + \Theta)^{-1}\, i_1} \tag{3.45}$$

$$M_{12} = \frac{(\Gamma + \Theta)^{-1}\, i_1}{i_1'\, (\Gamma + \Theta)^{-1}\, i_1} \tag{3.46}$$

$$M_{21} = \frac{i'\, (\Gamma + \Theta)^{-1}}{i_1'\, (\Gamma + \Theta)^{-1}\, i_1} \tag{3.47}$$

$$M_{22} = - \frac{1}{i_1'\, (\Gamma + \Theta)^{-1}\, i_1} \tag{3.48}$$

Subtracting A_{1t-1} from both sides of (3.43) and defining

$$\Delta A_{1t} = A_{1t} - A_{1t-1}$$

yield the bank's optimal portfolio adjustment equations:

$$\Delta A_{1t} = (M_{11}\, \Theta + M_{12}\, i_1')\, A_{1t} - (I - M_{11}\, \Gamma)\, A_{1t-1}. \tag{3.49}$$

From the definition of an inverse matrix

$$\begin{bmatrix} (\Gamma + \Theta) & i_1 \\ i_1' & 0 \end{bmatrix} \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0' & 1 \end{bmatrix} \tag{3.50}$$

and

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} (\Gamma + \Theta) & i_1 \\ i_1' & 0 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0' & 1 \end{bmatrix} \tag{3.51}$$

Expanding (3.50) and (3.51) we have

$$(\Gamma + \Theta)\ M_{11} + i_1\ M_{21} = I \tag{3.52}$$

$$(\Gamma + \Theta)\ M_{12} + i_1\ M_{22} = 0 \tag{3.53}$$

$$i_1'\ M_{11} = 0' \tag{3.54}$$

$$i_1'\ M_{12} = 1 \tag{3.55}$$

and

$$M_{11}\ (\Gamma + \Theta) + M_{12}\ i_1' = I \tag{3.56}$$

$$M_{11}\ i_1 = 0 \tag{3.57}$$

$$M_{21}\ (\Gamma + \Theta) + M_{22}\ i_1' = 0' \tag{3.58}$$

$$M_{21}\ i_1 = 1. \tag{3.59}$$

Rearranging (3.56) gives

$$I - M_{11}\ \Gamma = M_{11}\ \Theta + M_{12}\ i_1' \tag{3.60}$$

which enables us to rewrite (3.49) as the general stock adjustment form

$$\Delta A_{1t} = B\ (A_{1t} - A_{1t-1}) \tag{3.61}$$

where

$$B = M_{11}\ \Theta + M_{12}\ i_1' = I - M_{11}\ \Gamma. \tag{3.62}$$

(3.61) is truly in the spirit of a "general disequilibrium" framework for the dynamics of adjustment to a "general equilibrium" system advocated by Brainard and Tobin, since "the adjustment of any one asset holding" does indeed depend "not only on its own deviation but also on the deviations of other assets" [Brainard and Tobin (1968), p. 106]. In particular,

$$i_1' \, B = i_1' \, M_{11} \, \Theta + i_1' \, M_{12} \, i_1 = i_1' \qquad (3.63)$$

since $i_1' \, M_{11} \, \Theta = 0$. Thus, the columns of the adjustment coefficient matrix, all sum to unity. We may note in passing that B is in general not symmetric.

Substituting (3.24) - (3.27) for A_{1t} into (3.62) we obtain correspondingly four set of dynamic models of bank portfolio adjustment:

1. Dynamic Asset Demand Equations under the Assumption of Statistical Independency

Model D1A: Non-choice items as separate variables

$$\Delta A_{1t} = [(1/\beta)B \, G] \, e_1 - (B \, H^*) \, A_{2t} - B \, A_{1t-1}$$

$$= \Phi \, e_1 - \Lambda \, A_{2t} - B \, A_{1t-1} \qquad (3.64)$$

Model D1B: Non-choice items as a composite wealth variable

$$\Delta A_{1t} = [(1/\beta)B \, G] \, e_1 + (B \, H) \, \hat{w}_t - B \, A_{1t-1}$$

$$= \Phi \, e_1 + \Upsilon \, \hat{w}_t - B \, A_{1t-1} \qquad (3.65)$$

2. Dynamic Asset Demand Equations under the Assumption of Statistical Dependency

Model D1C: Non-choice items as separate variables

$$\Delta A_{1t} = [(1/\beta)]B\ G^*]\ e_1 - (B\ H^*)\ A_{2t} - B\ G\ \Omega_{\nu_1} - B\ A_{1t-1}$$

$$= K_1 + \Phi^*\ e_1 - \Lambda\ A_{2t} - B\ A_{1t-1} \qquad (3.66)$$

Model D1D: Non-choice items as a composite wealth variable

$$\Delta A_{1t} = [(1/\beta)B\ G^*]\ e_1 + (B\ H)\ \hat{w}_t - B\ G\ \Omega^*_{\nu} - B\ A_{1t-1}$$

$$= K_2 + \Phi^*\ e_1 + \Upsilon\ \hat{w}_t - B\ A_{1t-1} \qquad (3.67)$$

where

$$\Phi = (1/\beta)B\ G \qquad (3.68a)$$

$$\Phi^* = (1/\beta)B\ G^* \qquad (3.68b)$$

$$\Lambda = B\ H^* \qquad (3.68c)$$

$$\Upsilon = B\ H \qquad (3.68d)$$

$$K_1 = B\ G\ \Omega_{\nu_1} \qquad (3.68e)$$

$$K_2 = B\ G\ \Omega^*_{\nu}. \qquad (3.68f)$$

It is important to note that when a symmetric matrix is multiplied by another symmetric matrix the resulting matrix is generally non-symmetric. This implies that the coefficient matrix of e_1, Φ, will in general be non-symmetric and the symmetry restrictions which are embeded in Models S1A and S1B no longer hold in the dynamic models. It may be shown, however, that if a square matrix, each column of which

sums to c_1, is post-multiplied by a column vector, whose elements sum to c_2, then the element of the product vector will sum to c_1c_2. Using this result we can easily establish the following properties, similar to those pertaining to the static equilibrium models, of the above dynamic models:

(1) $i_1' \Phi = 0'$; (3.69a)

(2) $\Phi i_1 = 0$; (3.69b)

(3) $i_1' \Phi^* = 0'$; (3.69c)

(4) $\Phi^* i_1 = 0$; (3.69d)

(5) $i_1' \Lambda = - i_1'$; (3.69e)

(6) $i_1' \Upsilon = - 1$; (3.69f)

(7) $i_1' K_1 = 0$; (3.69g)

(8) $i_1' K_2 = 0$. (3.69h)

In addition, (3.63) implies that the impact of a change in the level of any one asset lagged one period must sum to unity across all equations:

(9) $i_1' B = i_1'$. (3.69i)

As with the static models we can rewrite the four dynamic models in the more compact form:

$$\Delta A_{1t} = K + \Phi\, e_1 + \Lambda\, A_{2t} - \mathbf{B}\, A_{1t-1} \qquad (3.70)$$

where K is an m × 1 vector of constants, Φ is an m × m matrix, Λ is an m × (n-m) matrix for Models D1A and D1C, and an m × 1 vector for Models D1B and D1D; similarly, A_{2t} is a (n-m) × 1 vector for Models D1A and D1C and a scalar for Models D1B and D1D; and $\mathbf{B}$ is an m × m matrix as defined in (3.62). For each of the four models the following restrictions are to be imposed upon their coefficients:

Model D1A:

(1) $K = 0$; (3.71a)

(2) $i_1' \Phi = i_1' \Phi = 0'$; (3.71b)

(3) $\Phi\, i_1 = \Phi\, i_1 = 0$; (3.71c)

(4) $i_1' \Lambda = i_1' \Lambda = - i_1'$; (3.71d)

(5) $i_1' \mathbf{B} = i_1' \mathbf{B} = i_1'$; (3.71e)

Model D1B:

(1) $K = 0$; (3.72a)

(2) $i_1' \Phi = i_1' \Phi = 0'$; (3.72b)

(3) $\Phi\, i_1 = \Phi\, i_1 = 0$; (3.72c)

(4) $i_1' \Lambda = i_1' \gamma = - 1$; (3.72d)

(5) $i_1' \mathbf{B} = i_1' \mathbf{B} = i_1'$; (3.72e)

Model D1C:

(1) $i_1' K = i_1' K_1 = 0$; (3.73a)

(2) $i_1' \Phi = i_1' \Phi^* = 0'$; (3.73b)

(3) $\Phi\, i_1 = \Phi^*\, i_1 = 0$; (3.73c)

(4) $i_1' \Lambda = i_1' \Lambda = - i_1'$; (3.73d)

(5) $i_1' \mathbf{B} = i_1' \mathbf{B} = i_1'$; (3.73e)

Model D1D:

(1) $i_1' K = i_1' K_2 = 0;$ (3.74a)

(2) $i_1' \Phi = i_1' \Phi^* = 0';$ (3.74b)

(3) $\Phi i_1 = \Phi^* i_1 = 0;$ (3.74c)

(4) $i_1' \Lambda = i_1' \gamma = -1;$ (3.74d)

(5) $i_1' B = i_1' B = i_1'.$ (3.74e)

3.5 AGGREGATION AND ECONOMETRIC SPECIFICATIONS

The models derived in Sections 3.3 and 3.4 apply to the theoretical portfolio behaviour of an individual bank. To estimate and test these theoretical models a number of problems arise. Firstly, the models include, in addition to the expected rate of return on each of the choice items, expected values of the non-choice items, which are also not observable. In order to produce empirically implementable equations assumptions must be made regarding these unobservable variables. More importantly, the theoretical models depict the portfolio behaviour of an individual bank while such data as generally available usually relate to groups of banks, and it will be necessary to perform aggregation over banks.

There are in general two approaches to the problem of aggregation over decision-making units. The oldest approach, commonly employed in the studies of consumer behaviour, is to ignore the problem all together and proceed to formulate the aggregate demand equations essentially on the assumption that the theory developed corresponds to the aggregate as being *the* decision-making unit [Brown and Deaton (1972)]. However, since our models are derived from the optimising behaviour of an individual bank and, indeed, as one of the aims of our study is to provide a suitable microeconomic foundation for the analysis of the banking sector, such an approach seems inappropriate. The alternative, as frequently espoused in portfolio studies, is to assume that all decision-making units have identical expectations of interest rates, attitude towards risk and perceived variance-covariance matrices, so that behaviour at the aggregate is equivalent to the behaviour at the individual level summed over the number of units forming the aggregate. It might be argued that such uniformity is hardly consistent with real economic life and accordingly inferences based on such assumptions are unlikely to conform with actual pattern of behaviour. However, such an argument needs not concern us unduly for the following reasons. With regard to uniformity the data against which we are to test our models are those for the London clearing banks the members of which have often been regarded as homogeneous. Indeed, as Revell (1973) points out,

"the balance sheets of London clearing banks may well differ in details and in some substantial points from those of Scottish clearing banks, but all London clearing banks will have very similar balance sheets..." [p. 135]. More importantly, as we shall see later, the assumptions necessary for aggregation need not to be so restrictive.

Specifically, we assume that

(1) All banks face the same decision period and maximise the same objective function of the form in (3.1);

(2) All banks have the same expectations about future rates of return on assets so that the expected rates of return and their subjective variance-covariance matrix is the same for all banks;

(3) All banks face the same transactions cost coefficients and the same disequilibrium cost coefficients.

Denoting the vectors A_1 and A_2 for the jth bank by A_1^j and A_2^j respectively, and defining

$$A_1 = \sum_{j=1}^{N} A_1^j \qquad \text{and} \qquad A_2 = \sum_{j=1}^{N} A_2^j$$

where N is the total number of banks in question, we may proceed to write the aggregate relation for the static models as

$$A_1 = \alpha + G\, e_1 + H\, A_2. \tag{3.75}$$

Consequently, the aggregate relation for the dynamic models can be written as

$$\Delta A_{1t} = K + \Phi\, e_1 + \Lambda\, A_{2t} - B\, A_{1t-1} \tag{3.76}$$

(3.75) and (3.76) describe the theoretical portfolio behaviour of the banks. We further assume that actual behaviour departs randomly from this theoretical behaviour, so that

$$A_1 = \alpha + G\, e_1 + H\, \hat{A}_2 + \epsilon_{st} \qquad (3.77)$$

$$\Delta A_{1t} = K + \Phi\, e_1 + \Lambda\, \hat{A}_{2t} - B\, A_{1t-1} + \epsilon_{dt} \qquad (3.78)$$

where ϵ_{st} and ϵ_{dt} are vectors of normally distributed random variables with means zero and covariance matrices Σ_s and Σ_d. We note that the disturbance terms in (3.77) and (3.78) can be interpreted as representing random errors resulting from the optimisation process. Therefore, we can assume that ϵ_{st} and ϵ_{dt} in the aggregate demand equations are independent of the other stochastic components in the model and are thus independent of the explanatory variables in the regression equations. It should be noted, however, that assuming error terms for both the static and dynamic models is theoretically inconsistent. If the static model specification in (3.77) is correct, then the dynamic model in (3.78) is incorrect, or at least redundant. Conversely, if the dynamic model is the appropriate specification, the static model is then misspecified. In other words, only one of the above specifications can be correct. Since which one to estimate is mainly an empirical matter and since one of the purposes in this study is to compare the models empirically, we shall estimate both the static and dynamic models.

Next arises the problem of defining A_1 and A_2. Each of the equations in (3.77) and (3.78) contains a constant term, the expected rates of return on all the choice items, the expected holdings of all non-choice items and the lagged variables for all the choice items. As we have seen in Chapter One, the balance sheet of a commercial bank usually contains many items, and the number of items differs not only from one bank to another but also for the same bank from one period to another, due to changes in the number of assets and liabilities available to commercial banks as well as changes in reporting and accounting practice. The large number of items in the balance sheet presents two problems for the estimation of the model in (3.77) and (3.78). First, data on the rates of return are not available for all the items in the balance sheet. Indeed, some of the items reported in the balance sheet might be themselves composites of

subgroups of assets and liabilities on the shares of which no information is available. Second, even if data relating to the rates of return on alternative assets and liabilities were obtained on a completely disaggregated basis, the large number of parameters in the model (3.77) and (3.78) are likely to preclude simultaneous estimation of all the choice items. Therefore, aggregation over the balance sheet items is necessary.

Our study covers the period between 1959 and 1983. During this period there were two major changes in UK banking statistics [see Bank of England (1972a, 1975a, 1985)]. The first major change took place in September 1971 in the wake of Competition and Credit Control. Before September 1971 there were only a small number of assets and liabilities in the balance sheets of the commercial banks, reflecting the fewer financial instruments available to the banks. During this period the assets of the London clearing banks were classified under three headings: liquid assets, investment and advances. Liquid assets included coins, notes and balances with the Bank of England, money at call and short notice, Treasury bills and commercial bills. Since 1960 the Special deposit with the Bank of England was added to the asset side of the balance sheet. The liabilities of the commercial banks were distinguished between current accounts, deposit accounts and other accounts. It should be noted that statistics during this period for the London and Scottish clearing banks and for Northern Ireland banks were arranged differently from those for other banks. A number of factors explain this difference. Firstly, the clearing banks' figures were based on monthly balance sheet statements published by the banks themselves, while the other banks' figures were derived from purpose-build statistical returns. More importantly, unlike the other banks, the clearing banks observed certain conventional cash and liquidity ratios. Furthermore, as the clearing banks concentrated on domestic branch banking, their assets and liabilities in foreign currencies were too small to warrant separate identification [Bank of England (1972a)].

Following the introduction in September 1971 of the new arrangements for the control of credit the Bank of England made substantial changes in the presentation of

banking statistics published in the statistical annexes to its quarterly bulletin. Further changes were made in May 1975 [see Bank of England (1975a)]. The changes occurred mainly in the following three aspects: (1) number and type of assets and liabilities shown, (2) definitions of some items and (3) treatment of assets and liabilities in foreign currencies. While some of the changes in the number and type of assets and liabilities were due to reclassification of assets and liabilities most of the changes in this respect largely reflect changes in the number and type of financial instruments available to the clearing banks. These changes themselves reflect *a priori* structural changes in the portfolio behaviour of the commercial banks and periods with different number and type of assets and liabilities should be estimated separately.

Changes in the definitions of some of the items present a problem for the estimation of the models and, if left untreated, they may introduce measurement errors and result in the misspecification of the models. It should be noted, however, these definitional changes usually occur between items within a broad group of assets and liabilities which have similar liquidity and risk characteristics. If only the aggregate of these assets and liabilities are to be used in the models these definitional changes should not affect the correct estimation of the models. Most of the changes in the definitions of items during our sample period fall into this category.

The third major difference in the presentation of balance sheet data of various periods is in the treatment of assets and liabilities in foreign currencies. Since October 1971 separate figures were given for assets and liabilities in currencies other than sterling. Theoretically, assets (liabilities) in different currencies should be regarded as different types of assets (liabilities) because of the presence of foreign exchange risks. However, the empirical implementation of such an idea would require disaggregated data of assets and liabilities on an individual currency basis. Even if data on an individual currency basis is available the large number of assets and liabilities in the models would once again preclude the simultaneous estimation of all the choice items. To avoid these problems we shall focus our attention on the assets and liabilities in sterling only and

treat foreign currency assets and liabilities as largely a separate portfolio. Following Spencer (1984) we define a bank's net foreign currency position (NP) as its foreign currency assets net of liabilities. Unlike Spencer (1984), however, we shall introduce NP as a non-choice variable into the bank's portfolio. Given that until 1979 foreign currency control was in existence and assets in foreign currencies had to be covered this approach can be justified. In fact, during the 1972-1983 period NP was only a tiny fraction of the commercial banks' sterling portfolio, suggesting that assets in foreign currencies were usually matched by foreign currency liabilities.

Based on these considerations A_1 and A_2 are defined as follows:

Before September 1971

A_1 = (CM TB GB CB AD)

A_2 = (CN SD EA DD TD)

After September 1971

A_1 = (CM TB GB CB AD CD LL)

A_2 = (CN SD SL NP)

where

CM = Money at call and short notice

TB = Treasury bills

GB = British government stocks and other investments

CB = Commercial bills and other UK bills

AD = Advances and loans other than to nationalised industries

CD = Negotiable sterling certificates of deposits

LL = Sterling loans to UK local authorities

SD = Special and supplementary deposits

CA = Cash, balances with the Bank of England and balancing items

EA = Advances to nationalised industries (for pre-1971 only)

DD = Demand deposit

TD = Time deposits

SL = Total sterling liabilities

NP = Net foreign currency position

Detailed definitions of these variables are contained the Appendix. Thus, the system in (3.77) and (3.78) will contain five equations for the period prior to September 1971 and seven equations for the period after September 1971.

However, before we can estimate the system of equations in (3.77) and (3.78) one more assumption is required. The vectors of expected rates of return on the choice variables, e_1, and the expected values of the non-choice variables, A_{2t}, are unobservable. We approximate the expected rates of return by the monthly averages of actual rates, $\hat{r}_1$ and replace the expected values of the non-choice items by their actual values A_{2t}. Since the time interval of the data is very short (monthly), it is believed that this simple formulation provides a reasonable approximation. Thus, (3.77) and (3.78) become

$$A_1 = \alpha + G\,\hat{r}_1 + H\,A_2 + \epsilon_{st} \qquad (3.79)$$

$$\Delta A_{1t} = K + \Phi\,\hat{r}_1 + \Lambda\,A_{2t} - B\,A_{1t-1} + \epsilon_{dt} \qquad (3.80)$$

with r_1 = (CMR TBR GBR ADR CBR LLR CDR) where

CMR = Rate of return on Money at call and short notice

TBR = Rate of return on treasury bills

GBR = Rate of return on government bonds and other investment

ADR = Rate of return on advances

CBR = Rate of return on commercial bills

LLR = Rate of return on loans to local authorities

CDR = Rate of return on certificate of deposits

During the pre-1971 period banks were subject to the Special Deposit Scheme and credit ceilings imposed by the Bank of England. While credit ceilings were abolished after the introduction of Competition and Credit Control the Special Deposits Scheme remained in the post-1971 period. In order to test whether the credit ceilings and the Special Deposit Scheme have had any effects on the portfolio behaviour of the commercial banks two dummy variables are added to the system of equations in (3.79) and (3.80) so that we have

$$A_1 = \alpha + G\,\hat{r}_1 + H\,A_2 + f\,D + \epsilon_{st} \tag{3.81}$$

$$\Delta A_{1t} = K + \Phi\,\hat{r}_1 + \Lambda\,A_{2t} - B\,A_{1t-1} + F\,D + \epsilon_{dt} \tag{3.82}$$

with $D = (D_{SD}\ D_{CC})'$ where D_{SD} and D_{CC} are the dummy variables for Special Deposits and credit ceilings respectively and f and F are m × 2 matrices of dummy coefficients. The dummy variables take the value of 1 when the schemes were in operation and 0 when they were not. It is obvious that

$$i_1'\,f = 0' \tag{3.83}$$

$$i_1'\,F = 0'. \tag{3.84}$$

Further, we note that the credit ceilings dummy applies only to the pre-1971 period.

Due to the presence of balance sheet constraint the system in (3.81) and (3.82) are

singular so that only m-1 of the m equations can be estimated independently. As shown by Barton (1969) and Berndt and Savin (1975) the full information maximum likelihood (FIML) estimators are numerically invariant to the choice of which of the m equations is dropped from the estimation. Let ϵ^{*}_{st}, ϵ^{*}_{dt}, Σ^{*}_{st} and Σ^{*}_{dt} denote respectively the disturbance vectors and covariance matrices after the deletion of one equation from the system (3.81) and (3.82). Given that the vectors ϵ_{st} and ϵ_{dt} are normally distributed ϵ^{*}_{st} and ϵ^{*}_{dt} are also independently normally distributed with mean vectors zero and non-singular covariance matrix Σ^{*}_{st} and Σ^{*}_{dt}. After the deletion of one equation from the systems the m-1 equations can be estimated independently by FIML and the estimates for the mth equation can then be derived using the balance sheet restrictions. In general, the FIML estimators are consistent, asymptotically efficient and asymptotically normally distributed. In addition, the use of FIML estimators provides a convenient way for employing the likelihood ratio statistic to test alternative hypotheses about the system.

3.6 SOME EMPIRICAL RESULTS FOR THE LONDON CLEARING BANKS: 1959-1983

There are five choice items in the pre-1971 period and seven choice items in the post-1971 period. To circumvent the singularity problem we drop one equation from our estimation. The same equation — the Government bond equation — is dropped from both the static and dynamic models in both periods. Estimates for the Government bond equation can then be derived from the balance sheet constraint. In some cases, even individual coefficient estimates for the Government bond equation can be derived from the restrictions. The data used are the monthly balance sheet data of the London clearing banks. Data sources and detailed definitions of the variables are given in Appendix I. Our main purpose in this section is not to provide final estimates for the four static and dynamic models developed in Sections 3.3 and 3.4, but rather to conduct various hypothesis tests. These include the testing of the theoretical restrictions of homogeneity and symmetry, the hypothesis of non-choice items as a single wealth variable and the validity of the Special and Supplementary Deposit and credit ceilings dummy variables. Since all these tests can be performed in the framework of nested hypothesis testing we shall employ in this section the classical likelihood ratio test only. Non-nested hypothesis tests, such as the testing of the liquid asset ratio constraints, will be deferred until Chapter Six. We discuss the static and the dynamic models separately.

3.6.1 Static Models

3.6.1.1 FIML Estimates and Hypothesis Tests Assuming No Vector Autocorrelation

The system of equations in (3.81) are estimated by FIML subject to the restrictions in (3.29) - (3.32) and (3.83). The restrictions (3.29) - (3.32) correspond to Models S1A - S1D and FIML results for these four models are presented in Tables 3.2A - 3.5B.

Table 3.2A
Maximum Likelihood Estimates for Model S1A (1959:01 to 1971:06)

	CM	TB	CB	AD	GB
CMR	98.88	20.78	48.03	-147.43	-20.26
	(1.96)	(0.51)	(2.06)	(3.16)	
TBR	20.78	-218.31	56.17	193.67	-52.31
	(0.51)	(2.72)	(1.84)	(3.14)	
CBR	48.03	56.17	1.96	-120.74	14.58
	(2.06)	(1.84)	(0.06)	(3.87)	
ADR	-147.43	193.67	-120.74	-36.16	110.66
	(3.16)	(3.14)	(3.87)	(0.36)	
GBR	-20.26	-52.31	14.58	110.66	-52.67
CN	-0.35	0.71	-0.32	-1.45	0.41
	(6.68)	(4.30)	(6.13)	(4.28)	
SD	0.53	0.09	0.23	-1.87	0.02
	(2.51)	(0.24)	(1.71)	(1.60)	
EA	0.19	-0.11	-0.37	-1.79	1.08
	(0.81)	(0.17)	(1.43)	(0.95)	
DD	-0.05	0.50	0.05	0.15	0.35
	(0.90)	(6.02)	(1.36)	(0.97)	
TD	0.48	-0.75	0.21	1.34	-0.28
	(9.00)	(7.81)	(5.91)	(6.33)	
D_{SD}	-110.94	150.45	-79.63	58.95	-18.83
	(2.93)	(2.01)	(3.34)	(0.36)	
D_{CC}	-12.02	-29.50	47.09	157.55	-163.12
	(0.33)	(0.56)	(2.93)	(1.21)	
SER	69.93	140.01	46.06	251.05	187.33

Note: (1) Figures in parentheses are the ratios of estimated coefficients to asymptotic standard errors. T-ratios for the Government bond equation are not provided because this equation is not directly estimated. (2) SER is the standard error of regression.

Table 3.2B
Maximum Likelihood Estimates for Model S1A (1971:10 to 1983:06)

	CM	TB	CB	AD	LL	CD	GB
CMR	-9.07	12.44	-9.15	88.04	-5.49	-5.21	-71.56
	(0.17)	(0.77)	(0.40)	(1.59)	(0.61)	(0.36)	
TBR	12.44	37.18	-22.82	84.76	-39.25	-30.38	-41.93
	(0.77)	(0.86)	(0.62)	(2.66)	(1.80)	(0.95)	
CBR	-9.15	-22.82	34.68	90.84	1.92	-71.06	-24.39
	(0.40)	(0.62)	(0.50)	(2.26)	(0.08)	(1.30)	
ADR	88.04	84.76	90.84	-279.59	-5.73	62.54	-40.86
	(1.59)	(2.66)	(2.26)	(2.47)	(0.25)	(1.78)	
LLR	-5.49	-39.25	1.92	-5.73	39.57	4.12	4.86
	(0.61)	(1.80)	(0.08)	(0.25)	(1.00)	(0.11)	
CDR	-5.21	-30.38	-71.06	62.54	4.12	16.50	23.49
	(0.36)	(0.95)	(1.30)	(1.78)	(0.11)	(0.29)	
GBR	-71.56	-41.93	-24.39	-40.86	4.86	23.49	150.39
CN	-0.10	0.05	-0.11	-1.00	-0.05	-0.06	0.26
	(1.03)	(1.78)	(2.29)	(7.92)	(3.55)	(2.41)	
SD	-1.56	0.52	-0.29	0.08	0.12	-0.01	0.14
	(1.80)	(2.58)	(0.68)	(0.09)	(1.16)	(0.02)	
NP	0.79	0.29	0.33	-1.38	0.21	0.68	0.08
	(0.97)	(0.99)	(1.10)	(1.94)	(1.22)	(2.70)	
TD	0.16	0.01	0.05	0.72	0.01	0.02	0.03
	(8.33)	(0.06)	(5.85)	(33.78)	(0.77)	(4.35)	
D_{SD}	800.26	-58.81	-57.39	-611.53	1.94	22.55	-97.02
	(1.81)	(0.49)	(0.29)	(1.29)	(0.03)	(0.16)	
SER	559.82	144.13	170.41	597.37	84.02	140.22	342.26

Note: (1) Figures in parentheses are the ratios of estimated coefficients to asymptotic standard errors. T-ratios for the Government bond equation are not provided because this equation is not directly estimated. (2) SER is the standard error of regression.

Table 3.3A
Maximum Likelihood Estimates for Model S1B (1959:01 to 1971:06)

	CM	TB	CB	AD	GB
CMR	182.73	114.90	73.00	-378.67	8.04
	(3.44)	(2.26)	(3.04)	(10.69)	
TBR	114.90	-548.22	78.62	587.32	-232.62
	(2.26)	(5.87)	(2.01)	(10.14)	
CBR	73.00	78.62	22.90	-211.86	37.34
	(3.04)	(2.01)	(0.51)	(9.58)	
ADR	-378.67	587.32	-211.86	-320.13	323.34
	(10.69)	(10.14)	(9.58)	(3.27)	
GBR	8.04	-232.62	37.34	323.34	-136.10
W	0.24	-0.06	0.11	0.61	0.10
	(14.48)	(2.34)	(11.06)	(18.62)	
D_{SD}	55.98	-4.81	-16.98	40.12	-74.31
	(1.35)	(0.08)	(0.62)	(0.46)	
D_{CC}	64.70	-282.45	81.31	424.77	-288.33
	(1.99)	(4.81)	(3.15)	(4.94)	
SER	96.77	195.57	57.61	303.96	192.00

Note: (1) Figures in parentheses are the ratios of estimated coefficients to asymptotic standard errors. T-ratios for the Government bond equation are not provided because this equation is not directly estimated. (2) SER is the standard error of regression.

Table 3.3B
Maximum Likelihood Estimates for Model S1B (1971:10 to 1983:06)

	CM	TB	CB	AD	LL	CD	GB
CMR	23.65	-5.72	-21.73	97.91	-11.64	-30.93	-51.54
	(0.52)	(0.39)	(1.09)	(1.56)	(1.20)	(2.30)	
TBR	-5.72	50.40	-40.14	99.29	-24.41	-49.38	-30.04
	(0.39)	(1.38)	(1.12)	(3.18)	(1.36)	(1.78)	
CBR	-21.73	-40.14	90.55	92.14	-30.70	-44.85	-45.27
	(1.09)	(1.12)	(1.31)	(2.40)	(1.17)	(1.03)	
ADR	97.91	99.29	92.14	-385.01	16.13	87.22	-7.68
	(1.56)	(3.18)	(2.40)	(3.20)	(0.71)	(2.73)	
LLR	-11.64	-24.41	-30.70	16.13	54.71	-1.07	-3.02
	(1.20)	(1.36)	(1.17)	(0.71)	(1.44)	(0.03)	
CDR	-30.93	-49.38	-44.85	87.22	-1.07	27.35	11.66
	(2.30)	(1.78)	(1.03)	(2.73)	(0.03)	(0.52)	
GBR	-51.54	-30.04	-45.27	-7.68	-3.02	11.66	125.89
W	0.16	0.01	0.03	0.71	0.01	0.01	0.07
	(30.38)	(1.07)	(18.25)	(134.17)	(6.07)	(2.91)	
D_{SD}	210.27	251.70	-231.52	-499.21	108.41	10.34	150.01
	(1.13)	(3.85)	(3.09)	(1.80)	(2.52)	(0.16)	
SER	632.16	174.88	187.44	768.75	110.38	159.54	442.09

Note: (1) Figures in parentheses are the ratios of estimated coefficients to asymptotic standard errors. T-ratios for the Government bond equation are not provided because this equation is not directly estimated. (2) SER is the standard error of regression.

Table 3.4A
Maximum Likelihood Estimates for Model S1C (1959:01 to 1971:06)

	CM	TB	CB	AD	GB
α_i	-834.39	1743.75	-717.63	-801.93	610.20
	(3985.39)	(11106.79)	(1599.73)	(8534.35)	
CMR	-19.94	44.07	5.91	33.13	-63.17
	(59.78)	(195.06)	(8.95)	(224.96)	
TBR	-89.57	8.30	-35.41	262.97	-146.29
	(437.65)	(59.38)	(90.48)	(2512.04)	
CBR	46.34	37.58	51.30	330.79	-466.01
	(242.22)	(227.89)	(121.19)	(5081.14)	
ADR	51.01	-76.32	-29.10	-559.52	613.93
	(357.63)	(838.34)	(108.42)	(9720.71)	
GBR	12.16	-13.63	7.30	-67.37	61.54
CN	-0.34	0.59	-0.24	-0.80	-0.21
	(0.01)	(0.01)	(0.01)	(0.01)	
SD	0.44	0.36	0.13	-2.28	0.35
	(0.01)	(0.01)	(0.01)	(0.16)	
EA	0.26	-0.30	-0.28	-1.41	0.73
	(0.01)	(0.02)	(0.01)	(0.16)	
DD	0.02	0.17	0.19	0.89	-0.27
	(0.01)	(0.01)	(0.01)	(0.01)	
TD	0.45	-0.59	0.12	0.67	0.35
	(0.01)	(0.01)	(0.01)	(0.01)	
D_{SD}	-77.86	54.90	-35.12	266.91	-208.83
	(416.41)	(380.28)	(84.70)	(3972.15)	
D_{CC}	-4.17	2.86	37.54	89.53	-125.76
	(25.07)	(20.97)	(107.27)	(1737.37)	
SER	65.37	118.66	36.85	196.03	138.99
R^2	0.97	0.84	0.96	0.96	0.50
D-W	1.01	0.79	0.54	0.46	0.30

Note: (1) Figures in parentheses are the ratios of estimated coefficients to asymptotic standard errors. T-ratios for the Government bond equation are not provided because this equation is not directly estimated. (2) SER is the standard error of regression.

Table 3.4B
Maximum Likelihood Estimates for Model S1C (1971:10 to 1983:06)

	CM	TB	CB	AD	LL	CD	GB
α_i	-728.97	471.26	276.50	-1704.50	204.08	197.42	1284.21
	(14977.00)	(5993.30)	(4662.80)	(42665.00)	(1200.30)	(2472.10)	
CMR	-101.35	21.17	-5.20	83.13	7.32	-1.76	-3.31
	(734.71)	(82.54)	(30.41)	(711.61)	(15.81)	(6.65)	
TBR	30.44	116.54	11.81	-315.23	-21.14	-4.70	182.28
	(338.25)	(838.66)	(93.68)	(4000.90)	(74.30)	(32.32)	
CBR	158.44	-45.51	31.41	32.15	-15.54	-65.85	-95.10
	(1880.40)	(371.85)	(266.64)	(419.18)	(75.03)	(516.17)	
ADR	-9.90	-12.19	30.54	138.52	-21.05	2.00	-127.92
	(80.43)	(71.01)	(179.61)	(1229.40)	(66.95)	(13.33)	
LLR	692.78	12.63	29.54	-462.27	-37.43	-62.51	-172.74
	(7817.40)	(96.10)	(215.57)	(5606.00)	(153.69)	(463.70)	
CDR	-672.78	-44.30	-75.07	505.51	81.40	106.20	99.04
	(7701.70)	(330.16)	(567.39)	(5233.40)	(326.44)	(763.52)	
GBR	-97.63	-48.34	-23.03	18.19	6.44	26.62	117.75
CN	-0.07	-0.07	-0.09	-1.09	-0.05	-0.04	0.41
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	
SD	-1.67	0.46	-0.35	0.32	0.11	-0.05	0.18
	(0.09)	(0.01)	(0.02)	(0.02)	(0.01)	(0.01)	
NP	1.24	-0.02	0.13	-0.40	0.08	0.54	-0.57
	(0.06)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	
TD	0.17	-0.01	0.04	0.76	-0.01	0.02	0.03
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	
D_{SD}	994.35	-84.97	-66.67	-530.12	-26.09	31.98	-318.48
	(27448.00)	(1263.20)	(1365.50)	(18952.00)	(184.66)	(511.75)	
SER	470.94	132.57	166.29	557.20	71.95	135.13	250.07
R^2	0.97	0.62	0.92	0.99	0.55	0.25	0.96
D-W	1.08	0.91	0.41	1.02	0.49	0.53	0.72

Note: (1) Figures in parentheses are the ratios of estimated coefficients to asymptotic standard errors. T-ratios for the Government bond equation are not provided because this equation is not directly estimated. (2) SER is the standard error of regression, R^2 is multiple correlation coefficient and D-W is the Durbin-Waston statistic.

Table 3.5A
Maximum Likelihood Estimates for Model S1D (1959:01 to 1971:06)

	CM	TB	CB	AD	GB
α_i	-1326.44	2347.52	-689.92	-342.66	11.50
	(2.89)	(4.00)	(3.18)	(0.31)	
CMR	-86.89	109.48	-0.22	113.24	-135.61
	(0.88)	(0.69)	(0.01)	(0.50)	
TBR	-101.26	44.65	-57.21	272.40	-158.58
	(1.20)	(0.37)	(1.10)	(1.28)	
CBR	179.87	-173.44	105.42	195.83	-307.68
	(1.82)	(2.00)	(2.55)	(0.86)	
ADR	-25.49	62.58	-53.79	-495.56	512.26
	(0.19)	(0.38)	(0.82)	(1.71)	
GBR	33.77	-43.27	5.80	-85.91	89.61
W	0.26	-0.18	0.14	0.74	0.04
	(8.47)	(6.07)	(11.47)	(8.48)	
D_{SD}	77.70	-12.96	-9.10	12.53	-68.17
	(1.51)	(0.26)	(0.33)	(0.12)	
D_{CC}	32.18	-43.34	27.87	61.24	-77.95
	(0.71)	(0.70)	(1.16)	(0.50)	
SER	85.19	132.72	47.97	214.15	152.55
R^2	0.94	0.80	0.94	0.95	0.40
D-W	0.47	0.66	0.24	0.28	0.13

Note: (1) Figures in parentheses are the ratios of estimated coefficients to asymptotic standard errors. T-ratios for the Government bond equation are not provided because this equation is not directly estimated. (2) SER is the standard error of regression, R^2 is multiple correlation coefficient and D-W is the Durbin-Waston statistic.

Table 3.5B
Maximum Likelihood Estimates for Model S1D (1971:10 to 1983:06)

	CM	TB	CB	AD	LL	CD	GB
α_i	-81.78	585.94	124.24	-2301.80	272.94	371.98	1028.48
	(1977.70)	(8763.50)	(2183.10)	(66069.00)	(1859.70)	(5333.50)	
CMR	-51.39	7.64	1.18	36.20	2.89	-9.40	12.88
	(402.65)	(35.36)	(7.72)	(353.69)	(7.05)	(40.12)	
TBR	191.26	125.57	1.61	-574.12	-1.83	-1.85	259.36
	(2683.20)	(974.65)	(16.83)	(10030.00)	(6.79)	(15.23)	
CBR	-60.61	-94.85	66.21	533.38	-70.14	-55.29	-318.67
	(904.20)	(862.51)	(708.58)	(9365.10)	(338.08)	(498.77)	
ADR	30.33	42.25	-12.09	-55.15	18.36	-16.59	-7.11
	(308.05)	(271.36)	(76.07)	(607.89)	(59.62)	(116.80)	
LLR	433.23	173.74	-36.55	-541.19	64.15	-4.42	-88.96
	(6011.80)	(1450.80)	(343.62)	(8899.30)	(261.40)	(38.82)	
CDR	-382.67	-218.95	16.65	528.37	-18.70	75.84	-0.54
	(5491.70)	(1772.90)	(164.48)	(9013.40)	(75.69)	(646.56)	
GBR	-160.15	-35.40	-37.01	72.51	5.27	11.71	143.07
W	0.16	-0.01	0.04	0.74	0.02	0.01	0.04
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	
D_{SD}	511.80	0.79	-124.46	-317.67	-28.80	-51.89	10.23
	(17971.00)	(17.35)	(3070.60)	(14490.00)	(333.00)	(1083.20)	
SER	515.71	177.36	200.68	696.03	95.08	143.58	322.50
R^2	0.96	0.32	0.88	0.99	0.22	0.15	0.93
D-W	0.68	0.62	0.27	0.56	0.34	0.46	0.67

Note: (1) Figures in parentheses are the ratios of estimated coefficients to asymptotic standard errors. T-ratios for the Government bond equation are not provided because this equation is not directly estimated. (2) SER is the standard error of regression, R^2 is multiple correlation coefficient and D-W is the Durbin-Waston statistic.

Examining the coefficients of the tables we note first of all that both the magnitude and the precision of the estimated coefficients vary greatly from one model to another. This is especially so for the interest rate coefficients. For the pre-1971 period, while all but one of the interest rate coefficients are significant in Models S1B and S1C, only few of them are statistically different from zero in Models S1A and S1D. For the post-1971 period, all but one of the interest rate coefficients are statistically different from zero in Models S1C and S1D, only a few of them are significant in Models S1A and S1B. Further, most of the interest rate coefficients change signs from one model to another. More disturbingly, at least one of the own interest rate coefficients are perversely signed in all of the four models for both periods. These features are probably due to the presence of multicollinearity among the regression variables. It is well known that interest rates are highly correlated. One serious consequence of multicollinearity is that the sampling variance of estimated coefficients rises with increasing collinearity between the explanatory variables and small variations in the data may produce substantial changes in the estimated coefficients. As for the coefficients of the non-choice items, quite a few of them are statistically different from zero in Model S1A, but none is significant in Model S1C, both in the pre-1971 period and in the post-1971 period. The coefficient patterns of the single wealth variable are less symmetric for the two periods. While all of them are significant in both Models S1B and S1D of the pre-1971 period, none of them is statistically different from zero in Model S1D of the post-1971 period. Two of the models, Models S1C and S1D, also have intercepts and all but one of the intercepts are statistically significant.

We now turn to a brief discussion of the empirical validity of the four alternative models. Before we conduct any formal test of the models it is important to note that, although Model S1B is a special case of both Model S1D and Model S1A, which in turn is a special case of Model S1C, neither Model S1A is nested within Model S1D, nor Model S1B nested within Model S1C. Nested tests can thus only be carried out between Models S1A and S1B, Models S1A and S1C, Models S1C and S1D and Models S1B and S1D. The test of Model S1A against Model S1C and Model S1B

against Model S1D is a test of the joint validity of the symmetry and no intercept restrictions, which derive from the assumption of statistical independence between asset/liability levels and their returns, while the test of Model S1A against Model S1B and Model S1C against Model S1D is a test of the non-choice items as a single wealth variable, namely, the assumption that all non-choice items have the same effect on bank's portfolio. Since likelihood ratio statistics are computed as twice the difference of the maximised sample log-likelihoods, we report in Table 3.6 the sample values of the maximised log-likelihood functions for the four models. For convenience, chi-square critical values at both the 1% and 5% significance levels are presented in Table 3.7.

The test results suggest that Model S1A and Model S1B must be rejected as empirically valid special cases of Model S1C and Model S1D respectively for both periods. The likelihood ratio statistics for the pre-1971 period are 166.96 and 207.38 respectively, while the 1% chi-square critical value with 10 degrees of freedom is only 23.209. The rejection is equally decisive for the post-1971 period: the likelihood ratio statistics are correspondingly 177.26 and 219.76, which far exceed the 1% chi-square critical value with 21 degrees of freedom. Based on the same information provided in Tables 3.6 and 3.7 we also find that the null hypothesis of non-choice items as a single wealth variable must be rejected for both the pre-1971 and the post-1971 periods. The rejection occurs regardless of whether the symmetry and no intercept restrictions are imposed. This does not imply, however, that Model S1C should be accepted as the empirically valid model of UK commercial bank portfolio behaviour for the periods in question. It is possible that Model S1C itself may be rejected when tested against a more general alternative. In fact, the above empirical results are based on the assumption of temporal independence of the disturbance vector. However, as we can see from Tables 3.5A - 3.6B, the Durbin-Watson statistics for all the equations are very low, indicating that autocorrelation may be present. We may note in passing that there are two important qualifications to the use of the Durbin-Watson statistic. First, it is necessary to have included a constant term in the regression. Second, it is strictly valid only when the explanatory variables are non-stochastic. When a constant term is

excluded from the regression, the correlation coefficient is also difficult to interpret. We have therefore omitted both the R^2 and the Durbin-Watson statistics from Tables 3.3A - 3.4B.

Table 3.6
Sample Values of the Maximised Log-likelihood Function for the Static Models Assuming No Autocorrelation

	Model S1A	Model S1B	Model S1C	Model S1D
Pre-1971	-3466.06 (38)	-3620.22 (22)	-3382.58 (48)	-3516.53 (32)
Post-1971	-5608.56 (51)	-5737.50 (33)	-5519.93 (72)	-5627.62 (54)

Note: Figures in parentheses are the number of free parameters estimated.

Table 3.7
Chi-square Critical Values

Degrees of Freedom	Significance Level 5%	1%
1	3.841	6.635
2	5.991	9.210
3	7.815	11.341
4	9.488	13.277
6	12.592	16.812
7	14.067	18.475
8	15.507	20.090
10	18.307	23.209
12	21.026	26.217
15	24.996	30.578
16	26.296	32.000
18	28.869	34.805
20	31.410	37.566
21	32.671	38.932
25	37.652	44.314
35	49.802	57.342
36	50.998	58.619
50	67.505	76.154

Source: Neave, H. R. and Worthington, P. L. (1988), Distribution-Free Tests, Unwin Hyman, London, p. 371.

3.6.1.2 Hypothesis Tests under the Assumption of First-order Vector Autocorrelation

To test whether first-order vector autocorrelation is present, we now specify that the $m \times 1$ disturbance vector ϵ_{st} follows the stationary autoregressive scheme

$$\epsilon_{st} = \forall\, \epsilon_{st-1} + \omega_{st}, \qquad t = 2, 3, \ldots, T \tag{3.85}$$

where ω_{st} is an independently identically distributed random vector and $\forall$ is an $m \times m$ matrix of unknown autoregressive coefficients. Since the system of equations in (3.81) is still singular, only m-1 of the disturbances ϵ_{st} in (3.85) are linearly independent. Berndt and Savin (1975) have shown that the singularity of the systems of equations in (3.81) and (3.85) imposes severe restrictions on the matrix of autoregressive coefficients, namely that all of the columns of $\forall$ must sum to a common constant. We note that if $\forall$ is presumed to be diagonal, then the above restriction implies that all the diagonal coefficients must be equal.

To circumvent the singularity problem we again delete one equation from (3.81) so that the m-1 disturbance vector ϵ^{*}_{st} follows the stationary autoregressive process

$$\epsilon^{*}_{st} = \forall^{*}\, \epsilon^{*}_{st-1} + \omega^{*}_{st}, \qquad t = 2, 3, \ldots, T \tag{3.86}$$

where ω^{*}_{st} is now independently normally distributed with mean vector 0 and non-singular covariance matrix Σ^{*}_{st}, and $\forall^{*}$ is the (m-1) $\times$ (m-1) matrix of the unknown autoregressive coefficients. It should be noted that unless additional restrictions are imposed on $\forall$, it is not possible to identify all of the elements of $\forall$ based on estimates of the elements in $\forall^{*}$ [Berndt and Savin (1975)]. In what follows we shall estimate the models under two alternative assumptions: $\forall$ = diagonal and $\forall$ = full. When $\forall$ is assumed to be diagonal all elements in $\forall$ can be identified from estimates of the elements in $\forall^{*}$. However, when $\forall$ is presumed to be full not all elements in $\forall$ can be identified from estimates of $\forall^{*}$.

The theoretical restrictions which are common to all the four static models are the Cournot condition (restrictions (2) in (3.29) - (3.32)), homogeneity (restrictions (3) in (3.29 - (3.32)) and the Engel condition (restrictions (4) in (3.29) - (3.32)). The Cournot and Engel conditions follow directly from the existence of the balance sheet constraint and are independent of the expected utility approach or any other assumptions made to derive the models. As the balance sheet constraint must be satisfied *a priori* these two restrictions will always hold. In two of the static models (Models S1A and S1B) there is also the symmetry restriction. Thus, of the theoretical restrictions only homogeneity and symmetry can be tested directly. The four models we have already discussed (Models S1A - S1D) are all estimated with homogeneity imposed and two of them (Models S1A and S1B) are estimated with additional symmetry restrictions. In order to test the validity of these theoretical restrictions and the validity of the intercepts and the dummy variables we have re-estimated under the assumption of first-order vector autocorrelation the system of equations in (3.81) with various combinations of the restrictions imposed. All together, we estimated 96 models for the pre-1971 period and 48 models for the post-1971 period. The maximised sample log-likelihood values for all the models are reported in Tables 3.8A and 3.8B.

The values in Tables 3.8A and 3.8B with superscript letters correspond to Models S1A - S1D. Comparing these values with those in Table 3.6 we find that the null hypothesis of no first-order vector autocorrelation must be rejected for all the four models in both periods. When tested against the alternative of $\forall$ = diagonal, the likelihood ratio statistics for the four models are 987.44, 1268.14, 850.56 and 1092.48 in the pre-1971 period, and 959.50, 1182.66, 822.32 and 1004.18 in the post-1971 period, all of which far exceed the 1% and 5% chi-square critical values with one degrees of freedom. The rejection of the null is equally decisive when the alternative is $\forall$ = full: the likelihood ratio statistics are correspondingly 1066.88, 1336.76, 919.00 and 1150.94 for the pre-1971 period, and 1009.22, 1234.28, 876.62 and 1014.52 for the post-1971 period, all of which are greater than the 1% chi-square critical values with sixteen (32.000)and thirty-six degrees of freedom (58.619).

Table 3.8A
Sample Values of the Maximised Log-likelihood Function
for the Static Models Assuming First-order Vector Autocorrelation: $\forall$ = diagonal

	Non-choice Items as Independent Variables			Non-choice Items as A Single Wealth Variable		
	U	H	SH	U	H	SH
			1959:01 to 1971:06			
Intercept SSD Dummy CC Dummy	-2951.07 (53)	-2957.30[C] (49)	-2959.28 (43)	-2964.62 (37)	-2970.29[D] (33)	-2971.66 (27)
Intercept SSD Dummy No CC Dummy	-2955.67 (49)	-2961.89 (45)	-2964.02 (39)	-2969.46 (33)	-2975.29 (29)	-2976.69 (23)
Intercept CC Dummy No SSD Dummy	-2952.25 (49)	-2958.48 (45)	-2960.51 (39)	-2965.57 (33)	-2971.19 (29)	-2972.77 (23)
SSD Dummy CC Dummy No Intercept	-2964.08 (49)	-2969.88 (45)	-2972.34[A] (39)	-2978.63 (33)	-2984.66 (29)	-2986.15[B] (23)
Intercept No SSD Dummy No CC Dummy	-2956.95 (45)	-2963.17 (41)	-2965.35 (35)	-2970.41 (29)	-2976.18 (25)	-2977.82 (19)
SSD Dummy No Intercept No CC Dummy	-2966.47 (45)	-2974.33 (41)	-2976.95 (35)	-2983.61 (29)	-2989.70 (25)	-2991.20 (19)
CC Dummy No Intercept No SSD Dummy	-2965.13 (45)	-2970.95 (41)	-2973.43 (35)	-2979.38 (29)	-2985.51 (25)	-2987.19 (19)
No Intercept No SSD Dummy No CC Dummy	-2969.71 (41)	-2975.50 (37)	-2978.13 (31)	-2984.27 (25)	-2990.56 (21)	-2992.27 (15)
			1971:10 to 1983:06			
Intercept SSD Dummy	-5108.19 (79)	-5108.77[C] (73)	-5117.13 (58)	-5124.98 (61)	-5125.53[D] (55)	-5132.73 (40)
Intercept No SSD Dummy	-5109.97 (73)	-5110.71 (67)	-5118.95 (52)	-5125.67 (55)	-5126.33 (49)	-5133.55 (34)
SSD Dummy No Intercept	-5118.24 (73)	-5119.82 (67)	-5128.81[A] (52)	-5136.06 (55)	-5138.53 (49)	-5146.17[B] (34)
No Intercept No SSD Dummy	-5119.30 (67)	-5120.79 (61)	-5129.56 (46)	-5136.16 (49)	-5138.75 (43)	-5146.32 (28)

Note: Figures in parentheses are the number of free coefficients estimated. The superscript letters correspond to Models S1A - S1D.

Table 3.8B
Sample Values of the Maximised Log-likelihood Function
for the Static Models Assuming First-order Vector Autocorrelation: $\forall$ = full

	Non-choice Items as Independent Variables			Non-choice Items as A Single Wealth Variable		
	U	H	SH	U	H	SH
			1959:01 to 1971:06			
Intercept SSD Dummy CC Dummy	-2916.72 (68)	-2923.08[C] (64)	-2925.87 (58)	-2933.69 (52)	-2941.06[D] (48)	-2943.63 (42)
Intercept SSD Dummy No CC Dummy	-2920.97 (64)	-2926.89 (60)	-2930.02 (54)	-2937.78 (48)	-2945.71 (44)	-2948.69 (38)
Intercept CC Dummy No SSD Dummy	-2919.18 (64)	-2926.89 (60)	-2929.25 (54)	-2935.07 (48)	-2942.82 (44)	-2945.49 (38)
SSD Dummy CC Dummy No Intercept	-2925.40 (64)	-2930.36 (60)	-2932.62[A] (54)	-2945.62 (48)	-2950.43 (44)	-2951.84[B] (38)
Intercept No SSD Dummy No CC Dummy	-2923.54 (60)	-2930.93 (56)	-2933.53 (50)	-2939.11 (44)	-2947.20 (40)	-2950.51 (34)
SSD Dummy No Intercept No CC Dummy	-2930.60 (60)	-2934.85 (56)	-2937.26 (50)	-2950.18 (44)	-2955.02 (40)	-2956.58 (34)
CC Dummy No Intercept No SSD Dummy	-2928.09 (60)	-2934.26 (56)	-2936.17 (50)	-2946.87 (44)	-2952.19 (40)	-2953.66 (34)
No Intercept No SSD Dummy No CC Dummy	-2933.41 (56)	-2938.86 (52)	-2940.86 (46)	-2951.45 (40)	-2956.84 (36)	-2958.47 (30)
			1971:10 to 1983:06			
Intercept SSD Dummy	-5081.20 (114)	-5081.62[C] (108)	-5093.02 (93)	-5100.59 (96)	-5100.95[D] (90)	-5109.75 (75)
Intercept No SSD Dummy	-5085.70 (108)	-5086.20 (102)	-5097.23 (87)	-5102.64 (90)	-5103.11 (84)	-5111.66 (69)
SSD Dummy No Intercept	-5091.80 (108)	-5092.21 (102)	-5103.95[A] (87)	-5111.10 (90)	-5111.98 (84)	-5120.36[B] (69)
No Intercept No SSD Dummy	-5094.57 (102)	-5094.95 (96)	-5106.97 (81)	-5111.94 (84)	-5112.84 (78)	-5120.62 (63)

Note: Figures in parentheses are the number of free coefficients estimated. The superscript letters correspond to Models S1A - S1D.

Table 3.9
Sample Likelihood Ratios for Testing the Diagonality of the Autoregressive Matrix

	Non-choice Items as Independent Variables			Non-choice Items as A Single Wealth Variable		
	U	H	SH	U	H	SH
			1959:01 to 1971:06			
Intercept SSD Dummy CC Dummy	68.70	68.44	66.82	61.86	58.46	56.06
Intercept SSD Dummy No CC Dummy	69.40	70.00	68.00	63.36	59.16	56.00
Intercept CC Dummy No SSD Dummy	66.14	63.18	62.52	61.00	57.90	54.56
SSD Dummy CC Dummy No Intercept	77.36	79.04	79.44	66.02	68.46	68.62
Intercept No SSD Dummy No CC Dummy	66.82	64.48	63.64	62.60	57.96	54.62
SSD Dummy No Intercept No CC Dummy	71.74	78.96	79.38	66.90	69.36	69.24
CC Dummy No Intercept No SSD Dummy	74.08	73.38	74.52	65.02	66.64	67.06
No Intercept No SSD Dummy No CC Dummy	72.60	73.28	74.54	65.64	67.44	67.60
			1971:10 to 1983:06			
Intercept SSD Dummy	53.98	54.30	48.22	48.78	49.16	45.96
Intercept No SSD Dummy	48.54	49.02	43.44	46.06	46.44	43.78
SSD Dummy No Intercept	52.88	55.22	49.72	49.92	53.10	51.62
No Intercept No SSD Dummy	49.46	51.68	45.18	48.44	51.82	51.40

Note: Under the null, there are fifteen and thirty-five degrees of freedom for the pre-1971 period and the post-1971 period respectively.

Table 3.10
Sample Likelihood Ratios for Testing the Validity of Intercepts, SSD and CC Dummies in the Static Models Assuming First-order Vector Autocorrelation

	Non-choice Items as Independent Variables			Non-choice Items as A Single Wealth Variable		
	U	H	SH	U	H	SH
			1959:01 to 1971:06			
Intercept	9.20	9.18	9.48	9.68	10.00	10.06
SSD Dummy	8.50	7.62	8.30	8.18	9.30	10.12
No CC Dummy	(4)	(4)	(4)	(4)	(4)	(4)
Intercept	2.36	2.36	2.46	1.90	1.80	2.22
CC Dummy	4.92	7.62	6.76	2.76	3.52	3.72
No SSD Dummy	(4)	(4)	(4)	(4)	(4)	(4)
SSD Dummy	26.06	25.16	26.12	28.02	28.74	28.98
CC Dummy	17.36	14.56	13.50	23.86	18.74	16.43
No Intercept	(4)	(4)	(4)	(4)	(4)	(4)
Intercept	11.76	11.74	12.14	11.58	11.78	12.32
No SSD Dummy	13.64	15.70	15.32	10.84	12.28	13.76
No CC Dummy	(8)	(8)	(8)	(8)	(8)	(8)
SSD Dummy	30.80	34.06	35.34	37.98	38.82	39.08
No Intercept	27.76	23.54	22.78	32.98	27.92	25.90
No CC Dummy	(8)	(8)	(8)	(8)	(8)	(8)
CC Dummy	28.12	27.30	28.30	29.52	30.44	31.06
No Intercept	22.74	22.36	20.60	26.36	22.26	20.06
No SSD Dummy	(8)	(8)	(8)	(8)	(8)	(8)
No Intercept	37.28	36.40	37.70	39.50	40.54	41.22
No SSD Dummy	33.38	31.56	29.98	35.52	31.56	29.68
No CC Dummy	(12)	(12)	(12)	(12)	(12)	(12)
			1971:10 to 1983:06			
Intercept	3.56	3.88	3.64	1.38	1.60	1.64
No SSD Dummy	9.00	9.16	8.42	4.10	4.32	3.82
	(6)	(6)	(6)	(6)	(6)	(6)
SSD Dummy	20.10	22.10	23.36	22.16	26.00	26.88
No intercept	21.20	21.18	21.86	21.02	22.06	21.22
	(6)	(6)	(6)	(6)	(6)	(6)
No Intercept	22.22	24.04	24.86	22.36	26.44	27.18
No SSD Dummy	26.74	26.66	27.90	22.70	23.78	21.74
	(12)	(12)	(12)	(12)	(12)	(12)

Note: (1) U stands for unconstrained (by the theoretical restrictions), H for homogeneity and SH for symmetry and homogeneity; (2) The first number in each cell is the sample likelihood ratio under the assumption $\forall$ = diagonal while second number is that under the assumption $\forall$ = full. (3) Figures in parentheses are the number of degrees of freedom under the null hypothesis.

Table 3.11
Sample Likelihood Ratios for Testing Symmetry and Homogeneity in the Static Models Assuming First-order Vector Autocorrelation

	Non-choice Items as Independent Variables		Non-choice Items as A Single Wealth Variable	
	Homogeneity	Sym. & Homo.	Homogeneity	Sym. and Homo.
Intercept	12.46	16.42	11.34	14.08
SSD Dummy	12.74	18.30	14.74	19.88
CC Dummy	(4)	(10)	(4)	(10)
Intercept	12.44	16.70	11.66	14.46
SSD Dummy	11.84	18.10	15.86	21.82
No CC Dummy	(4)	(10)	(4)	(10)
Intercept	12.46	16.52	11.24	14.40
CC Dummy	15.42	20.14	15.50	20.84
No SSD Dummy	(4)	(10)	(4)	(10)
SSD Dummy	11.60	16.52	12.06	15.04
CC Dummy	9.92	14.44	9.62	12.44
No Intercept	(4)	(10)	(4)	(10)
Intercept	12.44	16.80	11.54	14.82
No SSD Dummy	14.78	19.98	16.18	22.80
No CC Dummy	(4)	(10)	(4)	(10)
SSD Dummy	15.72	20.96	12.18	15.18
No Intercept	8.50	13.32	9.68	12.80
No CC Dummy	(4)	(10)	(4)	(10)
CC Dummy	11.64	16.60	12.26	15.62
No Intercept	12.34	16.16	10.64	13.58
No SSD Dummy	(4)	(10)	(4)	(10)
No Intercept	11.58	16.84	12.38	15.80
No SSD Dummy	10.90	14.90	10.78	14.04
No CC Dummy	(4)	(10)	(4)	(10)
		1971:10 to 1983:06		
Intercept	1.16(6)	17.88(21)	1.10(6)	15.50(21)
SSD Dummy	0.84	23.64	0.72	18.32
Intercept	1.48(6)	17.96(21)	1.32(6)	15.76(21)
No SSD Dummy	1.00	23.60	0.94	18.04
SSD Dummy	3.16(6)	21.14(21)	4.94(6)	20.22(21)
No Intercept	0.82	24.30	1.76	18.52
No Intercept	2.98(6)	20.52(21)	5.18(6)	20.32(21)
No SSD Dummy	0.76	24.80	1.80	17.36

Note: (1) The first number in each cell is the sample likelihood ratio under the assumption $\forall$ *= diagonal while second number is that under the assumption* $\forall$ *= full. (2) Figures in parentheses are the degrees of freedom under the null hypothesis.*

Table 3.12
Likelihood Ratios for Testing Non-choice Items as a Single Wealth Variable in the Static Models Assuming First-order Vector Autocorrelation

	Unrestricted	Homogeneity	Symmetry & Homogeneity
		1959:01 to 1971:06	
Intercept SSD Dummy CC Dummy	27.10 33.94	25.98 35.96	24.76 35.52
Intercept SSD Dummy No CC Dummy	27.58 33.62	26.80 37.64	25.34 37.34
Intercept CC Dummy No SSD Dummy	26.64 31.78	25.42 31.86	24.52 32.48
SSD Dummy CC Dummy No Intercept	29.10 40.44	29.56 40.14	27.62 38.44
Intercept No SSD Dummy No CC Dummy	26.92 31.14	26.02 32.54	24.94 33.96
SSD Dummy No Intercept No CC Dummy	34.28 39.16	30.74 40.34	28.50 38.64
CC Dummy No Intercept No SSD Dummy	28.50 37.56	29.12 35.86	27.52 34.98
No Intercept No SSD Dummy No CC Dummy	29.32 36.08	30.12 35.96	28.28 35.22
		1971:10 to 1983:06	
Intercept SSD Dummy	33.58 38.78	33.52 38.66	31.20 33.46
Intercept No SSD Dummy	31.40 33.88	31.24 33.82	29.20 28.86
SSD Dummy No Intercept	35.64 38.60	37.42 39.54	34.72 32.82
No Intercept No SSD Dummy	33.72 34.74	35.92 35.78	33.52 27.30

Note: (1) The first number in each cell is the sample likelihood ratio under the assumption $\forall$ = diagonal while second number is that under the assumption $\forall$ = full. (2) Under the null there are sixteen and eighteen degrees of freedom in the pre-1971 period and post-1971 period respectively.

Diagonality of the Autoregressive Matrix

Nested hypothesis tests can be conducted in numerous ways using the information in Tables 3.8A and 3.8B. One could, for example, test the joint validity of the symmetry, homogeneity restrictions, the intercept and the dummy variables. Alternatively, one can examine each of the main restrictions separately, and this is the approach we shall take. We shall test in turn the specification of the matrix of autoregressive coefficients, the validity of the intercepts and dummy variables, the theoretical restrictions (homogeneity and symmetry), and non-choice items as a single wealth variable. First we examine the specification of the matrix of autoregressive coefficients. In Table 3.9 we present the sample likelihood ratio statistics for testing the diagonality of the autoregressive matrix against the alternative of a full matrix of autoregressive coefficients for various models. We look at the test results for the two periods separately.

For the pre-1971 period , the null hypothesis of a diagonal matrix of autoregressive coefficients must be rejected in all the 48 cases in favour of the alternative of a full matrix at both the 5% and 1% significance levels. The 1% chi-square critical value with fifteen degrees of freedom is 30.578 while the lowest sample likelihood ratio statistic is 54.56. Thus, for the pre-1971 period the first-order vector autocorrelation models with a full matrix of autoregressive coefficients are empirically consistent.

The test results for the post-1971 period are less clear-cut. With thirty-five degrees of freedom the chi-square critical value is 57.342 at the 1% significance level while the highest sample likelihood ratio statistic is 53.98. Thus, at the 1% significance level we are unable to reject the null hypothesis of a diagonal matrix of autoregressive coefficients. However, at the 5% significance level the null hypothesis can only be accepted in 14 of the 24 cases. The lowest sample likelihood ratios are found in the two cases with the homogeneity and symmetry restrictions and with intercepts. Whether to accept the diagonal matrix of autoregressive coefficients as empirically consistent with the post-1971 data will depend on the combination of theoretical restrictions, the

intercepts and the dummy variables. We now turn to a discussion of the validity of the intercepts and the dummy variables.

The Intercepts and the Dummy Variables

Since the theoretical restrictions of symmetry and homogeneity have not been tested we shall report test results for both unrestricted models and models constrained by the theoretical restrictions under the alternative specifications of the matrix of autoregressive coefficients. Table 3.10 presents the likelihood ratio statistics for testing various combinations of the intercepts and the dummy variables. The first figure in each cell is the likelihood ratio statistic for testing the null hypothesis under the assumption of $\forall$ = diagonal while the second figure is the likelihood ratio statistic for testing the same null hypothesis under the assumption of $\forall$ = full. The degrees of freedom under the null are given in parentheses.

It is clear from Table 3.10 that, under both specifications of the matrix of the autoregressive coefficients, the null hypothesis of no intercept, either with or without the dummy variables, must be rejected for both the pre-1971 period and the post-1971 period. The rejection is decisive regardless of whether the non-choice items are considered as independent explanatory variables or as a consolidated wealth variable and regardless of whether the theoretical restrictions are imposed or not.

However, we are unable to reject the null hypothesis of no Special Deposit and credit ceilings dummies, either jointly or separately, for the pre-1971 period. When tested separately, the null hypothesis of no Special Deposit dummies have to be accepted at both the 1% and 5% significance levels in all the 12 cases. The test results for the null hypothesis of no credit ceilings dummies are less decisive: we can accept the null in all the 12 cases at the 1% significance level but in only 8 cases at the 5% level. However, this is probably due to the presence of the Special Deposit dummies since the null hypothesis of no Special Deposit and credit ceilings dummies jointly has to be

accepted in almost all the cases. The null hypothesis of no Special and Supplementary Deposit dummies also has to be accepted at both the 1% and 5% significance levels for the post-1971 period. Thus, for both periods the models with intercepts are empirically consistent within the static expected utility maximisation framework.

Homogeneity and Symmetry

We now turn to a discussion of the theoretical restrictions in the static models. We report in Table 3.11 the sample likelihood ratio statistics for testing homogeneity and symmetry restrictions under the two alternative specifications of the matrix of autoregressive coefficients. When testing homogeneity and symmetry we examine both models which regard non-choice items as independent explanatory variables and models which view non-choice items as a consolidated wealth variable. Similarly, when the null hypothesis of a single wealth variable is tested, all three sets of models are considered: unconstrained, constrained by homogeneity alone and constrained by both homogeneity and symmetry.

Under the assumption of $\forall$ = diagonal we have to reject for the pre-1971 period the homogeneity restrictions alone at the 5% significance level for the models; the rejection occurs regardless of whether non-choice items are treated as separate explanatory variables or as a single wealth variable. The lowest likelihood ratio is 11.24 while the 5% chi-square critical value with 4 degrees of freedom is 9.488. The homogeneity restrictions may, however, be accepted marginally in all but one of the models at the 1% significance level for the same period. When homogeneity is tested jointly with the symmetry restrictions the null hypothesis has to be accepted in all but one of the models at both the 1% and 5% significance levels. However, as we have seen earlier, the hypothesis of a diagonal matrix of autoregressive coefficients itself has been rejected as empirically consistent for the pre-1971 period.

When the matrix of the autoregressive coefficients is assumed to be full

homogeneity alone can be accepted in only 1 of the 16 cases for the post-1971 period. Even at the 1% significance level homogeneity has to be rejected in 6 cases, 2 of which are in the maintained models with intercepts and no dummy variables. The null hypothesis of homogeneity and symmetry jointly can be accepted at the 5% significance level in 9 of the 16 cases, but none of these is in the two maintained models with intercepts and no dummy variables, although the null may be accepted in all cases at the 1% significance level.

For the post-1971 period, we have to accept in all the cases the null hypothesis of homogeneity and symmetry, either separately or jointly, at both the 1% and 5% significance levels. The acceptance occurs regardless of whether the matrix of autoregressive coefficients is assumed to be diagonal or full and is far more decisive and unambiguous than for the pre-1971 period: the highest likelihood ratios are 6.06 with 6 degrees of freedom and 24.80 with 21 degrees of freedom, both of which are well below their corresponding chi-square critical values at both the 5% and 1% significance levels.

Non-choice Items as a Single Wealth Variable

Finally, we examine the test results for the null hypothesis of non-choice items being a single wealth variable. A comparison of the likelihood ratios in Table 3.12 with the chi-square critical values in Table 3.8 indicates that, with a diagonal matrix of autoregressive coefficients, we can accept for the pre-1971 period the null hypothesis of non-choice items as a single wealth variable in all but one of the 24 cases at the 1% significance level but in only 7 cases at the 5% significance level. However, with a full matrix of autoregressive coefficients the null hypothesis of non-choice variables as a single wealth variable is rejected in all cases at the 5% significance level; even at the 1% significance level the null may only be accepted very marginally in 3 cases. Thus, the null hypothesis of non-choice variables as a single wealth variable is not consistent with the pre-1971 data within the current framework.

For the post-1971 period, when $\forall$ = diagonal the null hypothesis of a single wealth variable can be accepted at the 1% significance level in 9 of the 12 cases, but has to be rejected in all the cases at the 5% significance level. With a full matrix of autoregressive coefficients, the null hypothesis can only be accepted at the 5% significance level in only 2 cases. In the model with intercepts and the homogeneity and symmetry restrictions which is the maintained model so far for the post-1971 period, the null hypothesis of non-choice items as a single wealth variable can be accepted at both the 1% and 5% significance levels, although the acceptance at the 5% significance level is only marginal. Given this and the fact that all the other models have already been rejected as empirically valid alternatives we regard the null of a single wealth variable as the maintained hypothesis for the post-1971 period.

Given the above test results we conclude that, when the disturbances in the static models are assumed to follow a first-order vector autoregressive process, a full matrix of the autoregressive coefficients is empirically consistent, for both the pre-1971 and post-1971 periods, within the current framework of expected utility maximisation. The intercepts are also consistent with the data from both periods while the dummy variables are rejected by both periods. However, the theoretical restrictions of homogeneity and symmetry can be accepted unambiguously for the post-1971 period only. As for the null hypothesis of non-choice items as a single wealth variable, it has to be rejected for the pre-1971 period but can be accepted for the post-1971 period. Therefore, we shall maintain at this stage the theoretically unconstrained (by homogeneity and symmetry) model with intercepts, independent non-choice items and a full matrix of autoregressive coefficients as the empirically consistent model for the pre-1971 period. For the post-1971 period, the theoretically constrained model with intercepts, a full matrix of autoregressive coefficients and a consolidated single wealth variable is maintained. FIML estimates of the maintained models are presented in Tables 3.13 and 3.14.

Table 3.13
FIML Estimates of the Unconstrained Static Model
with Independent Non-choice Variables, Intercepts and a Full Matrix of Autoregressive Coefficients
1959:01 to 1971:06

	CM	TB	CB	AD	GB
α_i	-1618.90	-749.11	-141.67	1923.20	586.48
	(2.19)	(0.18)	(0.16)	(0.53)	
CMR	9.18	-3.09	-0.29	12.66	-18.46
	(0.18)	(0.05)	(0.02)	(0.17)	
TBR	-56.27	61.11	7.18	-13.77	1.75
	(0.82)	(0.96)	(0.48)	(0.18)	
CBR	67.85	-154.82	-29.54	116.78	-0.27
	(0.59)	(1.26)	(0.80)	(0.84)	
ADR	-29.63	88.71	22.67	-94.09	12.34
	(0.26)	(0.69)	(0.62)	(0.65)	
GBR	44.71	-74.06	8.64	40.56	-15.85
	(1.50)	(1.29)	(0.66)	(0.64)	
CN	-0.36	-0.50	-0.02	-0.07	-0.05
	(3.80)	(3.94)	(0.48)	(0.36)	
SD	-0.20	0.54	-0.18	-0.96	-0.20
	(0.70)	(0.90)	(1.17)	(1.63)	
EA	-0.11	-0.64	-0.02	-0.21	-0.02
	(0.61)	(2.71)	(0.23)	(1.08)	
DD	0.26	0.71	-0.02	-0.09	0.14
	(2.55)	(6.25)	(0.59)	(0.63)	
TD	0.45	0.38	0.02	0.14	0.01
	(4.70)	(2.55)	(0.42)	(0.80)	
FIML estimates of autoregressive parameters in $\forall$					
	0.62	0.19	0.06	0.09	
	(5.34)	(1.26)	(1.84)	(0.50)	
	0.04	**0.97**	-0.01	-0.07	
	(0.59)	**(14.80)**	(0.05)	(0.88)	
	0.22	-0.36	**0.97**	0.02	
	(1.00)	(1.72)	**(19.44)**	(0.07)	
	-0.07	0.08	0.02	**0.94**	
	(1.19)	(1.21)	(1.21)	**(10.25)**	
SER	50.50	64.74	15.03	71.19	26.68
R^2	0.98	0.95	0.99	0.99	0.99
D-W	2.08	2.08	1.94	1.82	1.38

Note: (1) Figures in parentheses are the ratios of estimated coefficients to asymptotic standard errors. T-ratios for the Government bond equation are not provided because this equation is not directly estimated. (2) SER is the standard error of regression, R^2 is multiple correlation coefficient, and D-W is the Durbin-Waston statistic.

Table 3.14
FIML Estimates of the Constrained Static Model with a Single Wealth Variable, Intercepts and a Full Matrix of Autoregressive Coefficients
1971:10 to 1983:06

	CM	TB	CB	AD	LL	CD	GB
α_i	-116.85	426.02	-102.54	-1599.90	304.15	429.24	659.88
	(0.17)	(0.92)	(0.23)	(0.85)	(0.87)	(1.52)	
CMR	-41.10	-4.21	1.09	47.70	1.59	1.47	-6.54
	(0.79)	(0.30)	(0.07)	(0.59)	(0.50)	(0.20)	
TBR	-4.21	-13.97	-7.12	5.86	-27.30	40.50	6.23
	(0.30)	(0.18)	(0.19)	(0.10)	(1.38)	(1.72)	
CBR	1.09	-7.12	27.82	3.51	17.06	-27.57	-14.79
	(0.07)	(0.19)	(0.56)	(0.09)	(1.50)	(0.86)	
ADR	47.70	5.86	3.51	-37.45	-2.65	-2.07	-14.90
	(0.59)	(0.10)	(0.09)	(0.17)	(0.17)	(0.09)	
LLR	1.59	-27.30	17.06	-2.65	23.26	-12.57	0.61
	(0.50)	(1.38)	(1.50)	(0.17)	(0.92)	(0.55)	
CDR	1.47	40.51	-27.57	-2.07	-12.57	1.94	-1.70
	(0.20)	(1.72)	(0.86)	(0.09)	(0.55)	(0.06)	
GBR	-6.54	6.23	-14.79	-14.90	0.61	-1.70	31.09
W	0.16	-0.01	0.04	0.73	0.01	0.01	
	(11.07)	(0.13)	(4.21)	(18.95)	(0.05)	(0.47)	
	FIML estimates of autoregressive parameters in $\forall$						
	0.88	0.01	0.01	-0.02	-0.02	0.02	
	(4.24)	(0.18)	(0.09)	(0.06)	(0.80)	(0.41)	
	0.01	**0.84**	-0.05	0.09	-0.02	-0.07	
	(0.01)	**(7.28)**	(0.41)	(0.13)	(0.32)	(0.60)	
	-0.31	-0.03	**0.81**	0.48	-0.02	-0.04	
	(0.64)	(0.25)	**(9.23)**	(0.91)	(0.27)	(0.33)	
	0.08	-0.01	-0.01	**0.83**	-0.03	0.01	
	(0.38)	(0.07)	(0.12)	**(3.02)**	(1.25)	(0.17)	
	-0.09	0.06	-0.12	0.06	**0.85**	-0.03	
	(0.07)	(0.26)	(0.40)	(0.04)	**(9.56)**	(0.14)	
	-0.08	-0.05	0.06	0.21	-0.01	**0.80**	
	(0.15)	(0.48)	(0.47)	(0.33)	(0.13)	**(8.76)**	
SER	379.50	97.43	87.98	453.97	37.26	80.30	166.02
R^2	0.98	0.79	0.98	0.99	0.88	0.73	0.98
D-W	2.30	2.04	1.94	2.26	1.67	1.58	2.21

Note: (1) Figures in parentheses are the ratios of estimated coefficients to asymptotic standard errors. T-ratios for the Government bond equation are not provided because this equation is not directly estimated. (2) SER is the standard error of regression, R^2 is multiple correlation coefficient, and D-W is the Durbin-Waston statistic.

A few features of the tables deserve comment. First, it is apparent that none of the interest rate coefficients is significantly different from zero in either the pre-1971 model or the post-1971 model. This is not particularly surprising, however, since as we can see from the tables, the own equation autoregressive coefficients are quite high and significant in both periods. Indeed, three of the four own equation autoregressive coefficients are extremely high in the pre-1971 period so that almost all of the movements in the dependent variables are explained by their own lagged values. Second, as in the models estimated without assuming first-order vector autocorrelation, quite a number of the own interest rate coefficients are negative, although none of them is statistically significant. The presence of multicollinearity among the interest rates may be partly to blame for this perverse result. Finally, one of the coefficients on the wealth variable is also perversely signed in the post-1971 period, although once again, it is not statistically different from zero.

In Tables 3.13 and 3.14 we also report the standard error of regression (SER) for each of the equations. It is obvious that these standard errors of regression are much lower than those for any of the models previously estimated assuming zero-order vector autocorrelation. It is also noticable that, where applicable, the correlation coefficients are higher than previously recorded for both periods. In the above two tables we have also shown the Durbin-Watson statistic of each equation. As is well known, when the estimated equation contains the lagged dependent variable, the Durbin-Watson statistic is biased and the Durbin h is the more suitable test statistic for first-order vector autocorrelation. However, in most of the equations in Tables 3.13 and 3.14 the Durbin h statistic cannot be defined. Although other tests can be carried out to see whether higher-order vector autocorrelation is present, we shall not conduct any further tests here. It is possible that these two models may themselves be rejected when tested against other models to be developed in the following chapters, and we shall defer any further tests until Chapter Six. Finally, we may note that the ambiguity regarding the theoretical restrictions for the pre-1971 period is probably due to the inadequate account of the liquid asset ratios in the models, and it is to this issue we now turn.

3.6.1.3 The Cash and Liquid Asset Ratios

Prior to the introduction of Competition and Credit Control in September 1971, the London clearing banks were subject to the cash and liquid assets ratio imposed by the Bank of England, whereby the banks were required to hold a minimum of 30% before 16 October 1963 and 28% thereafter of their total deposits in liquid assets, at least 8% of which must be held in cash. The liquid assets of the banks were defined as comprising, in addition to cash, money at call and short notice, Treasury bills and commercial bills. An *a priori* decision as to whether these two constraints were binding or not will of course, as rightly pointed out by Parkin *et al* (1970), depend upon the knowledge of the parameters of the objective function. Unfortunately, such a knowledge is usually not available and this creates a problem.

A cursory inspection of the balance sheets of the London clearing banks for the period in question reveals that the banks had always held 8% of their total deposits in the form of cash. Thus, in most of the studies on the portfolio behaviour of UK commercial banks, such as Parkin *et al* (1970), Courakis (1974, 1980) and White (1975), the cash ratio constraint has always been regarded as binding and effective, while the liquid assets ratio has been treated as not binding on the grounds that the clearing banks had always maintained their holdings of liquid assets above the required minimum. However, the fact that the observed holdings of liquid assets above the legal minimum alone does not entail us to discard the liquid assets ratio constraint from our formal analysis of commercial bank portfolio behaviour. Indeed, as Ford (1983) points out, the historical data could be interpreted as suggesting that until the introduction of Competition and Credit Control in September 1971, the liquid assets ratio was binding and effective. We calculated that the average holdings of liquid assets by the London clearing banks was 33.50% of total deposits for the period from 1953 to October 1963 when the minimum liquid asset ratio was set at 30%. When the legal minimum was reduced by two percentage points in October 1963, average holdings of liquid assets in the subsequent period went down *pari passu* to 30.93% of total deposits, an average

reduction of more than two percentage points, suggesting that the portfolios of the clearing banks may indeed be influenced by the legal liquid assets ratio. Although one might argue that the observed liquid assets ratio does not contain any useful and extraneous information which can be incorporated in the estimation procedure, it is conceivable that in order to guarantee meeting the legal constraint and perhaps for additional precautionary reasons (see Chapter Five) the clearing banks had to maintain their actual holdings of liquid assets above the legal minimum. Indeed, as discussed in Chapter Two, the hypothesis that, in order to allow for disequilibria due to the portfolio adjustment process, the clearing banks operated during the concerned period with a desired exogenous liquid assets ratio above the legal minimum, was tested and subsequently accepted by Bewley (1981).

Let ζ be the cash ratio and τ be the liquid assets ratio. If the cash and liquid assets ratio was strictly binding and effective during the period, i.e. if the banks always held τ percent of their total deposits in the liquid assets, then as suggested by Ford (1983), the imposition of this constraint would mean that the bank's portfolio could be segregated into three separate portfolios. The first of these will contain only cash. As cash does not carry any interest no expected rates of return, no variance of return and no other non-choice asset variables could affect the holdings of cash and hence cash would be explained solely by ζ times total deposits. The second portfolio, consisting of money at call and short notice, Treasury bills and commercial bills, will depend upon (τ - ζ) times total deposits, any non-choice asset variables, and the expected rates of return and the variances and covariances of rates of return on these three assets alone. The last portfolio, comprising advances and Government bonds, would be explained by (1 - τ) times total deposits, any non-choice asset variables, the expected rates of return and the variances and covariances of rates of return on the two assets themselves and on no other. However, since we know *a priori* that the commercial banks held more than (τ - ζ) percent of their total deposits in the liquid assets other than cash, such an empirical implementation would result in the misspecification of the models. In order to accommodate the effects of the liquid asset ratio but at the same time avoid

misspecification we emend our static models in the following way. Let

$$R_r = (\tau - \zeta)\ (DD + TD)$$

be the required liquid assets other than cash and

$$W^* = W - R_r$$

be the total disposable fund net of the liquid assets. Further, write $A_2^* = (R_r\ W^*)'$, then the bank's asset demand equations can be written as

$$A_1 = \alpha + G_1\ e_1 + H_1\ A_2^* + f\ D + \epsilon_{st}. \tag{3.87}$$

where G_1 can be either symmetric or non-symmetric and

$$G_1 = \begin{bmatrix} g_{11} & g_{12} & g_{13} & 0 & 0 \\ g_{21} & g_{22} & g_{23} & 0 & 0 \\ g_{31} & g_{32} & g_{33} & 0 & 0 \\ 0 & 0 & 0 & g_{44} & g_{45} \\ 0 & 0 & 0 & g_{54} & g_{55} \end{bmatrix} \qquad H_1 = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{31} & h_{32} \\ 0 & h_{42} \\ 0 & h_{52} \end{bmatrix} \tag{3.88}$$

with

(1) $G_1\ i = 0$; (3.89a)

(2) $i'\ G_1 = 0'$; (3.89b)

(3) $i'\ H_1 = -\ i'$; (3.89c)

(4) $\alpha_1 + \alpha_2 + \alpha_3 = \alpha_4 + \alpha_5 = 0$; (3.89d)

(5) $i'\ f = 0'$. (3.89e)

The specification of G_1 in (3.88) ensures that only the expected rates of return on

money at call and short notice, Treasury bills and commercial bills appear in the demand equations for the liquid assets while only their own expected rates of returns appear in the advance and Government bond equations. Given the specification of G_I in (3.88) the restrictions embodied in (3.89a) and (3.89b) are thus the familiar homogeneity and Cournot conditions. The only difference between these conditions and those applicable to the static models without the liquid asset ratio constraints is that they apply to the interest rate coefficients within each sub-portfolio. The zero-sum restriction in (3.89d) applies also to the intercepts within each sub-portfolio. It is noteworthy that the above specification of G_I imposes much more severe restrictions upon the values of the interest rate coefficients than in the absence of the liquid asset ratio constraints. For example, the homogeneity restrictions imply that all the four interest rate coefficients in the second choice sub-portfolio will be the same in absolute values. The restrictions in (3.89c) together with specification H_I imply that the total required liquid assets will not affect the bank's demand for advance and Government bonds, but at the same time also allows for the fact that the bank's total demand for the liquid assets may, for precautionary and other reasons, exceed the required level. (3.89c) are therefore the Engel conditions. The restrictions in (3.89f) are the same as those in the static models without the liquid asset ratio constraints.

We estimated the system of equations in (3.87) with various combinations of the dummy variables and theoretical restrictions of homogeneity and symmetry imposed. Because of the particular specification of the model in (3.87), the model will not be tested against the alternative of non-choice items as independent variables. As before, the models are estimated under two alternative specifications of the matrix of autoregressive coefficients: $\forall$ = diagonal and $\forall$ = full. We also estimated the models under the assumption of zero-order vector autocorrelation, and once again, the models under the assumption of no first-order vector autocorrelation must be rejected as empirically valid. For brevity, we report in Table 3.15 only the sample values of the maximised log-likelihood function of the models under the assumption of first-order vector autocorrelation among the disturbances.

Table 3.15
Sample Values of the Maximised Log-likelihood Function for the Static Models with Liquid Asset Ratio Constraints Assuming First-order Vector Autocorrelation

	Unconstrained	Homogeneity	Symmetry and Homogeneity
		1959:01 to 1971:06	
Intercept SSD Dummy CC Dummy	-2970.99(27) -2938.84(42)	-2975.78(24) -2942.04(39)	-2975.99(23) -2943.84(38)
Intercept SSD Dummy No CC Dummy	-2975.86(23) -2944.01(38)	-2980.65(20) -2947.22(35)	-2981.02(19) -2949.26(34)
Intercept CC Dummy No SSD Dummy	-2972.20(23) -2941.50(38)	-2976.98(20) -2944.97(35)	-2977.36(19) -2946.35(34)
SSD Dummy CC Dummy No Intercept	-2981.62(23) -2949.80(38)	-2986.04(20) -2952.39(35)	-2986.35(19) -2952.92(34)
Intercept No SSD Dummy No CC Dummy	-2977.03(19) -2946.87(34)	-2981.81(16) -2950.12(31)	-2982.37(15) -2951.69(30)
SSD Dummy No Intercept No CC Dummy	-2986.66(19) -2954.49(34)	-2990.99(16) -2956.85(31)	-2991.51(15) -2957.60(30)
CC Dummy No Intercept No SSD Dummy	-2982.71(19) -2951.44(34)	-2987.21(16) -2954.33(31)	-2987.57(15) -2954.94(30)
No Intercept No SSD Dummy No CC Dummy	-2987.74(15) -2956.13(30)	-2992.15(12) -2958.81(27)	-2992.74(11) -2959.68(26)

Note: (1) The first number in each cell is the sample value of the maximised log-likelihood function under the assumption $\forall$ *= diagonal while second number is that under the assumption* $\forall$ *= full; (2) Figures in parentheses are the number of free coefficients estimated.*

Table 3.16
Sample Likelihood Ratios for Testing the Diagonality of the Autoregressive Matrix with Liquid Asset Ratio Constraint (1959:01 - 1971:06)

	Unconstrained	Homogeneity	Homogeneity and Symmetry
Intercept SSD Dummy CC Dummy	64.44	67.48	64.30
Intercept SSD Dummy No CC Dummy	63.70	66.86	63.52
Intercept CC Dummy No SSD Dummy	61.40	64.02	62.02
SSD Dummy CC Dummy No Intercept	63.64	67.30	66.86
Intercept No SSD Dummy No CC Dummy	60.48	63.38	61.36
SSD Dummy No Intercept No CC Dummy	64.34	68.28	67.82
CC Dummy No Intercept No SSD Dummy	62.54	65.76	65.26
No Intercept No SSD Dummy No CC Dummy	63.22	66.68	66.12

Note: Under the null, there are fifteen degrees of freedom.

Table 3.17
Sample Likelihood Ratios for Testing the Validity of Intercepts, SSD and CC Dummies in the Static Models with Liquid Asset Ratio Constraints Assuming First-order Vector Autocorrelation (1959:01 - 1971:06)

	Unconstrained	Homogeneity	Homogeneity and Symmetry
Intercept	9.47	9.74	10.06
SSD Dummy	10.34	10.36	10.84
No CC Dummy	(4)	(4)	(4)
Intercept	2.42	2.40	2.74
CC Dummy	5.32	5.80	5.02
No SSD Dummy	(4)	(4)	(4)
SSD Dummy	21.38	20.52	20.72
CC Dummy	21.29	20.70	18.16
No Intercept	(3)	(3)	(3)
Intercept	12.08	12.06	12.76
No SSD Dummy	16.06	16.16	15.70
No CC Dummy	(8)	(8)	(8)
SSD Dummy	31.34	30.42	31.04
No Intercept	31.30	29.62	27.52
No CC Dummy	(7)	(7)	(7)
CC Dummy	23.44	22.86	23.16
No Intercept	25.20	24.58	22.20
No SSD Dummy	(7)	(7)	(7)
No Intercept	33.50	32.74	33.50
No SSD Dummy	34.58	33.54	31.68
No CC Dummy	(11)	(11)	(11)

Note: (1) The first number in each cell is the sample likelihood ratio under the assumption $\forall$ *= diagonal while second number is that under the assumption* $\forall$ *= full. (2) Figures in parentheses are the number of degrees of freedom under the null hypothesis.*

Table 3.18
Sample Likelihood Ratios for Testing Symmetry and Homogeneity in the Static Models with Liquid Asset Ratio Constraints Assuming First-order Vector Autocorrelation (1959:01 - 1971:06)

	Homogeneity	Homogeneity and Symmetry
Intercept	9.58	10.00
SSD Dummy	6.40	10.08
CC Dummy	(3)	(4)
Intercept	9.58	9.96
SSD Dummy	6.42	10.50
No CC Dummy	(3)	(4)
Intercept	9.56	9.94
CC Dummy	6.94	9.70
No SSD Dummy	(3)	(4)
SSD Dummy	8.84	9.46
CC Dummy	5.18	6.24
No Intercept	(3)	(4)
Intercept	9.56	10.68
No SSD Dummy	6.50	9.64
No CC Dummy	(3)	(4)
SSD Dummy	8.66	9.70
No Intercept	4.72	6.22
No CC Dummy	(3)	(4)
CC Dummy	9.00	9.72
No Intercept	5.78	7.00
No SSD Dummy	(3)	(4)
No Intercept	8.82	10.00
No SSD Dummy	5.36	7.10
No CC Dummy	(3)	(4)

Note: (1) The first number in each cell is the sample likelihood ratio under the assumption $\forall$ *= diagonal while second number is that under the assumption* $\forall$ *= full. (2) Figures in parentheses are the degrees of freedom under the null hypothesis.*

It is clear from Table 3.16 that the null hypothesis of a diagonal matrix of autoregressive coefficients must be rejected at all reasonable significance levels in favour of the alternative of a full matrix. As in the models without the liquid asset ratio constraints, the rejection occurs in all models with or without intercepts, dummy variables and regardless of whether the theoretical restrictions of homogeneity and symmetry are imposed. The rejection is also very decisive: the lowest sample likelihood ratio statistic is 60.48 while with fifteen degrees of freedom the 1% chi-square critical value is only 30.578. Thus, once again, the full matrix specification of the autoregressive coefficients is consistent with the data.

Sample likelihood ratio statistics for testing various combinations of the intercepts and the dummy variables are presented in Table 3.17. As we can see from the table, the null hypothesis of no intercepts must be rejected under both specification of the matrix of autoregressive coefficients. Indeed, the rejection is universal: in both theoretically constrained and unconstrained models, with or without the dummy variables. As for the dummy variables, the null hypothesis of no Special Deposit dummies must be accepted in all the 6 cases, at both the 5% and 1% significance levels. However, the test results for the credit ceilings dummy variables are less unambiguous: they can be rejected at the 5% significance level but not at the 1% level. This ambiguity is also reflected in the test statistics for the joint hypothesis of no Special Deposit and credit ceilings dummies. When the matrix of autoregressive coefficients is presumed to be diagonal the joint hypothesis may be accepted at both 1% and 5% significance levels; but when the matrix is assumed to be full the joint hypothesis may only be accepted at the 1% significance level. Given these ambiguities, we shall maintain the models with intercepts and credit ceilings dummy variables as empirically consistent within the class of models with liquid asset ratio constraints.

Table 3.18 contains the sample likelihood ratio statistics for testing the homogeneity and symmetry restrictions. The test results are quite different under the two alternative specifications of the matrix of autoregressive coefficients. Under the assumption of a

diagonal matrix the homogeneity and symmetry restrictions, either separately or jointly, must be rejected at the 5% significance level in almost all cases, although they may be accepted at the 1% significance level. Under the assumption of a full matrix of autoregressive coefficients, however, the homogeneity restrictions must be accepted at both the 5% and 1% significance levels. This result is in sharp contrast to that obtained in the models without the liquid asset ratio constraints where the homogeneity restrictions are rejected in most cases. With $\forall$ = full, the joint restrictions of homogeneity and symmetry can also be accepted in all the cases at the 1% significance level, although they can only be accepted in 4 of the 8 cases. In the maintained model with intercepts and credit ceilings dummies the sample likelihood ratio statistic for the joint homogeneity and symmetry restrictions is 9.94, slightly higher than the 5% chi-square critical value but well within 1% critical region.

Given the above test results we shall regard the theoretically restricted model with intercepts, credit ceilings dummies and a full matrix of first-order autoregressive coefficients as the maintained hypothesis within the set of static models constrained by the liquid asset ratio. The FIML estimates of this model are presented in Table 3.19. We note briefly from the table that again one of the own interest rate coefficients, the Commercial bill rate, is perversely signed. The coefficient on the required liquid asset variable in the commercial bill equation is also negatively signed, but this is mainly due to the rounding of values. Both directly estimated coefficients on the required liquid assets are statistically significant. The standard errors of regression are slightly higher than those in the maintained model without the liquid asset ratio constraints. Indeed, the R^2 and hence the D-W statistics are quite similar to those in the previously maintained. This suggests that the imposition of the liquid asset ratio constraints has not improved the efficiency of the model. As the two maintained models are non-nested we shall defer the formal test until Chapter Six.

Table 3.19
FIML Estimates of the Theoretically Restricted Model with Intercepts, Credit Ceiling Dummy Variables, and the Liquid Asset Ratio Constraints ($\forall$ = Full)
(1959:01 to 1971:06)

	CM	TB	CB	AD	GB
α_i	-1508.30	3193.80	-1685.50	-775.03	775.03
	(4.12)	(3.48)		(1.26)	
CMR	12.82	-19.55	6.70		
	(0.45)	(0.65)			
TBR	-19.55	2.18	17.37		
	(0.65)	(0.06)			
CBR	6.70	17.37	-24.07		
ADR				2.66	-2.66
				(0.28)	(0.28)
GBR				-2.66	2.66
				(0.28)	(0.28)
R_r	0.48	0.53	-0.01		
	(3.89)	(4.48)			
W^*	0.23	0.61	0.01	0.02	0.13
	(4.92)	(7.96)	(0.35)	(0.27)	
D_{CC}	53.02	-11.47	13.90	-41.85	-13.62
	(1.22)	(0.25)	(1.47)	(0.62)	
	FIML estimates of autoregressive parameters in $\forall$				
	0.97	0.07	0.01	-0.05	
	(30.51)	(1.38)	(0.81)	(0.84)	
	0.04	**0.99**	-0.01	-0.09	
	(1.02)	**(20.40)**	(0.04)	(1.49)	
	0.10	-0.18	**0.98**	-0.07	
	(0.84)	(0.60)	**(29.94)**	(0.37)	
	0.02	0.03	0.01	**0.91**	
	(0.46)	(0.60)	(0.42)	**(13.65)**	
SER	52.44	70.03	15.43	73.86	27.49
R^2	0.98	0.94	0.99	0.99	0.98
D-W	2.12	2.24	2.00	1.89	1.33

Note: Figures in parentheses are the ratios of estimated coefficients to asymptotic standard errors. Some t-ratios for the Commercial bill and Government bond equations are not provided because some of parameters are not directly estimated. Those provided are given by the symmetry restrictions.

Before we turn to a discussion of the empirical results for the dynamic models we may note that the maintained hypothesis is inconsistent with our theoretical models none of which allows the coexistence of the intercepts and the symmetry restrictions. It is possible that the acceptance of the intercepts may be simply due to the assumption concerning the expected rates of return. In our econometric specification of the models we assumed that the expected rates of return e_1 is given by

$$e_1 = \hat{r}_1. \tag{3.90}$$

If, however, the true expected rates of return is given by

$$e_1 = \hat{r}_1 + k \tag{3.91}$$

where k is an unknown vector of constants, then the system of asset demand equations will include an intercept term even in the absence of statistical dependence between the asset/liability levels and their rates of return. Since the maintained hypothesis itself may be rejected as empirically valid when tested against other non-nested alternatives we shall not discuss this issue here any further.

3.6.2 Dynamic Models

In this sub-section we report the empirical results based on the dynamic models. For reasons of economy, the FIML estimates of the four dynamic models will not be presented as we did with the four static models. Instead, we shall first test the validity of the theoretical restrictions and the dummy variables in the dynamic models and then report the FIML estimates of only those models which are maintained. The theoretical restrictions which are common to all the four dynamic models are the Cournot (restrictions (2) in (3.71) - (3.74)) and Engel conditions (restrictions (4) and (5) in (3.71) - (3.74)), which must be imposed *a priori* because of the balance sheet constraint, and homogeneity. In two of the static models (Models S1A and S1B) there is also the

symmetry restriction. However, the symmetry restriction no longer applies to any of the dynamic models as we have already noted in Section 3.4. Thus, of the theoretical restrictions only homogeneity can be tested directly in the dynamic models. Both dynamic models with and without the liquid assets ratio constraints are estimated and tested. We first discuss the test results for the dynamic models without the liquid asset ratio constraints.

3.6.2.1 Dynamic Models Without the Liquid Assets Ratio Constraints

As with the static models we estimate the dynamic system of equations in (3.82) with and without homogeneity restrictions and with various combinations of the intercepts and the dummy variables. Both models which treat non-choice items as separate independent explanatory variables and models which regard non-choice items as a single consolidated wealth variable are estimated. Sample values of the maximised likelihood function are contained in Table 3.25. As before, when testing the homogeneity restrictions we shall consider both models which regard non-choice items as independent explanatory variables and models which treat non-choice items as a single wealth variable. Similarly, both models with and without the homogeneity restrictions will be discussed when we examine the validity of the single wealth variable hypothesis.

Table 3.20
Sample Values of the Maximised Log-likelihood Function
for the Dynamic Models

	Non-choice Items as Independent Variables		Non-choice Items as A Single Wealth Variable	
	Unconstrained	Homogeneity	Unconstrained	Homogeneity
		1959:01 to 1971:06		
Intercept SSD Dummy CC Dummy	-2885.91 (72)	-2895.89 (68)	-2909.42 (56)	-2922.83 (52)
Intercept SSD Dummy No CC Dummy	-2888.50 (68)	-2898.93 (64)	-2910.78 (52)	-2924.38 (48)
Intercept CC Dummy No SSD Dummy	-2896.45 (68)	-2904.21 (64)	-2917.68 (52)	-2928.84 (48)
SSD Dummy CC Dummy No Intercept	-2891.84 (68)	-2900.37 (64)	-2917.72 (52)	-2929.66 (48)
Intercept No SSD Dummy No CC Dummy	-2898.98 (64)	-2906.72 (60)	-2919.19 (48)	-2931.48 (44)
SSD Dummy No Intercept No CC Dummy	-2894.59 (64)	-2903.60 (60)	-2918.86 (48)	-2931.21 (44)
CC Dummy No Intercept No SSD Dummy	-2902.26 (64)	-2908.84 (60)	-2926.07 (48)	-2934.61 (44)
No Intercept No SSD Dummy No CC Dummy	-2904.96 (60)	-2911.56 (56)	-2927.32 (44)	-2937.23 (40)
		1971:10 to 1983:06		
Intercept SSD Dummy	-5062.07 (120)	-5064.49 (114)	-5076.06 (102)	-5077.57 (96)
Intercept No SSD Dummy	-5071.33 (114)	-5073.81 (108)	-5084.50 (96)	-5086.10 (90)
SSD Dummy No Intercept	-5072.12 (114)	-5076.24 (108)	-5084.65 (96)	-5088.03 (90)
No Intercept No SSD Dummy	-5078.09 (108)	-5081.62 (102)	-5090.05 (90)	-5093.26 (84)

Note: Figures in parentheses are the number of free coefficients estimated.

In Table 3.21 we report the sample likelihood ratio statistics for testing the homogeneity restrictions and the single wealth variable. It is immediately clear from the table that we have to reject homogeneity for the pre-1971 period at the 5% significance level for all the models; this decisive rejection of homogeneity occurs regardless of whether non-choice items are treated as separate explanatory variables or as a single wealth variable. At the 1% significance level we are unable to reject the homogeneity restrictions in only two occasions. But even in these two cases the acceptance is very marginal: the sample likelihood ratios are 13.16 and 13.20 while the chi-square critical value is shade higher at 13.277. This result is contrary to that for the static models where homogeneity is accepted as empirically consistent with the pre-1971 data. In contrast, for the post-1971 period homogeneity has to be accepted at both the 1% and 5% significance levels in all the models. The highest sample likelihood ratio is 8.24, far below the 5% and 1% chi-square critical values with 6 degrees of freedom. This result is in accordance with the conclusion reached in the static models for the post-1971 period.

We now turn to a discussion of the test results for the null hypothesis of non-choice items being a single wealth variable. A comparison of the sample likelihood ratios in the last two columns of Table 3.21 with the Chi-square critical values in Table 3.7 suggests that the models which treat non-choice items as a single wealth variable must be rejected for pre-1971 period as an empirically valid special case of the models which regard non-choice items as separate explanatory variables, at both the 5% and 1% significance levels. The rejection is unambiguous and occurs regardless of whether the homogeneity restrictions are imposed or not. The lowest sample likelihood ratio statistic is 40.42, while the 1% Chi-square critical value with sixteen degrees of freedom is 32.00. We note that the rejection of the non-choice items as a single choice variable in the dynamic models is consistent with the static model result for the same period. Also, the null hypothesis of a single wealth variable has to be accepted for the post-1971 period with or without the homogeneity restrictions. The highest sample likelihood ratio is 27.98, still lower than the 5% chi-square critical value with 18 degrees of freedom.

This result is once again consistent with that of the static models for the same period.

Finally, we examine the validity of the intercepts and the dummy variables. Although the null hypothesis of non-choice items being a single wealth variable and the homogeneity restrictions have been rejected by the pre-1971 data, for completeness we report the sample likelihood ratio statistics for all the models in Table 3.22. It is clear from the table that, for the pre-1971 period, the null hypothesis of no credit ceilings can be accepted at both the 5% and 1% significance levels regardless of whether homogeneity restrictions is imposed or not and regardless of whether the non-choice items are treated as independent variables or as a consolidated single wealth variable. This result is in line with that for the static models without the liquid asset ratio constraints. Unlike the static models, however, the null hypothesis of no Special deposit dummy variables must be rejected in all the four cases at 5% significance level and in 3 cases at the 1% significance level. As for the intercepts, the test results are less clear-cut. At the 1% significance level the null hypothesis of no intercepts can be accepted in two of the four cases. This ambiguity is reflected in the test results for the jointly hypothesis of no intercepts and credit ceiling dummies: the null can be accepted at the 1% significance level in all four cases and must be rejected at the 5% significance level in three cases. However, given that the joint hypothesis of intercepts and Special deposit dummy variables have been accepted, the intercepts can be regarded as consistent with the data. Since both homogeneity and a single wealth variable are rejected by the pre-1971 data, we maintain the unconstrained (by homogeneity) model with intercepts, Special Deposits dummies and independent non-choice items as empirically valid for the pre-1971 period. The test results for the post-1971 period are unambiguous and decisive. The null hypothesis of no intercepts and no Special and Supplementary Deposits dummies, either individually or jointly, must be rejected in all the models at both the 5% and 1% significance levels. Given that non-choice items as a single wealth variable and homogeneity are consistent with the data in this period we regard Model D1D as the maintained model for the post-1971 period.

Table 3.21
Likelihood Ratios for Testing Homogeneity and Non-choice Items as a Single Wealth Variable in the Dynamic Models

	Homogeneity		Single Wealth Variable	
	Single Wealth Variable	Independent Variables	Unconstrained	Homogeneity
	1959:01 to 1971:06			
Intercept SSD Dummy CC Dummy	19.96 (4)	26.82 (4)	47.02 (16)	53.88 (16)
Intercept SSD Dummy No CC Dummy	20.86 (4)	27.20 (4)	44.56 (16)	50.90 (16)
Intercept CC Dummy No SSD Dummy	15.52 (4)	22.32 (4)	42.46 (16)	49.26 (16)
SSD Dummy CC Dummy No Intercept	17.06 (4)	23.88 (4)	51.76 (16)	58.58 (16)
Intercept No SSD Dummy No CC Dummy	15.48 (4)	24.58 (4)	40.42 (16)	49.52 (16)
SSD Dummy No Intercept No CC Dummy	18.02 (4)	24.70 (4)	48.54 (16)	55.22 (16)
CC Dummy No Intercept No SSD Dummy	13.16 (4)	17.08 (4)	47.62 (16)	51.54 (16)
No Intercept No SSD Dummy No CC Dummy	13.20 (4)	44.72 (4)	44.72 (16)	51.34 (16)
	1971:10 to 1983:06			
Intercept SSD Dummy	4.84 (6)	3.02 (6)	27.98 (18)	26.16 (18)
Intercept No SSD Dummy	4.96 (6)	3.20 (6)	26.34 (18)	24.58 (18)
SSD Dummy No Intercept	8.24 (6)	6.76 (6)	25.06 (18)	23.58 (18)
No Intercept No SSD Dummy	7.06 (6)	6.42 (6)	23.92 (18)	23.28 (18)

Note: Figures in parentheses are the number of degrees of freedom *under the null hypothesis.*

Table 3.22
Likelihood Ratios for Testing the Validity of the Intercepts, SSD and CC Dummies in the Dynamic Models

	Non-choice Items as Independent Variables		Non-choice Items as A Single Wealth Variable	
	Unconstrained	Homogeneity	Unconstrained	Homogeneity
	1959:01 to 1971:06			
Intercept SSD Dummy No CC Dummy	5.18 (4)	6.08 (4)	2.72 (4)	3.10 (4)
Intercept CC Dummy No SSD Dummy	21.08 (4)	16.64 (4)	16.52 (4)	12.02 (4)
SSD Dummy CC Dummy No Intercept	11.86 (4)	8.96 (4)	16.60 (4)	13.66 (4)
Intercept No SSD Dummy No CC Dummy	26.14 (8)	21.66 (8)	19.54 (8)	17.30 (8)
SSD Dummy No Intercept No CC Dummy	17.36 (8)	15.42 (8)	18.88 (8)	16.76 (8)
CC Dummy No Intercept No SSD Dummy	32.70 (8)	25.90 (8)	33.30 (8)	23.56 (8)
No Intercept No SSD Dummy No CC Dummy	38.10 (12)	31.34 (12)	35.80 (12)	28.80 (12)
	1971:10 to 1983:06			
Intercept No SSD Dummy	18.52 (6)	18.64 (6)	16.88 (6)	17.06 (6)
SSD Dummy No Intercept	20.10 (6)	23.50 (6)	17.18 (6)	20.92 (6)
No Intercept No SSD Dummy	32.04 (12)	34.26 (12)	27.98 (12)	31.38 (12)

Note: Figures in parentheses are the number of degrees of freedom under the null hypothesis.

Table 3.23
FIML Estimates for the Dynamic Model with Intercepts, SSD Dummy and No Homogeneity — Non-choice Items as Independent Variables (1959:01 to 1971:06)

	ΔCM	ΔTB	ΔCB	ΔAD	ΔGB
K_i	-502.35	587.29	-70.45	-47.24	32.75
	(934.20)	(78.92)	(71.40)	(101.37)	
CMR	-20.76	1.68	9.36	16.24	-6.52
	(7.43)	(71.40)	(1.81)	(6.67)	
TBR	-46.44	79.46	-17.98	-10.76	-4.28
	(14.94)	(1.81)	(3.13)	(3.98)	
CBR	50.17	-49.70	-2.40	0.66	1.27
	(12.53)	(3.13)	(0.33)	(0.19)	
ADR	21.39	-31.58	9.02	-9.93	11.10
	(4.90)	(0.33)	(1.11)	(2.61)	
GBR	16.29	-22.72	7.41	21.34	-22.32
	(4.58)	(1.11)	(1.16)	(6.92)	
CN_t	-0.37	-0.43	-0.01	-0.10	-0.09
	(0.01)	(1.16)	(0.01)	(0.01)	
SD_t	-0.14	-0.44	0.10	-0.36	-0.16
	(0.01)	(0.01)	(0.01)	(0.01)	
EA_t	0.03	-0.68	-0.04	-0.18	-0.13
	(0.01)	(0.01)	(0.01)	(0.01)	
DD_t	0.27	0.57	0.02	0.04	0.10
	(0.01)	(0.01)	(0.01)	(0.01)	
TD_t	0.41	0.43	-0.01	0.05	0.12
	(0.01)	(0.01)	(0.01)	(0.01)	
CM_{t-1}	0.66	0.42	-0.01	-0.10	0.03
	(0.01)	(0.01)	(0.01)	(0.01)	
TB_{t-1}	0.23	0.76	0.01	-0.05	0.05
	(0.01)	(0.01)	(0.01)	(0.01)	
CB_{t-1}	0.23	0.74	0.11	-0.10	0.02
	(0.01)	(0.01)	(0.01)	(0.01)	
AD_{t-1}	0.26	0.53	-0.01	0.10	0.12
	(0.01)	(0.01)	(0.01)	(0.01)	
GB_{t-1}	0.23	0.58	0.01	0.02	0.17
	(0.01)	(0.01)	(0.01)	(0.01)	
D_{SD}	-30.07	37.77	-22.91	-11.57	26.84
	(63.40)	(78.92)	(27.76)	(27.40)	
SER	47.27	63.93	14.54	70.64	23.80
R^2	0.42	0.54	0.19	0.15	0.47
D-W	2.05	2.09	2.04	1.83	1.57

Note: Figures in parentheses are the ratios of estimated coefficients to asymptotic standard errors. T-ratios for the Government bond equation are not provided since this equation is not directly estimated.

Table 3.24
FIML Estimates for the Dynamic Model with Intercepts, SSD Dummy and Homogeneity — Non-choice Items as a Single Wealth Variable
(1971:10 to 1983:06)

	ΔCM	ΔTB	ΔCB	ΔAD	ΔLL	ΔCD	ΔGB
K_i	203.53	96.21	32.11	-768.18	-34.48	293.68	177.13
	(3234.31)	(762.63)	(252.62)	(13406.85)	(117.32)	(1974.45)	
CMR	-53.86	-5.38	1.39	59.24	5.57	2.80	-9.76
	(270.91)	(12.91)	(4.45)	(327.72)	(6.50)	(5.30)	
TBR	89.68	43.79	1.58	-166.03	-32.41	29.91	33.48
	(649.45)	(176.16)	(7.22)	(1311.55)	(66.40)	(105.75)	
CBR	-153.03	-2.24	9.44	179.79	20.54	-25.95	-28.55
	(1067.96)	(10.74)	(52.10)	(1351.64)	(56.86)	(90.98)	
ADR	127.01	2.34	-1.17	-132.86	1.93	1.07	1.68
	(613.29)	(8.17)	(4.33)	(669.56)	(3.34)	(3.14)	
LLR	317.16	-45.93	31.90	-106.73	-20.70	-43.49	-132.21
	(2005.33)	(202.47)	(154.53)	(722.41)	(51.08)	(143.47)	
CDR	-294.11	15.97	-37.31	182.76	23.04	28.86	80.79
	(1910.16)	(68.80)	(188.25)	(1285.24)	(53.71)	(93.29)	
GBR	-32.85	-8.55	-5.83	-16.17	2.03	6.80	54.57
W_t	0.34	0.01	0.04	0.54	-0.01	0.01	0.09
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	
CM_{t-1}	0.51	-0.01	0.03	0.52	-0.01	-0.03	-0.01
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	
TB_{t-1}	0.52	0.20	0.10	0.23	-0.03	0.08	-0.10
	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	
CB_{t-1}	0.73	0.05	0.23	-0.28	-0.01	0.13	0.15
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	
AD_{t-1}	0.28	0.01	0.04	0.62	0.01	0.01	0.03
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	
LL_{t-1}	0.39	-0.04	0.13	0.68	0.15	-0.05	-0.26
	(0.02)	(0.01)	(0.01)	(0.03)	(0.01)	(0.01)	
CD_{t-1}	0.43	0.02	-0.03	0.11	-0.03	0.27	0.23
	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	
GB_{t-1}	0.41	0.01	0.03	0.20	-0.03	0.08	0.30
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	
D_{SD}	-31.51	19.33	-7.61	196.09	-1.36	-75.15	-99.79
	(630.91)	(179.72)	(108.99)	(4629.31)	(5.45)	(627.05)	
SER	344.95	95.15	87.71	420.78	37.49	76.92	151.73
R^2	0.43	0.13	0.23	0.53	0.22	0.25	0.24
D-W	2.28	2.16	1.95	2.22	1.76	1.65	2.26

Note: Figures in parentheses are the ratios of estimated coefficients to asymptotic standard errors. T-ratios for the Government bond equation are not provided since this equation is not directly estimated.

FIML estimates of the maintained models for the two periods are given in Tables 3.23 and 3.24. Notwithstanding their different structures the estimates of the two models have very common features. First of all, unlike the estimates in the two maintained static models in Tables 3.13 and 3.14, most of the interest rate coefficients in both dynamic models are significant. In fact, all of the directly estimated interest rate coefficients in the post-1971 model are statistically different from zero and in general the asymptotic t ratios in the post-1971 model are much higher than those in the pre-1971 model. This is probably due partly to the lower multicollinearity among the interest rates in the later period. Moreover, all the intercepts and the Special and Supplementary Deposit dummy variables are very significant in both models. On the other hand, neither the coefficients of the separate non-choice items in Table 3.23 nor those of the single wealth variable in Table 3.24 are significant. One of the coefficients of the single wealth variable is even negatively signed.

It is also noteworthy that none of the adjustment coefficients in the two models are significantly different from zero. Indeed, in terms of goodness-of-fit both dynamic models perform badly. This is, however, not due to the presence of first-order vector autocorrelations among the disturbances, as was the case in the static models. Although the Durbin-Watson statistic for some of the equations are slightly low, the presence of first-order vector autocorrelation is not obvious. In fact, we reestimated the dynamic models under the assumption of first-order vector autocorrelation and likelihood ratio tests suggest that we have to accept the null hypothesis of zero-order vector autocorrelation among the disturbances (for economy of space these tests are not reported here). Indeed, one might argue that if the dynamic models in (3.82) are adequate, they should be able to capture at least partly the effects of the autoregressive process. Given that the data we employ are the monthly balance sheet data, the unusually low R^2s in the dynamic models, together with the fact that first-order vector autocorrelation is present in the static models, may simply indicate that the adjustment process developed in Section 3.4 is inappropriate for describing such short-run behaviour of commercial banks.

3.6.2.2 Dynamic Models With the Liquid Assets Ratio Constraints

In this sub-section we briefly discuss the test results for the dynamic models with the liquid asset ratio constraints. Given the static specification in (3.87) and the adjustment process discussed in Section 3.4 we can write the bank's dynamic model with the liquid asset ratio constraints as

$$\Delta A_{1t} = K + \Phi_1 r_1 + \Lambda_1 A^*_{2t} - B A_{1t-1} + F D + \epsilon_{dt} \qquad (3.92)$$

where

$$\Phi_1 = \begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} & 0 & 0 \\ \varphi_{21} & \varphi_{22} & \varphi_{23} & 0 & 0 \\ \varphi_{31} & \varphi_{32} & \varphi_{33} & 0 & 0 \\ 0 & 0 & 0 & \varphi_{44} & \varphi_{45} \\ 0 & 0 & 0 & \varphi_{54} & \varphi_{55} \end{bmatrix} \qquad \Lambda_1 = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \\ \lambda_{31} & \lambda_{32} \\ 0 & \lambda_{42} \\ 0 & \lambda_{52} \end{bmatrix} \qquad (3.93)$$

with

(1) $\Phi_1 \, i = 0$; (3.94a)

(2) $i' \, \Phi_1 = 0'$; (3.94b)

(3) $i' \, \Lambda_1 = - \, i'$; (3.94c)

(4) $K_1 + K_2 + K_3 = K_4 + K_5 = 0$; (3.94d)

(5) $i' \, F = 0'$; (3.94e)

(6) $i' \, B = i'$. (3.94f)

(3.94a) and (3.94b) are the homogeneity and Cournot conditions while (3.94c) are the Engel conditions. The restrictions embodied in (3.94d) to (3.94f) have similar interpretations as those in the dynamic models without the liquid asset ratio constraints except that the zero-sum restriction in (3.94d) applies to the intercepts in each

sub-portfolio. As in the dynamic models without the liquid asset ratio constraints, only the homogeneity restrictions can be tested directly. In order to test the validity of the homogeneity conditions the system of equations in (3.92) are estimated both with and without the homogeneity restrictions, together with several different combinations of the intercept and dummy variables. Sample values of the maximised log-likelihood function for the various models are presented in Table 3.25.

As in the dynamic models without the liquid asset ratio constraints, homogeneity must be rejected at the 5% significance level as empirically valid in the eight cases considered: the lowest likelihood ratio under the null hypothesis is 8.96 while 5% chi-square critical value with 3 degrees of freedom is 7.815. Even at the 1% significance level, homogeneity can be accepted in only two cases where the likelihood ratios are 9.44 and 8.96. Turning to the intercept and dummy variables we observe that the intercept and the Special and Supplementary Deposit dummies have to be accepted with or without the homogeneity restrictions, while the credit ceilings dummies are once again found to be inconsistent with the data. We therefore regard the theoretically unrestricted model with intercept and SSD dummies as the maintained hypothesis among the set of dynamic models constrained by the liquid asset ratio and the FIML estimates are reported in Table 3.26.

Table 3.25
Sample Values of the Maximised Log-likelihood Function
for the Dynamic Models with Liquid Asset Ratio Constraints

	Unconstrained	Homogeneity
	1959:01 to 1971:06	
Intercept SSD Dummy CC Dummy	-2910.98 (47)	-2923.30 (44)
Intercept SSD Dummy No CC Dummy	-2912.09 (43)	-2924.74 (40)
Intercept CC Dummy No SSD Dummy	-2921.07 (43)	-2929.16 (40)
SSD Dummy CC Dummy No Intercept	-2927.21 (43)	-2937.20 (40)
Intercept No SSD Dummy No CC Dummy	-2923.00 (39)	-2931.19 (36)
SSD Dummy No Intercept No CC Dummy	-2928.29 (39)	-2938.52 (36)
CC Dummy No Intercept No SSD Dummy	-2938.22 (39)	-2942.94 (36)
No Intercept No SSD Dummy No CC Dummy	-2940.18 (35)	-2944.66 (32)

Note: Figures in parentheses are the number of free coefficients estimated.

Table 3.26
FIML Estimates for the Dynamic Model with Liquid Asset Ratio Constraints, Intercepts, SSD Dummy and No Homogeneity (1959:01 to 1971:06)

	ΔCM	ΔTB	ΔCB	ΔAD	ΔGB
K_i	-641.75	593.78	47.97	-71.50	71.50
	(3.99)	(3.35)		(1.06)	
CMR	-20.25	11.12	9.13		
	(0.67)	(0.33)			
TBR	-54.10	51.96	2.14		
	(1.80)	(1.60)			
CBR	91.21	-82.17	-9.04		
	(2.56)	(2.09)			
ADR				-1.11	1.11
				(0.15)	
GBR				19.97	-19.97
				(1.74)	
R_r	0.52	0.46	0.02		
	(6.09)	(4.39)			
W^*	0.21	0.53	0.01	0.12	0.13
	(4.00)	(5.64)	(0.48)	(1.22)	
CM_{t-1}	0.55	0.46	-0.01	-0.01	0.01
	(8.71)	(5.07)	(0.41)	(0.05)	
TB_{t-1}	0.20	0.74	0.02	0.01	0.03
	(3.47)	(7.75)	(0.52)	(0.02)	
CB_{t-1}	0.06	0.75	0.02	0.03	0.14
	(0.42)	(3.44)	(0.42)	(0.14)	
AD_{t-1}	0.20	0.54	0.02	0.14	0.10
	(3.92)	(5.85)	(0.67)	(1.52)	
GB_{t-1}	0.14	0.61	0.03	0.07	0.15
	(2.42)	(5.82)	(1.01)	(0.76)	
D_{SD}	2.50	28.84	-11.63	-47.28	27.57
	(0.12)	(1.20)	(2.13)	(1.63)	
SER	48.84	65.46	15.40	71.99	24.70
R^2	0.38	0.52	0.09	0.12	0.42
D-W	2.09	2.08	1.95	1.76	1.43

Note: Figures in parentheses are the ratios of estimated coefficients to asymptotic standard errors. Some t-ratios for the Commercial bill and Government bond equations are not provided because they are not directly estimated.

Comparing the estimates in Table 3.26 with those in Table 3.23 we note first of all that while most of the interest rate coefficients in the latter are significant only two of the eight directly estimated interest rate coefficients are statistically different from zero in the former table. This is rather surprising, since given the fact that interest rates are highly correlated, the smaller number of interest rates appearing in each of the equations in the dynamic model with the liquid asset ratio constraints should help alleviate the problem of multicollinearity, hence reducing the estimated variances of the interest rate coefficients. Of course, it should be noted that the two tables are not strictly comparable because of different number of non-choice items in the systems of equations. In fact, while none of the non-choice items in Table 3.23 is statistically different from zero, four of the six directly estimated coefficients on the required liquid assets and the other wealth variable are very significant. Moreover, almost half of the adjustment coefficients are significantly different from zero in the model with the liquid asset ratio constraints compared with none in the model without the constraints. The equations in Table 3.23 have in general slightly lower standard errors of regression and higher $\bar{R}^2$s than those in Table 3.26, once again, suggesting that the imposition of the liquid asset ratio constraints does not lead to improvement in the performance of the model. However, this does not mean that the model without the liquid asset ratio constraints is consistent with the pre-1971 date. We shall leave the formal test of the two models till Chapter Six.

3.7 CONCLUDING REMARKS

In this chapter we developed a large number of of static and dynamic models of bank portfolio behaviour under the assumption of risk aversion. The static models are derived within the traditional mean-variance framework while the dynamic models are constructed employing the general stock adjustment process. The theoretical models are then estimated and tested using balance sheet data of the London clearing banks for the period 1959:01 to 1983:06. The whole sample period is divided into the pre-1971 period and the post-1971 period because of the significant changes in the number of items on the banks' balance sheet. Our main purpose as set out in the introduction to this chapter is to test the alternative models and the various restrictions rather than to provide estimates of a single model which can be judged as "best". One general and important conclusion emerging from the many tables of statistics reported in this chapter is that hypothesis test results differ greatly for the two sub-periods. We summerise our findings below.

(1) The theoretical restrictions of homogeneity are consistently and decisively being accepted by the post-1971 data, either in the static models or in the dynamic models. The symmetry restrictions are also consistent with the post-1971 data. On the other hand, the homogeneity and symmetry restrictions can only be accepted marginally in one case by the pre-1971 data. Indeed, the homogeneity restrictions are consistently being rejected in the dynamic models by the pre-1971 data, either with or without the liquid asset ratio constraints. Given the different environments in which the London clearing banks operated during the pre-1971 and the post-1971 periods the differing results in the two periods are not surprising. Compared with the pre-1971 period when a number of regulatory restrictions (e.g., credit ceilings, the liquid asset ratios and the Special deposit scheme) were imposed on the London clearing banks, the banks were relatively free to allocate their disposable funds among the available assets in the post-1971 period, and were thus able to respond more quickly to changes in interest rates. However, for the reasons to be discussed

below these conclusions should be regarded as very tentative.

(2) The models with a consolidated single wealth variable are consistently accepted by the post-1971 data in both the static and the dynamic models, but are always rejected by the pre-1971 data.

(3) For the dummy variables the test results are inconclusive. While the Special and Supplementary deposit dummy variables are all insignificant in the static models they are consistent with the data in the dynamic models for both the pre-1971 period and the post-1971 period. The credit ceilings dummy variables apply only to the pre-1971 period and they are consistently being rejcted by the data in the dynamic models, with or without the liquid asset ratio constraints. In the static models, however, they are consistent with the data when the model is constrained by the liquid asset ratio restrictions, although they are once again being rejected in the model without the liquid asset ratio restrictions.

(4) Notwithstanding the above differences in the test results among the models there is one feature which is common to both the static and dynamic models in both the pre-1971 period and the post-1971 period, namely, the consistent acceptance of the intercepts by the data. As we discussed earlier, the coexistence of intercepts and the symmetry restrictions are not consistent with any of the models developed in this chapter. One possible reason for this inconsistency is the use of actual as expected rates of return on assets. Indeed, the crux of the mean-variance approach to portfolio behaviour is the expectations formation process of the banks (regarding especially the rates of return) and most of the test results depend critically on the validity of the proxies for the expected rates of return. Ideally, before estimating the models and testing the theoretical restrictions one should model explicitly the banks' expectations process regarding the rates of return. Although such a task is beyond the scope of this study we shall nevertheless return to this issue briefly in the final chapter.

(5) In terms of goodness of fit, the models without the liquid asset ratio constraints perform better in general than those with the constraints. This is probably due to the way the liquid asset ratio constraints are incorporated into the models, which may not capture the effects on the banks' portfolio behaviour correctly. Further research in this area is obviously required.

(6) The performance of the dynamic models are in general very poor. This is probably due to restrictive assumptions about the functional form of the disequilibrium and transactions costs.

Given the many auxiliary but important assumptions employed to derive the models in this chapter one should not regard the rejection of the homogeneity and symmetry restrictions in the models for the pre-1971 period as a rejection of the mean-variance approach to bank portfolio behaviour *perse*. The rejection may be due to the use of proxies for the expected rates of return or to the inappropriate treatment of the liquid asset ratio constraints. Moreover, the acceptance of the homogeneity and symmetry restrictions in the post-1971 period does not necessarily imply that the models derived within the mean-variance framework are the "correct" models of the portfolio behaviour of the London clearing banks in the post-1971 period. What we are able to conclude from the test results is simply that the homogeneity and symmetry restrictions are consistent with the post-1971 data but not with the pre-1971 data. It is possible that the maintained models in this chapter themselves are rejected when tested against non-nested alternatives, and it is for this reason that no further policy conclusions will be contemplated at this stage. We develop one such set of non-nested alternative models in the next chapter.

CHAPTER FOUR

THE PRINCIPLE OF SAFETY FIRST AND BANK PORTFOLIO BEHAVIOUR UNDER UNCERTAINTY

4.1 INTRODUCTION

In Chapter Three the traditional mean-variance approach based on expected utility maximisation has been applied to the analysis of the portfolio behaviour of UK commercial banks. However, such an approach is not without its problems. First of all, as we have discussed in Chapter Two, unless the aggregate risk borne by the decision-maker is small compared to its total wealth, the mean-variance approach is only strictly applicable if either the utility function is quadratic or actual return is normally distributed [see, for example, Tsiang (1972)]. When neither of these assumptions, which have been frequently criticised in the literature, is satisfied, the mean-variance analysis can only be viewed as an approximation to the expected utility maximisation procedure[1].

More fundamentally, the mean-variance approach based on expected utility maximisation requires that the decision-maker, be it an individual investor or an institution, is able to list all the possible "states of nature" that could occur and knows the probability of each of these "states of nature" occurring. In other words, it assumes that decisions by individuals or institutions are made on the basis of infinitely repeatable experiments. In economic life, however, as Professor Shackle (1957) has reminded us, any decision is unique. An individual or an institution does not have the opportunity or the luxury to indulge in infinitely repeatable experiments. What might happen on the average, or for a large group of individuals or of institutions taken collectively, is of no relevance for the individual decision-maker. Indeed, "an experiment can be such as to destroy for ever the circumstances in which it was made. When he looks upon his action as a unique non-repeatable experiment, the decision-maker must abandon the idea

of frequency ..., and ask himself not what will happen in the long run but what can happen, at the best and at the worst, on this unique occasion" [Shackle (1957), p. 107]. Thus, in the case of an individual who endeavours to choose the best possible portfolio, his decision is necessarily constrained by the fact that he has only a limited amount of wealth and he stands to lose that wealth if he makes a totally wrong choice of portfolio.(2)

Finally, one might argue that, if economists themselves have difficulties in choosing an appropriate approach to model individual decision making under uncertainty, it will not be reasonable to assume that decision-makers in reality have a precise knowledge of all possible outcomes of a given course of action together with their respective probabilities. This is not to deny, however, the usefulness of probability theory in analysing portfolio behaviour, as probability can be interpreted as subjective beliefs [Savage (1950, 1951, 1954)], but to suggest that in reality expectations are generally framed in a much more vague, and, perhaps more conservative manner in the sense that decision-makers will strive to avoid the worst that can occur when making a decision. It is often argued that economic life bears close resemblance to navigation in poorly charted waters or manoeuver in a hostile jungle [Roy (1952)]. Thus, "decisions taken in practice are less concerned with whether a little more of this or of that will yield the largest net increase in satisfaction than with avoiding known rocks of uncertain position or with deploying forces so that, if there is an ambush round the next corner, total disaster is avoided" [Roy (1952), p. 432]. Such a rule of decision as propounded by Roy (1952) in the context of asset choice is known as the Principle of Safety First, which asserts that, when making a decision, the decision-maker will seek to minimise the probability of the target variable falling below a disaster level by suitable choice of the control variables.

The Principle of Safety First has been applied to the theory of risk in insurance companies by Cramér (1930), to the theory of the firm under price uncertainty by Arzac (1976), and recently to the problem of revenue raising under uncertainty by Dickinson,

Driscoll and Ford (1984). Such a rule of decision may be particularly relevant to a commercial bank, whose main resources are deposit liabilities. As Professor Hicks points out, the existence of liabilities as well as assets would ensure that the bank's attitude towards the chances of favourable or unfavourable outcomes cannot be symmetric. "A worse than expected outcome must be dreaded more than a better than expected outcome is desired ..., because of the impact which such unfavourable outcome may have upon the non-liquid elements in the situation" [Hicks (1962), p. 794]. There are many reasons why a bank might adopt the Principle of Safety First when it chooses its portfolio. Most of all, the nature of its liabilities presupposes that the very survival of the bank depends upon its ability to meet at any time deposit withdrawals. Moreover, because of the special role played by the banking system, monetary authorities may require that banks operate on a prudent basis. Failure of one bank may undermine the confidence of depositors in the whole banking system.

In this chapter, we explore the implications of applying the Principle of Safety First to the modelling of bank portfolio behaviour. The main advantage of employing the Principle of Safety First to model bank portfolio behaviour is that the concept of an expected value and the variance can be given an everyday interpretation which is more in line with real economic life. By appealing to the Tchebycheff's inequality, as we shall see shortly, the bank's optimal asset demand equations can be derived without specifying the exact probability distribution of actual return, hence avoiding some of the distributional problems with the mean-variance approach based on expected utility maximisation (see Chapter Two).

The rest of the chapter is organised as follows. Section 4.2 presents a brief description of the Principle of Safety First and its application to the individual's demand for assets. The bank's optimal static and dynamic asset demand equations as well as their static properties under the Principle of Safety First are discussed in Section 4.3. The bank's static and dynamic asset demand equations derived under the Principle of Safety First are highly nonlinear and the problems of estimating these nonlinear systems

of equations are dealt with in Section 4.4. Some empirical results using the Safety First model for UK commercial banks are reported in Section 4.5. Section 4.6 provides a summary of the chapter.

4.2 THE PRINCIPLE OF SAFETY FIRST AND INDIVIDUAL DEMAND FOR ASSETS

4.2.1 The Principle of Safety First

Suppose that, when choosing his portfolio, the investor is concerned that actual return at the end of a decision period from a given portfolio, $\prod$, should not be less than a predetermined level, D. Because the yields of assets at the end of each decision period are not known at the moment of decision, actual return from a given portfolio will be uncertain, and the investor will have to form expectations about it. Let us further assume that the investor is able to estimate from past data only the expected value of $\prod$, μ_{π}, and its variance, σ_{π}^2, which, assuming that the investor operates in a perfectly competitive market and thus can hold whatever level of each asset he chooses, are defined as

$$\mu_{\pi} = a' \; e \tag{4.1}$$

$$\sigma_{\pi}^2 = a' \; \Omega \; a \tag{4.2}$$

where, as before, a is an n × 1 vector of asset levels, e an n × 1 vector of expected rates of return and Ω is the n × n variance-covariance matrix of the n asset returns.

The investor's objective is then to minimise the probability that actual return from a given portfolio is less than the critical level, D, i.e., the investor will seek to minimise

$$\Pr \; (\prod \leqslant D) \tag{4.3}$$

where Pr denotes probability. Without specifying the exact distributional form of actual return, however, it will not be possible to determine the precise probability that actual return at the end of the decision period is less than or equal to the predetermined

critical level, for a given pair of values of μ_π and σ_π^2. Fortunately, we may appeal to the Tchebycheff's inequality which permits us to place an upper bound on the probability of disaster. The Tchebycheff's inequality states that if $\prod$ is a random variable with mean μ_π and variance σ_π^2, then for any real number $\tau > 0$

$$\Pr\ (|\prod - \mu_\pi| \geqslant \tau) \leqslant \frac{\sigma_\pi^2}{\tau^2} \qquad (4.4)$$

namely, the probability that the random variable, $\prod$, will differ from its mean, μ_π, by more than a fixed number, τ, is less than or equal to the ratio of its variance to the square of the fixed number. Let $\tau = \mu_\pi - D$, then we can rewrite (4.4) as

$$\Pr\ (|\prod - \mu_\pi| \geqslant \mu_\pi - D) \leqslant \frac{\sigma_\pi^2}{(\mu_\pi - D)^2} .$$

Since the inequality above applies to both tails of a probability distribution it must be the case that

$$\Pr\ (\mu_\pi - \prod \geqslant \mu_\pi - D) \leqslant \Pr\ (|\prod - \mu_\pi| \geqslant \mu_\pi - D) \leqslant \frac{\sigma_\pi^2}{(\mu_\pi - D)^2} .$$

Hence

$$\Pr\ (\prod \leqslant D) \leqslant \frac{\sigma_\pi^2}{(\mu_\pi - D)^2} . \qquad (4.5)$$

We may note that if $\prod$ is assumed to follow a symmetric distribution (4.5) can be written as

$$\Pr\ (\prod \leqslant D) \leqslant \frac{\sigma_\pi^2}{2(\mu_\pi - D)^2} .$$

Indeed, if $\prod$ is normally distributed the inequality becomes an equality:

$$\Pr\ (\prod \leqslant D) = \frac{\sigma_{\pi}^2}{2(\mu_{\pi} - D)^2} .$$

However, since we have argued that decision makers may not have a precise knowledge of all possible outcomes of a given course of action or their respective probabilities we shall not in the ensuing analysis assume any specific probability distribution for the actual return, $\prod$.

Equation (4.5) expresses the upper bound of the probability that actual return will fall short of the critical level in terms of the mean value and the variance of actual return and the critical value itself. If we assume that the investor seeking to reduce the probability of disaster endeavours to minimise the upper bound of this probability, the minimisation of (4.3) is then equivalent to the maximisation of

$$Q = \frac{(\mu_{\pi} - D)^2}{\sigma_{\pi}^2} . \qquad (4.6)$$

It is interesting to note that, if σ_{π}^2 is constant for all values of μ_{π}, then the maximisation of equation (4.6) is equivalent to maximising expected return for any given disaster level, suggesting that the above procedure based upon the Principle of Safety First may be regarded as a generalisation of profit maximisation under uncertainty. More encouragingly, equation (4.6) involves only two parameters of the probability distribution of $\prod$, namely the mean and the variance. This indicates that the optimal portfolio derived from the Principle of Safety First can also be expressed, as in the traditional mean-variance analysis of portfolio selection based on expected utility maximisation, in terms of the best combination of μ_{π} and σ_{π}^2 [for a comparison of the Safety First principle and expected utility maximisation in mean-variance portfolio analysis see Pyle and Turnovsky (1970)]. Indeed, given the values of μ_{π} and σ_{π}^2 for all possible portfolios there exists a functional relationship such that

$$f\ (\mu_\pi,\ \sigma_\pi^2\ ) = 0$$

which is the efficient mean-variance frontier.

4.2.2 The Efficient Mean-Variance Frontier

To obtain the efficient mean-variance frontier[3] we choose a to

$$\text{minimise} \qquad \sigma_\pi^2 = a'\ \Omega\ a \tag{4.7}$$

$$\text{subject to} \qquad \mu_\pi = a'\ e \tag{4.8}$$

$$W = a'\ i \tag{4.9}$$

where W is total wealth. Equation (4.9) is of course the wealth constraint, namely, the sum of choice assets must equal total wealth. Forming the Lagrangean

$$L = a'\ \Omega\ a + 2\lambda_1\ (\mu_\pi - a'\ e) + 2\lambda_2\ (W - a'\ i)$$

where λ_1 and λ_2 are the Lagrangean multipliers, and differentiating it with respect to the choice vector a and λ_1 and λ_2 we obtain the following first-order conditions:

$$\frac{\partial L}{\partial a} = 2\Omega\ a - 2\lambda_1\ e - 2\lambda_2\ i = 0 \tag{4.10}$$

$$\frac{\partial L}{\partial \lambda_1} = 2\mu_\pi - 2a'\ e = 0 \tag{4.11}$$

$$\frac{\partial L}{\partial \lambda_2} = 2W - 2a'\ i = 0. \tag{4.12}$$

Rearranging (4.10) gives

$$a = \Omega^{-1} (\lambda_1 e + \lambda_2 i) \tag{4.13}$$

where Ω^{-1} is the inverse of Ω. However, before this solution vector can be substituted back into (4.7) to obtain the efficient mean-variance frontier the two undetermined Lagrangean multipliers must be eliminated. To follow the procedure suggested by Roy (1952) and developed by Ford (1983) we substitute (4.13) for a into equations (4.7) - (4.9), so that

$$\sigma_\pi^2 = (\lambda_1 e + \lambda_2 i)' \Omega^{-1} (\lambda_1 e + \lambda_2 i) \tag{4.14}$$

$$\mu_\pi = (\lambda_1 e + \lambda_2 i)' \Omega^{-1} e \tag{4.15}$$

$$W = (\lambda_1 e + \lambda_2 i)' \Omega^{-1} i. \tag{4.16}$$

Expanding (4.14) yields

$$\sigma_\pi^2 - \lambda_1 (\lambda_1 e + \lambda_2 i)' \Omega^{-1} e - \lambda_2 (\lambda_1 e + \lambda_2 i)' \Omega^{-1} i = 0$$

which upon utilising equations (4.15) and (4.16) can be expressed as

$$\sigma_\pi^2 - \lambda_1 \mu_\pi - \lambda_2 W = 0. \tag{4.17}$$

But from equations (4.15) and (4.16) we deduce

$$\lambda_1 = (1/d)[i' \Omega^{-1} i \mu_\pi - i' \Omega^{-1} e W] \tag{4.18}$$

$$\lambda_2 = (1/d)[e' \Omega^{-1} e W - e' \Omega^{-1} i \mu_\pi] \tag{4.19}$$

where

$$d = e' \Omega^{-1} e \; i' \Omega^{-1} i - e' \Omega^{-1} i \; i' \Omega^{-1} e. \qquad (4.20)$$

Substituting these expressions into (4.17) for λ_1 and λ_2 we obtain the efficient mean-variance frontier as

$$d \sigma_{\pi}^2 - i' \Omega^{-1} i \mu_{\pi}^2 + 2i' \Omega^{-1} e \mu_{\pi} W - e' \Omega^{-1} e W^2 = 0. \qquad (4.21)$$

Along this frontier all possible portfolios are efficient. In fact, this efficient frontier is a hyperbolic function in the μ_{π}-σ_{π} space since, by rearranging (4.21), we have

$$d\eta \; (\sigma_{\pi}^2 - \eta W^2) = (\mu_{\pi} - \delta W)^2 \qquad (4.22)$$

where

$$\delta = \frac{i' \Omega^{-1} e}{i' \Omega^{-1} i}$$

$$\eta = \frac{1}{i' \Omega^{-1} i} .$$

Once the investor has located his preferred position along the mean-variance frontier, (4.22) will determine the optimal value of the mean return, μ_{π}, which can then be used in equations (4.18), (4.19) and (4.13) to yield the investor's optimal asset holdings, *a*. But how does the investor locate his optimal position along the mean-variance frontier? We answer this question in the next subsection.

4.2.3 The Investor's Optimal Asset Holdings

As we have just seen, the efficient mean-variance frontier is a hyperbolic function in the μ_{π}-σ_{π} space. This function is drawn as the curve AB in Figure 4.1. The critical level D is ploted on the μ_{π}-axis as point (0, D). The investor's objective is to

choose the portfolio which maximises the quantity in (4.10). The slightest acquaintance with trigonometry will suggest that this is the same as maximising the slope of the line from the point (0, D) to the efficient set of portfolios since the slope, being the tangent of the angle θ°, is simply the ratio of PE to DE which is equal to the square root of the quantity in (4.6). The investor will then choose the portfolio at point P where the line DP is tangential to the efficient mean-variance frontier [see Ford (1983)].

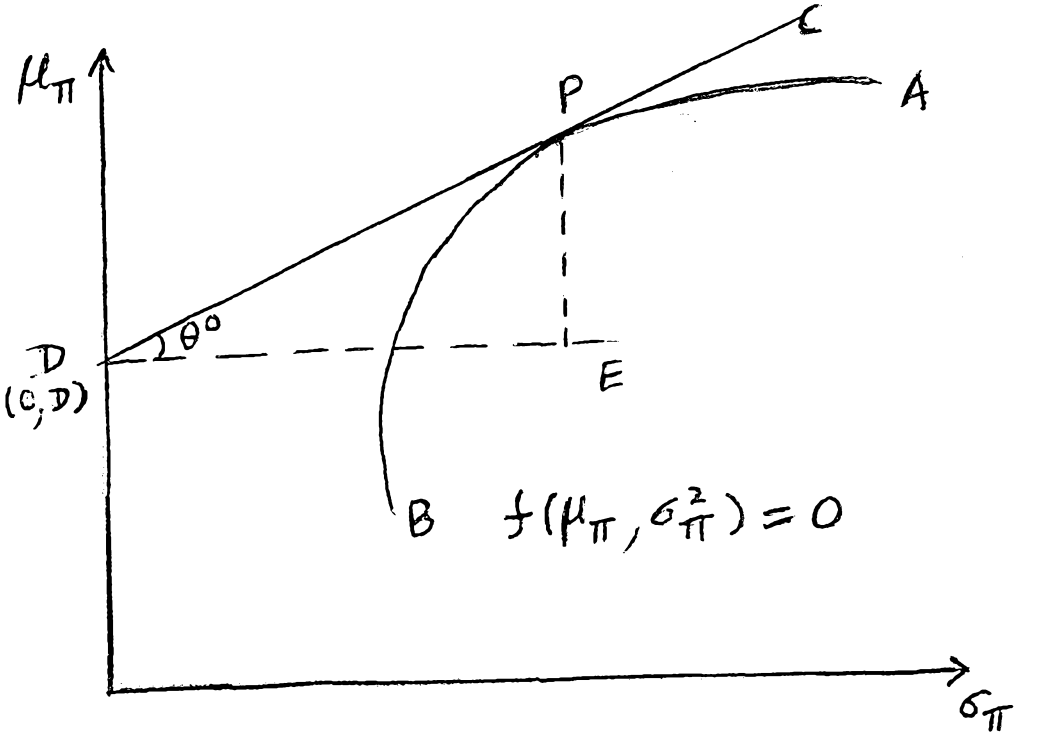


Figure 4.1

The Geometric Determination of Optimal Asset Holdings

From Figure 4.1 it is clear that the line DE can be represented by the linear function

$$\mu_\pi = D + k\sigma_\pi \tag{4.23}$$

where

$$k = \frac{\mu_\pi - D}{\sigma_\pi} \tag{4.24}$$

is the slope to be maximised. To obtain the value of σ_π at which (4.23) is tangential to the efficient mean-variance frontier we simply set μ_π in the frontier equation (4.22) equal to the μ_π in (4.23):

$$d\eta\ (\sigma_\pi^2 - \eta\ W^2) = (D + k\sigma_\pi - \delta W)^2$$

which can be written as a quadratic function of σ_π

$$a\sigma_\pi^2 + b\sigma_\pi + c = 0 \tag{4.25}$$

where

$$a = d\eta - k^2$$

$$b = 2k\ (\delta W - D)$$

$$c = 2\delta DW - \delta^2\ W^2 - d\eta^2\ W^2 - D^2.$$

There will be in general two solutions for σ_π, which are given by

$$\sigma_\pi = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} . \qquad (4.26)$$

However, at the point of tangency it must be the case that only one of these roots is allowed; and as we can appreciate from the above formula this requires that

$$b^2 - 4ac = 0$$

or

$$4k^2 (\delta W - D)^2 = 4 (k^2 - d\eta)[(D - \delta W)^2 + d\eta^2 W^2]$$

from which we deduce the optimal value for k^2 as

$$k^2 = d\eta[1 + \frac{(D - \delta W)^2}{d\eta^2 W^2}]$$

which, given the definitions of d, δ and η, can be written as

$$k^2 = e' \Omega^{-1} e - 2 e' \Omega^{-1} i \frac{D}{W} + i' \Omega^{-1} i \frac{D^2}{W^2}$$

$$= (e - \frac{D}{W} i)' \Omega^{-1} (e - \frac{D}{W} i). \qquad (4.27)$$

The reciprocal of k^2 is of course the upper bound of the probability that actual return will be less than or equal to the critical level D when the optimal portfolio is chosen. We may note that, since k^2 is quadratic in $[e - (D/W) i]$, the upper bound of the probability, $1/k^2$, will be zero if and only if the determinant of Ω, $|\Omega|$, is zero. This implies, however, as we shall see later, that an optimal solution for asset holdings cannot be obtained. We therefore rule out such a possibility.

Now we are at a position to solve for the optimal asset holdings from the efficient mean-variance frontier by utilising the expression in (4.27) for k^2 [see Roy (1952) and Ford (1983)]. From (4.23) we have

$$\frac{\partial \mu_\pi}{\partial \sigma_\pi} = k \tag{4.28}$$

and

$$\sigma_\pi = \frac{\mu_\pi - D}{k} \, . \tag{4.29}$$

At the point of tangency (4.28) must equal the slope of the efficient mean-variance frontier which from (4.22) is

$$\frac{\partial \mu_\pi}{\partial \sigma_\pi} = \frac{d\eta}{\mu_\pi - \delta W} \sigma_\pi$$

$$= \frac{d\eta}{\mu_\pi - \delta W} \left(\frac{\mu_\pi - D}{k} \right). \tag{4.30}$$

Setting (4.28) equal to (4.30) gives the optimal value of μ_π:

$$\mu_\pi = \frac{k^2 \delta W - d\eta D}{k^2 - d\eta} \tag{4.31}$$

which upon substituting for k^2 from (4.27) and δ and η from (4.22) becomes

$$\mu_\pi = \frac{[e - (D/W)\, i]' \, \Omega^{-1} e}{[e - (D/W)\, i]' \, \Omega^{-1} i} W. \tag{4.32}$$

We slot this value for μ_π into equations (4.18) and (4.19), so that

$$\lambda_1 = \frac{W}{[e - (D/W)\ i]'\ \Omega^{-1}\ i} \tag{4.33}$$

$$\lambda_2 = \frac{-D}{[e - (D/W)\ i]'\ \Omega^{-1}\ i}. \tag{4.34}$$

Finally, substituting these values for λ_1 and λ_2 into equation (4.13) we obtain the investor's optimal asset holdings as

$$a = \frac{\Omega^{-1}\ [e - (D/W)\ i]}{[e - (D/W)\ i]'\ \Omega^{-1}\ i}\ W. \tag{4.35}$$

A cursory inspection of (4.35) shows that these asset demand equations differ from those derived from expected utility maximisation in Chapters Two and Three.

Let ω_{ij} be the cofactor of the (j,i)th element of Ω, then

$$\Omega^{-1} = \frac{1}{|\Omega|}\ M$$

where $|\Omega|$, the determinant of Ω, is positive semi-definite, and M is the adjugate matrix of Ω with ω_{ij} being its (i,j)th element. We assume here that $|\Omega|$ is positive definite. That is, all the rates of return are not linearly dependent in the mathematical sense. In particular, we assume $\sigma_{ii} > 0$ for all i. This implies that there exists no riskless assets in the choice set. Since Ω, being the variance-covariance matrix, is symmetric, M is also symmetric. Further we define

$$\omega_i = \sum_j^n \omega_{ij} \tag{4.36}$$

as the row sum of the adjugate matrix M. In the special case where Ω is diagonal so that M is also diagonal, we have

$$\omega_i = \omega_{ii}. \tag{4.37}$$

Additionally, from the symmetry of M we infer that

$$\omega_i = \sum_j^n \omega_{ij} = \sum_j^n \omega_{ji}. \tag{4.38}$$

Using these notations we may express the investor's optimal holding of asset i as

$$a_i = \frac{\omega_{i1}[e_1 - (D/W)] + \omega_{i2}[e_2 - (D/W)] + \ldots + \omega_{in}[e_n - (D/W)]}{\omega_1[e_1 - (D/W)] + \omega_2[e_2 - (D/W)] + \ldots + \omega_n[e_n - (D/W)]} W. \tag{4.39}$$

Therefore, the investor's optimal demand for asset i will depend upon the expected rates of return on all assets as well as his total wealth and the level of his predetermined disaster return. However, unlike the asset demand equations derived from expected utility maximisation, which are linear in expected returns and total wealth, the asset demand equations in (4.39) are nonlinear in the expected returns as well as in total wealth. In fact, we note that all the asset demand equations in (4.39) will have the same denominator. This nonlinearity is very interesting. For example, if all the asset returns are not correlated with each other, i.e., if Ω is diagonal, then, given the definition of ω_i above we find

$$a_i = \frac{\omega_{ii}[e_i - (D/W)]}{\omega_{11}[e_1 - (D/W)] + \omega_{22}[e_2 - (D/W)] + \ldots + \omega_{nn}[e_n - (D/W)]} W. \tag{4.40}$$

Thus, even when asset returns are not correlated with each other, the demand for one asset will still depend upon his own expected rate of return as well as upon the expected rates of return on all other assets in the choice set.

More importantly, we note that in (4.39) the ratio of disaster return to total wealth,

D/W, is subtracted from each and every expected rate of return on assets. Suppose we were to hold all our wealth in only one asset and its price at the end of the decision period fell to or below D/W, then the disaster we seek to avoid would have happened. Thus, the ratio D/W, called the *critical price* by Roy (1952), is the criterion which we can use to assess the attractiveness of each asset. To decide which asset to hold in the portfolio we consider whether the expected rate of return on the asset under examination is greater or less than the critical price, if the expected rates of return on all other assets fell to the critical price. Should the expected rate of return on this particular asset be greater than the critical price, we should then hold some of our total wealth in this asset. Conversely, when the expected rate of return on the concerned asset is less than the critical price we should then exclude this asset from our portfolio, or indeed, contract liabilities in this form, if this is possible. How much of each asset to hold in the portfolio will then depend on both the difference between the expected rate of return on the particular asset and the critical price and the riskiness of this asset, i.e. its variance [for a formulation and interpretation of the disaster rate of return within the expected utility maximisation see Hicks (1977a)].

In general, the investor will hold a diversified portfolio. Consider for example the special case where the expected rates of return on all n assets in the choice set are the same. Will the investor be indifferent as to which assets to hold? The answer is clearly no as Roy (1952) noted, for, although the investor cannot affect the expected return by varying his holdings, he can alter the uncertainty about the return. If the investor holds all his wealth in only one of the n assets with the same expected rate of return, the upper bound of the probability that actual return at the end of the decision period will fall below the critical level is, according to (4.27)

$$\Pr\ (\prod \leq D) = \frac{\sigma^2}{e - (D/W)}$$

where e is the identical expected rate of return and σ^2 its variance. However, if the investor spreads his total wealth among the n assets in equal amounts the upper bound

of the probability reduces to

$$\Pr\ (\prod \leq D) = \frac{\sigma^2}{n[e - (D/W)]}$$

which, as can be shown, is the least of the upper bound of the probability the investor can expect under these conditions.

4.3 THE PRINCIPLE OF SAFETY FIRST AND BANK PORTFOLIO BEHAVIOUR

We assume that, when choosing its portfolio, the bank is concerned that actual return at the end of a decision period from a given portfolio, $\prod$, should not be less than a predetermined level, R. This predetermined level of return could be the target set by the bank to assess the performance of its management team, or it could be that level of return the management deems necessary for the bank to break even. Whatever this level may be, it suffices for the moment to assume that such a target or critical level does exist for the bank. Unlike the individual investor the bank is not able to determine the level of each and every asset/liability. As in Chapter Three, we partition the bank's balance sheet into choice and non-choice sets. Treating liabilities as negative assets, then the bank's actual return from a given portfolio is

$$\prod = A_1' r_1 + A_2' r_2$$

where A_1 is the m × 1 vector of choice items, r_1 the m × 1 vector of rates of return on the choice items, A_2 the (n-m) × 1 vector of non-choice items and r_2 the (n-m) × 1 vector of rates of return on the non-choice items. Because the actual levels of assets and liabilities and their yields at the end of each decision period are not known at the moment of decision, actual return from a given portfolio will be uncertain, and the bank will have to form expectations about it. Let us further assume that the bank is able to estimate from past data only the expected value of $\prod$, μ_π, and its variance, σ_π^2.

Assuming that the rates of return on assets/liabilities are not correlated with the levels of these assets/liabilities and that only the rates of return within the choice set are correlated with each other, we can then write the expected return μ_π and its variance σ_π^2 as

$$\mu_\pi = \hat{A}_1' e_1 + \hat{A}_2' e_2 \qquad (4.41)$$

$$\sigma_{\pi}^2 = \hat{A}_1' \, \Omega_{11} \, \hat{A}_1 \qquad (4.42)$$

where $\hat{A}_1$, $\hat{A}_2$, e_1 and e_2 are the expected values of A_1, A_2, r_1 and r_2 respectively, and Ω_{11} is the variance-covariance matrix of the rates of return on the choice items.

4.3.1 The Bank's Static Asset Demand Equations

If the bank's objective is to minimise the probability that actual return from a given portfolio is less than the critical level, R, then, as we have seen in the last section, the bank will seek to maximise

$$Q = \frac{(\mu_{\pi} - R)^2}{\sigma_{\pi}^2} \qquad (4.43)$$

which is the reciprocal of the upper bound of the probability. However, as we mentioned in the last section, a direct maximisation of the above quantity is not possible because it involves higher than second-order powers of the choice vector A_1. To solve for the bank's optimal portfolio we follow the method employed in the last section: we first derive the bank's efficient mean-variance frontier along which the optimal value of μ_{π} is located; after we have determined the optimal value of μ_{π} we can then use the first-order conditions for the efficient mean-variance frontier to arrive at the optimal portfolio, A_1.

To obtain the efficient mean-variance frontier we choose A_1 to

minimise $\qquad \sigma_{\pi}^2 = \hat{A}_1' \, \Omega_{11} \, \hat{A}_1 \qquad (4.44)$

subject to $\qquad \mu_{\pi} = \hat{A}_1' \, e_1 + \hat{A}_2' \, e_2 \qquad (4.45)$

$$\hat{A}_1' \, i_1 + \hat{A}_2' \, i_2 = 0. \qquad (4.46)$$

Equation (4.46) is the balance sheet constraint, namely, the sum of all assets must equal the sum of all liabilities. Forming the Lagrangean

$$L = \hat{A}_1' \Omega_{11} \hat{A}_1 + 2\lambda_1 (\mu_\pi - \hat{A}_1' e_1 - \hat{A}_2' e_2) + 2\lambda_2 (\hat{A}_1' i_1 + \hat{A}_2' i_2)$$

where λ_1 and λ_2 are, as in the last section, the Lagrangean multipliers, and differentiating it with respect to the choice vector $\hat{A}_1$ and λ_1 and λ_2 we obtain the following first-order conditions:

$$\frac{\partial L}{\partial \hat{A}_1} = 2\Omega_{11} \hat{A}_1 - 2\lambda_1 e_1 - 2\lambda_2 i_1 = 0 \qquad (4.47)$$

$$\frac{\partial L}{\partial \lambda_1} = 2\mu_\pi - 2\hat{A}_1' e_1 - 2\hat{A}_2' e_2 = 0 \qquad (4.48)$$

$$\frac{\partial L}{\partial \lambda_2} = 2A_1' i_1 + 2\hat{A}_2' i_2 = 0. \qquad (4.49)$$

Rearranging (4.47) gives

$$\hat{A}_1 = \Omega_{11}^{-1} (\lambda_1 e_1 - \lambda_2 i_1). \qquad (4.50)$$

However, this solution vector involves the two undetermined Lagrangean multipliers. To eliminate λ_1 and λ_2 we first substitute (4.50) for $\hat{A}_1$ into equations (4.44) - (4.46), so that

$$\sigma_\pi^2 = (\lambda_1 e_1 - \lambda_2 i_1)' \Omega_{11}^{-1} (\lambda_1 e_1 - \lambda_2 i_1) \qquad (4.51)$$

$$\mu_\pi = (\lambda_1 e_1 - \lambda_2 i_1)' \Omega_{11}^{-1} e_1 + \hat{A}_2' e_2 \qquad (4.52)$$

$$\hat{A}_2' = - (\lambda_1 e_1 - \lambda_2 i)' \Omega_{11}^{-1} i_1. \qquad (4.53)$$

Expanding (4.51) and utilising (4.52) and (4.53) we have

$$\sigma_{\pi}^2 - \lambda_1 (\mu_{\pi} - \hat{A}_2' e_2) - \lambda_2 \hat{A}_2' i_2 = 0. \tag{4.54}$$

But from equations (4.52) and (4.53) we can solve for λ_1 and λ_2 as

$$\lambda_1 = (1/d)[i_1' \Omega_{11}^{-1} i_1 (\mu_{\pi} - d_1) + i_1' \Omega_{11}^{-1} e_1 d_2] \tag{4.55}$$

$$\lambda_2 = (1/d)[e_1' \Omega_{11}^{-1} e_1 d_2 + e_1' \Omega_{11}^{-1} i_1 (\mu_{\pi} - d_1)] \tag{4.56}$$

where

$$d = e_1' \Omega_{11}^{-1} e_1 \, i_1' \Omega_{11}^{-1} i_1 - e_1' \Omega_{11}^{-1} i_1 \, i_1' \Omega_{11}^{-1} e_1$$

$$d_1 = \hat{A}_2' e_2$$

$$d_2 = \hat{A}_2' i_2. \tag{4.57}$$

Substituting these expressions into (4.54) for λ_1 and λ_2 we obtain the efficient mean-variance frontier as

$$d \sigma_{\pi}^2 - i_1' \Omega_{11}^{-1} i_1 \mu_{\pi}^2 + 2(i'_1 \Omega_{11}^{-1} i_1 d_1 - i_1' \Omega_{11}^{-1} e_1 d_2) \mu_{\pi}$$

$$- i_1' \Omega_{11}^{-1} i_1' d_1^2 + 2 i_1' \Omega_{11}^{-1} e_1 d_1 d_2 - e_1' \Omega_{11}^{-1} e_1 d_2^2 = 0. \tag{4.58}$$

Along this frontier all possible portfolios are efficient. Dividing (4.58) through by $i_1' \Omega_{11}^{-1} i_1$ and letting

$$\delta = \frac{i_1' \, \Omega_{11}^{-1} e_1}{i_1' \, \Omega_{11}^{-1} i_1}$$

$$\gamma = \frac{e_1' \, \Omega_{11}^{-1} \, e_1}{i_1' \, \Omega_{11}^{-1} \, i_1}$$

$$\eta = \frac{1}{i_1' \, \Omega_{11}^{-1} \, i_1}.$$

we can rewrite the efficient frontier as a hyperbolic function in the μ_π-σ_π space:

$$d\eta \, \sigma_\pi^2 + (\delta^2 - \gamma) \, d_2^2 = [\mu_\pi - (d_1 - \delta d_2)]^2. \qquad (4.59)$$

Once the bank has located its preferred position along the mean-variance frontier, (4.59) will determine the optimal value of the mean return, μ_π, which can then be used in equations (4.55), (4.56) and (4.50) to yield the bank's optimal asset holdings, A_1. After some algebraic manipulation we obtain the bank's optimal static asset demand equations as

$$A_1 = - \frac{\Omega_{11}^{-1}[e_1 - (d_1 - R)/d_2 \, i_1]}{[e_1 - (d_1 - R)/d_2 \, i_1]' \, \Omega_{11}^{-1} \, i_1} \, d_2. \qquad (4.60)$$

A detailed derivation of the above system of equations is provided in the Appendix to this chapter. A non-trivial solution requires that the inverse matrix Ω_{11}^{-1} exists. Let m_{ij} be the cofactor of the (j,i)th element of Ω_{11}, then

$$\Omega_{11}^{-1} = \frac{1}{|\Omega_{11}|} M_{11} \qquad (4.61)$$

where $|\Omega_{11}|$ is the determinant of Ω_{11} and M_{11} is the adjugate matrix of Ω_{11} with m_{ij} being its (i,j)th element. The inverse of Ω_{11} exists if and only if $|\Omega_{11}|$ is positive definite. This requires that all the rates of return on the choice items are not linearly dependent in the mathematical sense. In particular, we assume $\sigma_{ii} > 0$ for all i. This implies that there exists no riskless assets in the choice set. Further we define

$$m_i = \sum_{j=1}^{m} m_{ij} \tag{4.62}$$

as the row sum of the adjugate matrix M_{11}. Since Ω_{11}, being the covariance matrix, is symmetric, M_{11} is also symmetric, so that we have

$$m_i = \sum_{j=1}^{m} m_{ij} = \sum_{j=1}^{m} m_{ji} \tag{4.63}$$

which, in the special case where Ω_{11} is diagonal so that M_{11} is also diagonal, reduces to

$$m_i = m_{ii}.$$

Denoting the ith elements of A_1 and e_1 by $\hat{a}_{1i}$ and e_{1i} respectively, we may express the bank's optimal holding of asset i as

$$\hat{a}_{1i} = -\frac{m_{i1}[e_{11}-(d_1-R)/d_2]+ m_{i2}[e_{12}-(d_1-R)/d_2] + \ldots + m_{im}[e_{1m}-(d_1-R)/d_2]}{m_1[e_{11}-(d_1-R)/d_2] + m_2[e_{12}-(d_1-R)/d_2] + \ldots + m_m[e_{1m}-(d_1-R)/d_2]}\, d_2. \tag{4.64}$$

Unlike the asset demand equations derived in Chapter Three from expected utility maximisation, which are linear in expected rates of return and the non-choice items, these assets demand equations are highly nonlinear and resemble those derived in the last section for the individual investor. As in the case of the individual investor the denominator in (4.64) is independent of the subscript i, so that all asset demand equations will have the same denominator. It is interesting to note that the quantity $(d_1 - R)/d_2$ is subtracted from each and every expected rate of return on the choice items. Recall that $d_1 = A_2' e_2$ and $d_2 = A_2' i_2$ where A_2 is the m × 1 vector of non-choice items including both assets and liabilities. If, and as will be assumed, all liabilities of the bank are contained in A_2, then given that liabilities are treated as negative assets, d_2

is thus the total disposable fund the bank can invest in the choice assets. However, unlike the individual case, the bank has to pay interest on its liabilities and the net cost of these liabilities is d_1. How does the bank decide which asset to hold in its portfolio? Letting

$$c_n = d_1/d_2 = (\hat{A}_2' e_2)/(\hat{A}_2' i_2) \tag{4.65}$$

be the *net unit cost* of disposable funds and

$$r_c = - R/d_2 \tag{4.66}$$

be the *disaster rate of return* we can then rewrite the bank's demand for assets i as

$$\hat{a}_{1i} = - \frac{m_{i1}[(e_{11}-c_n) - r_c] + m_{i2}[(e_{12}-c_n) - r_c] + \ldots + m_{im}[(e_{1m}-c_n) - r_c]}{m_1[(e_{11}-c_n) - r_c] + m_2[(e_{12}-c_n) - r_c] + \ldots + m_m[(e_{1m}-c_n) - r_c]} d_2. \tag{4.67}$$

The terms in the parentheses are then the *net expected rates of return*, i.e. the gross expected rates of return minus the *net unit cost* of disposable funds. In deciding which asset to hold in its portfolio the bank considers whether the *net expected rate of return* on the asset in question is greater than the *disaster rate of return*, if the *net expected rates of return* on all other assets fell to the *disaster rate of return*. Should the *net expected rate of return* on this particular asset be greater than the *disaster rate of return*, the bank would then hold some of its total disposable funds in this asset. Conversely, when the *net expected rate of return* on the concerned asset is less than the *disaster rate of return* the bank would then exclude this asset from its portfolio, or indeed, contract liabilities in this form, if this is possible. How much of each asset to hold in the portfolio will then depend on both the difference between the *net expected rate of return* on the particular asset and the *disaster rate of return* and the riskiness of

this asset, i.e. its variance. Similar to the case of an individual investor the bank's *disaster rate of return* depends only upon the predetermined level of return, R and total disposable funds. However, unlike the case of the individual investor, the *net rate of return* on an asset depends not only on the gross expected rate of return on that particular asset but also on the *net unit cost* of the bank's disposable funds, which in turn depends upon the levels of the non-choice items as well as their expected rates of return. In other words, both the levels of non-choice items and the expected rates of returns on them will affect the bank's optimal holdings for the choice items. In the case of *net unit cost* of disposable funds being zero, i.e. $c_n = 0$, the bank's optimal demand for asset i reduces to

$$\hat{a}_{1i} = - \frac{m_{i1}[e_{11} - r_c] + m_{i2}[e_{12} - r_c] + \ldots + m_{im}[e_{1m} - r_c]}{m_1[e_{11} - r_c] + m_2[e_{12} - r_c] + \ldots + m_m[e_{1m} - r_c]} d_2$$

which, assuming that $W = -d_2$ and $D = R$, is the same as the individual investor's demand for asset i in (4.39).

Another important difference between these asset demand equations and those derived under the assumption of expected utility maximisation occurs when the rates of return on the choice items are not correlated with each other. Under the assumption of expected utility maximisation the bank's demand for an asset will depend only upon the expected rate of return on this particular asset when the rates of return on the choice items are not correlated with each other. However, this property no longer holds under the Principle of Safety First. If the rates of return on the choice items are not correlated with each other, i.e. if Ω_{11} is diagonal, then, given the definition of m_i above, (4.67) collapses to

$$\hat{a}_{1i} =$$

$$- \frac{m_{ii}[e_{1i} - c_n) - r_c]}{m_{11}[e_{11} - c_n) - r_c] + m_{22}[e_{12} - c_n) - r_c] + \ldots + m_{mm}[e_{1m} - c_n) - r_c]} d_2. \qquad (4.68)$$

Thus, even when the rates of return on the choice assets are not correlated with each other, the demand for one asset will still depend upon its own expected rate of return as well as the expected rates of return on all other assets in the choice set.

As in the case of the individual investor, the bank will in general hold a diversified portfolio. Consider again the special case where the gross (and hence the net) expected rates of return on all m assets in the choice set are the same. If the bank holds all its disposable funds in only one of the m assets with the same expected rate of return, the upper bound of the probability that actual return at the end of the decision period will fall below the critical level, R, is

$$\Pr\ (\prod \leq R) = \frac{\sigma^2}{(e_s - c_n) - r_c}$$

where e_s is the identical expected rate of return and σ^2 its variance. However, if the bank spreads its total disposable funds among the m assets in equal amounts the upper bound of the probability reduces to

$$\Pr\ (\prod \leq R) = \frac{\sigma^2}{m[(e_s - c_n) - r_c]}$$

which, as in the case of individual investor, is the least of the upper bound of the probability the bank can expect under these conditions.

Using the definitions in (4.61), (4.65) and (4.66) we can rewrite the bank's optimal static asset demand in matrix form as

$$A_1 = - \frac{M_{11}\ [e_1 - (c_n + r_c)\ i_1]}{[e_1 - (c_n + r_c)\ i_1]'\ M_{11}\ i_1}\ d_2. \qquad (4.60')$$

We note that both c_n and r_c are scalars so that the same constant will be subtracted from each and every expected rate of return on the choice items of the portfolio.

Dividing both sides of (4.60') by $-d_2$ and defining $\hat{s}_1 = -A_1/d_2$, we can rewrite the bank's system of asset demand equations under the Principle of Safety First as

$$\hat{s}_1 = \frac{M_{11}\,[e_1 - (c_n + r_c)\,i_1]}{[e_1 - (c_n + r_c)\,i_1]'\,M_{11}\,i_1}. \qquad (4.69)$$

Given the interpretation of d_2 as the total disposable fund the bank can invest in the choice assets, $\hat{s}_1$ is the vector of the share of each asset in the choice portfolio. (4.69) is, therefore, the system of asset demand equations in terms of shares.

4.3.2 Comparative Statics

In order to discuss the asset demand equations given in (4.60) and (4.64) in more detail we now turn to an examination of the comparative statics of these equations. For convenience we shall use in the following discussion both equations (4.60) and (4.64). We are interested in the response of the bank's demand for choice assets to changes in the expected rates of return on the choice items, e_1, changes in their risk characteristics, Ω_{11}, changes in the levels of non-choice items, A_2, and their expected rates of return, e_2, and changes in the predetermined level of return, R. Given the equations for optimal asset demand this exercise is a straightforward one.

4.3.2.1 Response of Asset Demand to Changes in Expected Rates of Return

We first investigate the effects of a change in the expected rates of return on the optimal demand for assets. Let

$$N_i = m_{i1}[e_{11} - (d_1 - R)/d_2] + m_{i2}[e_{12} - (d_1 - R)/d_2]$$

$$+ \ldots + m_{im}[e_{1m} - (d_1 - R)/d_2]$$

and

$$T = m_1[e_{11} - (d_1 - R)/d_2] + m_2[e_{12} - (d_1 - R)/d_2]$$

$$+ \ldots + m_m[e_{1m} - (d_1 - R)/d_2]$$

then (4.64) can be written as

$$\hat{a}_{1i} = - (N_1/T)d_2$$

and

$$\frac{\partial N_i}{\partial e_{1i}} = m_{ii}, \qquad \frac{\partial T}{\partial e_{1i}} = m_i .$$

Thus, from (4.64) we have

$$\frac{\partial \hat{a}_{1i}}{\partial e_{1i}} = - \frac{d_2}{T^2} [\, m_{ii}\, T - m_i\, N_i \,] = - \frac{1}{T} [\, m_{ii}\, d_2 + m_i\, \hat{a}_{1i} \,] \qquad (4.70)$$

$$\frac{\partial \hat{a}_{1i}}{\partial e_{1j}} = - \frac{d_2}{T^2} [\, m_{ij}\, T - m_j\, N_i \,] = - \frac{1}{T} [\, m_{ij}\, d_2 + m_j\, \hat{a}_{1i} \,] \qquad (4.71)$$

$$\frac{\partial \hat{a}_{1j}}{\partial e_{1i}} = - \frac{d_2}{T^2} [\, m_{ji}\, T - m_i\, N_j \,] = - \frac{1}{T} [\, m_{ji}\, d_2 + m_i\, \hat{a}_{1j} \,] . \qquad (4.72)$$

From (4.70) and (4.71) it is clear that in general $\partial \hat{a}_{1i}/\partial e_{1j} \neq \partial \hat{a}_{1j}/\partial e_{1i}$. This differs from the symmetric result of asset demand equations derived from expected utility maximisation.

If it is assumed that the common denominator of the asset demand equations T is positive then

$$\frac{\partial \hat{a}_{1i}}{\partial e_{1i}} \gtrless 0 \qquad \text{as} \qquad -\frac{\hat{a}_{1i}}{d_2} \gtrless \frac{m_{ii}}{m_i} \tag{4.73}$$

$$\frac{\partial \hat{a}_{1i}}{\partial e_{1j}} \gtrless 0 \qquad \text{as} \qquad -\frac{\hat{a}_{1i}}{d_2} \gtrless \frac{m_{ij}}{m_j} \tag{4.74}$$

$$\frac{\partial \hat{a}_{1j}}{\partial e_{1i}} \gtrless 0 \qquad \text{as} \qquad -\frac{\hat{a}_{1j}}{d_2} \gtrless \frac{m_{ji}}{m_i}. \tag{4.75}$$

Clearly, the signs of the response to changes in the expected rates of return on the choice items depend on the initial proportion of total disposable funds held in a particular asset. We may also note

$$\sum_j^m \frac{\partial \hat{a}_{1j}}{\partial e_{1i}} = -\frac{1}{T}\left[d_2 \sum_j^m m_{ji} + m_i \sum_j^m \hat{a}_{1j} \right] = 0 \tag{4.76}$$

since

$$\sum_j^m m_{ji} = m_i \text{ and } \sum_j^m \hat{a}_{1j} = -d_2.$$

(4.76) is thus the equivalent of the Cournot condition which states that the effect on demand for choice assets of a change in the expected rate of return on a particular asset in the choice set sums to zero. This result follows directly from the existence of the balance sheet constraint: if total disposable funds are held constant an increase in one choice asset must be offset by a reduction in one or more of the remaining assets in the choice set.

4.3.2.2 Response of Demand for Choice Assets to Changes in Risk

Let us now turn to the effect of the change in risk on the demand for assets. Let

$$N = \Omega_{1\bar{1}}^{1} \, [e_1 - (d_1 - R)/d_2 \, i_1]$$

and

$$T^* = [e_1 - (d_1 - R)/d_2 \, i_1]' \, \Omega_{1\bar{1}}^{1} \, i_1$$

and note that N is an $m \times 1$ vector and T^* is a scalar, then

$$\hat{A}_1 = - (N/T^*) \, d_2$$

and

$$\frac{\partial N}{\partial \sigma_{ij}} = - \Omega_{1\bar{1}}^{1} \frac{\partial \Omega_{11}}{\partial \sigma_{ij}} \Omega_{1\bar{1}}^{1} [e_1 - (d_1 - R)/d_2 \, i_1] = - \Omega_{1\bar{1}}^{1} \frac{\partial \Omega_{11}}{\partial \sigma_{ij}} N$$

$$\frac{\partial T^*}{\partial \sigma_{ij}} = - [e_1 - (d_1 - R)/d_2 \, i_1]' \, \Omega_{1\bar{1}}^{1} \frac{\partial \Omega_{11}}{\partial \sigma_{ij}} \Omega_{1\bar{1}}^{1} \, i_1 .$$

Thus, differentiating (4.60) with respect to σ_{ij} we derive

$$\frac{\partial \hat{A}_1}{\partial \sigma_{ij}} = - \frac{d_2}{T^{*2}} \left[T^* \frac{\partial N}{\partial \sigma_{ij}} - N \frac{\partial T^*}{\partial \sigma_{ij}} \right]$$

$$= \frac{1}{T^*} \{ \Omega_{1\bar{1}}^{1} \frac{\partial \Omega_{11}}{\partial \sigma_{ij}} N \, d_2 + [e_1 - (d_1 - R)/d_2 \, i_1]' \, \Omega_{1\bar{1}}^{1} \frac{\partial \Omega_{11}}{\partial \sigma_{ij}} \Omega_{1\bar{1}}^{1} \, i_1 \, \hat{A}_1 \}$$

$$= \frac{1}{T^*} \{ [e_1 - (d_1 - R)/d_2 \, i_1]' \, \Omega_{1\bar{1}}^{1} \frac{\partial \Omega_{11}}{\partial \sigma_{ij}} \Omega_{1\bar{1}}^{1} \, i_1 \, \hat{A}_1 - \Omega_{1\bar{1}}^{1} \frac{\partial \Omega_{11}}{\partial \sigma_{ij}} T^* \, \hat{A}_1 \} .$$

(4.77)

We note that, for $i = j$, $\partial \Omega_{11}/\partial \sigma_{ij}$ is a matrix with 1 at position (i,i) and 0 everywhere else, while, for $i \neq j$, $\partial \Omega_{11}/\partial \sigma_{ij}$ has 1 at positions (i,j) and (j,i) and 0 elsewhere. Therefore, using previous notation we obtain

$$\frac{\partial \hat{a}_{1k}}{\partial \sigma_{ii}} = \frac{1}{T\,|\Omega_{11}|}\{m_i\,\hat{a}_{1k}\{m_{1i}\,[e_{11} - (d_1 - R)/d_2] + m_{2i}[e_{12} - (d_1 - R)/d_2]$$

$$+ \ldots + m_{mi}\,[e_{1m} - (d_1 - R)/d_2]\} - m_{ki}\,\hat{a}_{1i}\{m_1\,[e_{11} - (d_1 - R)/d_2]$$

$$+ m_2\,[e_{12} - (d_1 - R)/d_2] + \ldots + m_m\,[e_{1m} - (d_1 - R)/d_2]\}\}$$

$$= -\frac{(1/d_2)\,m_i\,\hat{a}_{1i}\,\hat{a}_{1k} + m_{ki}\,\hat{a}_{1i}}{|\Omega_{11}|} = -\frac{\hat{a}_{1i}}{|\Omega_{11}|d_2}[\,m_{ki}\,d_2 + m_i\,\hat{a}_{1k}\,]$$

$$= \frac{T\,\hat{a}_{1i}}{|\Omega_{11}|\,d_2}\frac{\partial \hat{a}_{1k}}{\partial e_{1i}}. \qquad (4.78)$$

Similarly,

$$\frac{\partial \hat{a}_{1k}}{\partial \sigma_{ij}} = \frac{1}{T|\Omega_{11}|}\{m_i\,\hat{a}_{1k}\{m_{1j}\,[e_{11} - (d_1 - R)/d_2] + m_{2j}\,[e_{12} - (d_1 - R)/d_2]$$

$$+ \ldots + m_{mj}\,[e_{1m} - (d_1 - R)/d_2]\} + m_j\,\hat{a}_{1k}\{m_{1i}\,[e_{11} - (d_1 - R)/d_2]$$

$$+ m_{2i}\,[e_{12} - (d_1 - R)/d_2] + \ldots + m_{mi}\,[e_{1m} - (d_1 - R)/d_2]$$

$$- T\{m_{kj}\,\hat{a}_{1i} + m_{ki}\,\hat{a}_{1j}\}\}.$$

But by symmetry of M_{11} we know that $m_{ij} = m_{ji}$ for i,j = 1, 2, ..., m, so that

$$\frac{\partial \hat{a}_{1k}}{\partial \sigma_{ij}} = \frac{m_i\,\hat{a}_{1k}\,N_j + m_j\,\hat{a}_{1k}\,N_i - T\,[\,m_{kj}\,\hat{a}_{1i} + m_{ki}\,\hat{a}_{1j}\,]}{T\,|\Omega_{11}|}$$

$$= -\frac{(1/d_2)\,m_i\,\hat{a}_{1k}\,\hat{a}_{1j} + (1/d_2)\,m_j\,\hat{a}_{1k}\,\hat{a}_{1i} + m_{kj}\,\hat{a}_{1i} + m_{ki}\,\hat{a}_{1j}}{|\Omega_{11}|}$$

$$= -\frac{(\hat{a}_{1i}/d_2)\,[\,m_{kj}\,d_2 + m_j\,\hat{a}_{1k}\,] + (\hat{a}_{1j}/d_2)\,[\,m_{ki}\,d_2 + m_i\,\hat{a}_{1k}\,]}{|\Omega_{11}|}$$

$$= \frac{T}{|\Omega_{11}|\,d_2}\,[\,\hat{a}_{1i}\frac{\partial \hat{a}_k}{\partial e_{1j}} + \hat{a}_{1j}\frac{\partial \hat{a}_{1k}}{\partial e_{1i}}\,]. \qquad (4.79)$$

For k = i, we have

$$\frac{\partial \hat{a}_{1i}}{\partial \sigma_{ii}} = \frac{T\,\hat{a}_{1i}}{|\Omega_{11}|\,d_2}\,\frac{\partial \hat{a}_{1i}}{\partial e_{1i}} \tag{4.80}$$

$$\frac{\partial \hat{a}_{1i}}{\partial \sigma_{ij}} = \frac{T}{|\Omega_{11}|\,d_2}\left[\,\hat{a}_{1i}\,\frac{\partial \hat{a}_{1i}}{\partial e_{1j}} + \hat{a}_{1j}\,\frac{\partial \hat{a}_{1i}}{\partial e_{1i}}\,\right] \tag{4.81}$$

and for k = j, we find

$$\frac{\partial \hat{a}_{1j}}{\partial \sigma_{ij}} = \frac{T}{|\Omega_{11}|\,d_2}\left[\,\hat{a}_{1i}\,\frac{\partial \hat{a}_{1j}}{\partial e_{1j}} + \hat{a}_{1j}\,\frac{\partial \hat{a}_{1j}}{\partial e_{1i}}\,\right]. \tag{4.82}$$

It is obvious that the signs of these partial derivatives are closely related to the signs of the partial derivatives with respect to the expected rates of return. In particular, given that $\hat{a}_{1i} > 0$ for i = 1, 2, ..., m, and that $|\Omega_{11}|$, being the determinant of a *positive definite* matrix, is also positive, we may deduce that

$$\frac{\partial \hat{a}_{1i}}{\partial \sigma_{ii}} \gtrless 0 \qquad \text{as} \qquad \frac{\partial \hat{a}_{1i}}{\partial e_{1i}} \lessgtr 0 \tag{4.83}$$

$$\frac{\partial \hat{a}_{1k}}{\partial \sigma_{ii}} \gtrless 0 \qquad \text{as} \qquad \frac{\partial \hat{a}_{1k}}{\partial e_{1i}} \lessgtr 0 \tag{4.84}$$

$$\frac{\partial \hat{a}_{1i}}{\partial \sigma_{ij}} \gtrless 0 \qquad \text{as} \qquad \hat{a}_{1i}\,\frac{\partial \hat{a}_{1i}}{\partial e_{1j}} \lessgtr -\,\hat{a}_{1j}\,\frac{\partial \hat{a}_{1i}}{\partial e_{1i}} \tag{4.85}$$

$$\frac{\partial \hat{a}_{1j}}{\partial \sigma_{ij}} \gtrless 0 \qquad \text{as} \qquad \hat{a}_{1i}\,\frac{\partial \hat{a}_{1j}}{\partial e_{1j}} \lessgtr -\,\hat{a}_{1j}\,\frac{\partial \hat{a}_{1j}}{\partial e_{1i}} \tag{4.86}$$

$$\frac{\partial \hat{a}_{1k}}{\partial \sigma_{ij}} \gtrless 0 \qquad \text{as} \qquad \hat{a}_{1i}\,\frac{\partial \hat{a}_{1k}}{\partial e_{1j}} \lessgtr -\,\hat{a}_{1j}\,\frac{\partial \hat{a}_{1k}}{\partial e_{1i}}. \tag{4.87}$$

Therefore, if a rise in the own expected rate of return increases the demand for the

asset, then the demand for that asset is reduced when it becomes more risky. Similarly, if a rise in the expected rate of return on asset i decreases the demand for asset k ($i \neq k$), then the demand for asset k is increased when asset i becomes more risky.

4.3.2.3 Response of Asset Demand to Changes in Disaster Return

Differentiating (4.64) with respect to R we obtain

$$\frac{\partial \hat{a}_{1i}}{\partial R} = - \frac{d_2}{T^2} \left[T \frac{\partial N_i}{\partial R} - N_i \frac{\partial T}{\partial R} \right].$$

But from the definitions of N_i and T

$$\frac{\partial N_i}{\partial R} = m_i \frac{1}{d_2}$$

$$\frac{\partial T}{\partial R} = m \frac{1}{d_2}$$

where

$$m = \sum_{i}^{m} m_i .$$

Therefore,

$$\frac{\partial \hat{a}_{1i}}{\partial R} = - \frac{1}{T} \left[m_i + m \frac{\hat{a}_{1i}}{d_2} \right]. \qquad (4.88)$$

It is clear that the direction of response of asset demand to a change in the disaster level R depends upon the initial proportion of total disposable deposits held in a particular asset. In particular, assuming that T is positive, then

$$\frac{\partial \hat{a}_{1i}}{\partial R} \gtrless 0 \qquad \text{as} \qquad -\frac{\hat{a}_{1i}}{d_2} \gtrless \frac{m_i}{m}. \tag{4.89}$$

Summing the effects over the n assets in the choice set we find that

$$\sum_{1}^{n} \frac{\partial \hat{a}_{1i}}{\partial R} = 0. \tag{4.90}$$

(4.90) informs us that if only the disaster level changes the effect of that change on asset demand must sum to zero. This result, similar to the Cournot condition, follows directly from the existence of the balance sheet constraint. If total disposable funds are held constant, an increase in one choice asset must be offset by a reduction in one or more of the remaining assets in the choice set.

4.3.2.4 Response of Asset Demand to Changes in Non-choice Items and Their Expected Rates of Return

Finally, we examine the effects on demand for choice assets of changes in the levels of non-choice items, A_2, and their expected rates of return, e_2. Recalling that $d_1 = A_2' e_2$ and $d_2 = A_2' i_2$ we have for j = m+1, m+2, ... n,

$$\frac{\partial d_1}{\partial \hat{a}_{2j}} = e_{2j}, \qquad \frac{\partial d_2}{\partial \hat{a}_{2j}} = 1, \qquad \frac{\partial d_1}{\partial e_{2j}} = \hat{a}_{2j}.$$

Further, let

$$N_i^* = \{ m_{i1}[e_{11} - (d_1 - R)/d_2] + m_{i2}[e_{12} - (d_1 - R)/d_2]$$

$$+ \ldots + m_{im}[e_{1m} - (d_1 - R)/d_2]\} d_2$$

and

$$T = m_1[e_{11} - (d_1 - R)/d_2] + m_2[e_{12} - (d_1 - R)/d_2]$$

$$+ \ldots + m_m[e_{1m} - (d_1 - R)/d_2]$$

so that

$$\frac{\partial N_i^*}{\partial \hat{a}_{2j}} = m_{i1}\,[e_{11} - e_{2j}] + m_{i2}\,[e_{12} - e_{2j}] + \ldots + m_{im}\,[e_{1m} - e_{2j}]$$

$$\frac{\partial T}{\partial \hat{a}_{2j}} = (1/d_2)^2\,[\,d_1 - R - d_2\,e_{2j}]\,m$$

$$\frac{\partial N_i^*}{\partial e_{2j}} = -\,m_i\,\hat{a}_{2j}$$

$$\frac{\partial T}{\partial e_{2j}} = -\,m\,(\hat{a}_{2j}\,/\,d_2).$$

Differentiating (4.64) with respect to $\hat{a}_{2j}$ and e_{2j} we obtain

$$\frac{\partial \hat{a}_{1i}}{\partial e_{2j}} = -\,\frac{1}{T^2}\left[\,T\,\frac{\partial N_i^*}{\partial e_{2j}} - N_i^*\,\frac{\partial T}{\partial e_{2j}}\,\right]$$

$$= (\hat{a}_{2j}/T)\,[\,m_i + m\,(\hat{a}_{1i}/d_2)] \qquad (4.91)$$

$$\frac{\partial \hat{a}_{1i}}{\partial \hat{a}_{2j}} = -\,\frac{1}{T^2}\left[\,T\,\frac{\partial N_i^*}{\partial \hat{a}_{2j}} - N_i^*\,\frac{\partial T}{\partial \hat{a}_{2j}}\,\right]$$

$$= -\,\frac{1}{T}\{\,m_{i1}\,[e_{11} - e_{2j}] + m_{i2}\,[e_{12} - e_{2j}] + \ldots + m_{im}\,[e_{1m} - e_{2j}]$$

$$+ \hat{a}_{1i}\,(1/d_2)^2\,[\,d_1 - R - d_2\,e_{2j}]\,m\}$$

$$= (1/d_2)\left[\,\hat{a}_{1i} - \frac{\partial \hat{a}_{1i}}{\partial e_{2j}}\,\frac{d_1 - R - d_2 e_{2j}}{\hat{a}_{2j}}\,\right]. \qquad (4.92)$$

A cursory inspection of (4.91) and (4.92) reveals that the signs of $\partial\hat{a}_{1i}/\partial\hat{a}_{2j}$ will depend upon the signs of $\partial\hat{a}_{1i}/\partial e_{2j}$. In particular, from (4.91) and (4.92) we observe that

$$\frac{\partial\hat{a}_{1i}}{\partial e_{2j}} \gtrless 0 \qquad \text{as} \qquad \frac{m_i}{m} \gtrless -\frac{\hat{a}_{1i}}{d_2} \tag{4.93}$$

$$\frac{\partial\hat{a}_{1i}}{\partial\hat{a}_{2j}} \gtrless 0 \qquad \text{as} \qquad \frac{\partial\hat{a}_{1i}}{\partial e_{2j}} \gtrless \frac{\hat{a}_{1i}\,\hat{a}_{2j}}{d_1 - R - d_2\, e_{2j}}. \tag{4.94}$$

Summing over all the (n-m) assets we observe that

$$\sum_{m+1}^{n} \frac{\partial\hat{a}_{1i}}{\partial\hat{a}_{2j}} = -1 \tag{4.95}$$

$$\sum_{m+1}^{n} \frac{\partial\hat{a}_{1i}}{\partial e_{2j}} = 0. \tag{4.96}$$

(4.95) and (4.96) are the two familiar aggregation conditions, the former being the Engel condition, the latter being the Cournot condition. We recall that liabilities are treated as negative assets so that $\hat{a}_{2j}$ can be either positive or negative. (4.95) states that if only the level of one non-choice item changes, the total change must be apportioned across all the assets in the choice set. Once again, the Cournot condition is a direct result of the budget constraint: if total disposable funds are constant, a change in one choice asset must be compensated by changes in one or more of the other assets in the choice set.

4.3.3 The Bank's Portfolio Adjustment

The bank's asset demand equations in (4.60') are the static equilibrium solutions. We assume that the bank's adjustment process is as that developed in Chapter Three:

$$\Delta A_{1t} = B\,(\hat{A}_{1t} - A_{1t-1}) \tag{4.97}$$

where $\Delta A_{1t} = A_{1t} - A_{1t-1}$ and B is an m × m matrix of adjustment coefficients as defined in (3.57).

Substitution of (4.60) into (4.97) yields the bank's dynamic asset demand equations as:

$$\Delta A_{1t} = - \frac{B\ \Omega_{1\bar{1}}^{1}\ [e_1 - (d_1 - R)/d_2\ i_1]}{[e_1 - (d_1 - R)/d_2\ i_1]'\ \Omega_{1\bar{1}}^{1}\ i_1}\ d_2 - B\ A_{1t-1}$$

$$= - \frac{\Phi^{*}\ [e_1 - (d_1 - R)/d_2\ i_1]}{[e_1 - (d_1 - R)/d_2\ i_1]'\ \Omega_{1\bar{1}}^{1}\ i_1}\ d_2 - B\ A_{1t-1} \qquad (4.98)$$

which in the form of (4.60') is

$$\Delta A_{1t} = - \frac{\Phi\ [e_1 - (c_n + r_c)\ i_1]}{[e_1 - (c_n + r_c)\ i_1]'\ M_{11}\ i_1}\ d_2 - B\ A_{1t-1} \qquad (4.98')$$

where

$$\Phi^{*} = B\ \Omega_{1\bar{1}}^{1} \qquad (4.99a)$$

$$\Phi = B\ M_{11} \qquad (4.99b)$$

and from (3.58)

$$i'\ B = i. \qquad (4.100)$$

It also follows from (4.99) and (4.100) that

$$i'\ \Phi^{*} = i'\ \Omega_{1\bar{1}}^{1} \qquad (4.101a)$$

and

$$i' \Phi = i' M_{11}. \qquad (4.101b)$$

We note that, although $\Omega_{1\bar{1}}^{-1}$ and M_{11} are symmetric, Φ^* and Φ will in general be non-symmetric.

4.4 PROBLEMS OF ESTIMATION

The models in (4.60') and (4.98) describe the optimal static and dynamic portfolio behaviour of an individual bank. In order to estimate and test these models we shall employ the same definitions of A_1 and A_2 as in Chapter Three and make similar assumptions regarding aggregation over banks and regarding the expected values of rates of return and non-choice items. Specifically, we assume that

(1) all banks face the same decision period which is one calender month and maximise the same objective function of the form in (4.43);

(2) all banks have the same expectations about future rates of return on assets so that the expected rates of return and the subjective variance-covariance matrix is the same for all banks;

(3) all banks face the same transactions cost coefficients and the same disequilibrium cost coefficients;

(4) the expected rates of return on assets are given by their monthly averages while the expected values of non-choice items are approximated by their actual levels;

(5) the disaster return R is zero for all banks.

The first four assumptions are similar to those made in Chapter Three and their justifications have already been discussed. The last assumption implies that every bank regards break-even as the critical point and will try to minimise the probability that actual return from their portfolios fall short of the costs of funding. Such an assumption is not as restrictive as it may seem at first sight. As we can see from the asset demand equations in (4.60), the same quantity R/d_2 is subtracted from each and every expected rate of return of choice asset. Thus, whatever the value of R it will not alter the *relative* attractiveness of the assets in the portfolio.

Given the above assumptions we can write the aggregate optimal static and dynamic

asset demand equations of all banks under consideration as

$$A_1 = - \frac{M_{11} [\hat{r}_1 - c_n i_1]}{[\hat{r}_1 - c_n i_1]' M_{11} i_1} d_2 \qquad (4.102)$$

$$\Delta A_{1t} = - \frac{\Phi [\hat{r}_1 - c_n i_1]}{[\hat{r}_1 - c_n i_1]' M_{111} i_1} d_2 - B A_{1t-1} \qquad (4.103)$$

where

$$c_n = d_1/d_2 = (A_2' \hat{r}_2)/(A_2' i_2) \qquad (4.104)$$

and $\hat{r}_1$ and $\hat{r}_2$ are as defined in Chapter Three.

(4.102) and (4.103) describe the theoretical or optimal aggregate portfolio behaviour of the banks. Actual behaviour may depart from the theoretical behaviour randomly and we assume that the vector of random errors enter the demand equations additively. As in Chapter Three we shall introduce into the above demand equations the credit ceilings and the Special and Supplementary Deposit Scheme dummy variables which are also presumed to be additive. Therefore, actual asset demand equations of the banks can be written as

$$A_1 = - \frac{M_{11} [\hat{r}_1 - c_n i_1]}{[\hat{r}_1 - c_n i_1]' M_{11} i_1} d_2 + f D + \epsilon_{st} \qquad (4.105)$$

$$\Delta A_{1t} = - \frac{\Phi [\hat{r}_1 - c_n i_1]}{[\hat{r}_1 - c_n i_1]' M_{11} i_1} d_2 - B A_{1t-1} + F D + \epsilon_{dt} \qquad (4.106)$$

where ϵ_{st} and ϵ_{dt} are the vectors of normally distributed random variables with means zero and covariance matrices Σ_s and Σ_d. D is the two-dimensional vector of dummy variables and f and F are their coefficient matrices. It is obvious that the restrictions in (3.83) and (3.84) also apply to (4.105) and (4.106).

Denoting the ith elements of A_{1t}, ΔA_{1t}, and $\hat{r}_1$ by a_{1it}, Δa_{1it} and $\hat{r}_{1i}$ respectively, we may write the ith equations of (4.105) and (4.106) as

$$a_{1it} = - \frac{m_{i1}[\hat{r}_{11} - c_n] + m_{i2}[\hat{r}_{12} - c_n] + \ldots + m_{im}[\hat{r}_{1m} - c_n]}{m_1[\hat{r}_{11} - c_n] + m_2[\hat{r}_{12} - c_n] + \ldots + m_m[\hat{r}_{1m} - c_n]} d_2$$

$$+ f_{i1} D_{SD} + f_{i2} D_{CC} + \epsilon_{ist} \qquad (4.107)$$

and

$$\Delta a_{1it} = - \frac{\varphi_{i1}[\hat{r}_{11} - c_n] + \varphi_{i2}[\hat{r}_{12} - c_n] + \ldots + \varphi_{im}[\hat{r}_{1m} - c_n]}{m_1[\hat{r}_{11} - c_n] + m_2[\hat{r}_{12} - c_n] + \ldots + m_m[\hat{r}_{1m} - c_n]} d_2$$

$$- b_{i1} a_{11t-1} - b_{i2} a_{12t-1} - \ldots b_{im} a_{1mt-1} +$$

$$+ F_{i1} D_{SD} + F_{i2} D_{CC} + \epsilon_{idt} \qquad (4.108)$$

where m_i is as defined in (4.62). The following restrictions apply to the system of equations in (4.107) and (4.108):

$$(1)\ m_{ij} = m_{ji} \qquad \text{for all } i,j = 1, 2, \ldots, m \qquad (4.109a)$$

$$(2)\ m_i = \sum_{j=1}^{m} m_{ij} = \sum_{j=1}^{m} m_{ji} \qquad \text{for all } i = 1, 2, \ldots, m \qquad (4.109b)$$

$$(3)\ \sum_{i=1}^{m} b_{ij} = 1 \qquad \text{for all } j = 1, 2, \ldots, m \qquad (4.109c)$$

$$(4)\ \sum_{i=1}^{m} \varphi_{ij} = m_j \qquad \text{for all } j = 1, 2, \ldots, m \qquad (4.109d)$$

$$(5)\ \sum_{i=1}^{m} f_{ij} = \sum_{i=1}^{m} F_{ij} = 0 \qquad \text{for all } j = 1, 2. \qquad (4.109e)$$

(4.109a) is the symmetry restriction which derives from the symmetry of the

variance-covariance matrix Ω_{11}. (4.109b) may be termed the adding-up restriction which requires that m_i equal the sum of all elements in the ith row of the adjugate matrix M_{11}. Given the symmetry property of the adjugate matrix the adding-up restriction also implies that m_i be equal to the sum of all elements in the ith column of M_{11}. (4.109c) is the familiar Engel condition in the dynamic model which states that the effect of a change in the level of any choice asset lagged one period must sum to unity across all equations. The restriction specified in (4.109d) follows directly from those in (4.109b) and (4.109c). The final equation in (4.109) imposes restrictions on the dummy variables, namely, the impact of any dummy variable must sum to zero across all the choice asset equations. The restrictions in (4.109b) to (4.109e) are imposed to satisfy the balance sheet constraint and must hold *a priori*. The symmetry restrictions in (4.109a) can, however, be tested.

The systems of equations in (4.107) and (4.108) are, however, not directly estimatable. In addition to the singularity problem which arises from the balance sheet constraint, some of the parameters cannot be identified. As in Chapter Three, the singularity problem can be solved easily by dropping one of the equations in the system and the m-1 equations can then be estimated independently by FIML estimator. The lack of identifiability stems from the structure of the models in (4.107) and (4.108) and will remain even after one of the equations is dropped from estimation. This is so because, as we can see from (4.107) and (4.108) above, the denominator is the same for all equations in the choice set and the static interest rate parameters in the numerators of all equations appear in the denominator of each and every equation. In fact, all the static interest rate parameters also appear in the numerator of each and every equation in (4.108).

Formally, in the nonlinear model $y = f(\mathbf{x};\theta)$, the lack of identifiability is defined as occurring when there exists two sets of values θ_1 and θ_2 such that $f(\mathbf{x};\theta_1) = f(\mathbf{x};\theta_2)$ for all $\mathbf{x}$ [see, for example, Chapter 3 in Seber and Wild (1989)]. In (4.107), $\theta = (M_{11}, f)$. It is clear that $\theta_c = (cM_{11}, f)$ gives that same value of $f(\mathbf{x};\theta)$ for any nonzero c. It is

immediately obvious that these features also carry through to the system of equations in (4.108). One way out of this problem would be to reparametrize the systems in (4.107) and (4.108) using, say

$$g_{ij} = m_{ij} \; / \; m_m \tag{4.110a}$$

$$g_i = m_i \; / \; m_m \tag{4.110b}$$

$$q_{ij} = \varphi_{ij} \; / \; m_m \tag{4.110c}$$

so that we have

$$a_{1it} = - \frac{g_{i1}[\hat{r}_{11} - c_n] + g_{i2}[\hat{r}_{12} - c_n] + \ldots + g_{im}[\hat{r}_{1m} - c_n]}{g_1[\hat{r}_{11} - c_n] + g_2[\hat{r}_{12} - c_n] + \ldots + [\hat{r}_{1m} - c_n]} d_2$$

$$+ f_{i1} D_{SD} + f_{i2} D_{CC} + \epsilon_{ist} \tag{4.111}$$

and

$$\Delta a_{1it} = - \frac{q_{i1}[\hat{r}_{11} - c_n] + q_{i2}[\hat{r}_{12} - c_n] + \ldots + q_{im}[\hat{r}_{1m} - c_n]}{g_1[\hat{r}_{11} - c_n] + g_2[\hat{r}_{12} - c_n] + \ldots + [\hat{r}_{1m} - c_n]} d_2$$

$$- b_{i1} a_{11t-1} - b_{i2} a_{12t-1} - \cdots b_{im} a_{1mt-1}$$

$$+ F_{i1} D_{SD} + F_{i2} D_{CC} + \epsilon_{idt}. \tag{4.112}$$

Accordingly, the restrictions in (4.109a) to (4.109e) now become

$$(1)\; g_{ij} = g_{ji} \qquad \text{for all } i,j = 1, 2, \ldots, m \tag{4.109a'}$$

$$(2)\; g_i = \sum_{j=1}^{m} g_{ij} = \sum_{j=1}^{m} g_{ji} \qquad \text{for all } i = 1, 2, \ldots, m \tag{4.109b'}$$

$$(3)\quad \sum_{i=1}^{m} b_{ij} = 1 \qquad \text{for all } j = 1, 2, \ldots, m \qquad (4.109c')$$

$$(4)\quad \sum_{i=1}^{m} q_{ij} = g_j \qquad \text{for all } j = 1, 2, \ldots, m \qquad (4.109d')$$

$$(5)\quad \sum_{i=1}^{m} f_{ij} = \sum_{i=1}^{m} F_{ij} = 0 \qquad \text{for all } j = 1, 2. \qquad (4.109e')$$

In more detail, the two models in (4.111) and (4.112) can be written as

$$a_{1it} = -\{(g_{i1}[\hat{r}_{11} - c_n] + g_{i2}[\hat{r}_{12} - c_n] + \ldots + g_{im}[\hat{r}_{1m} - c_n]) /$$

$$((g_{11} + g_{12} + \ldots + g_{1m})[\hat{r}_{11} - c_n] +$$

$$(g_{21} + g_{22} + \ldots + g_{2m})[\hat{r}_{12} - c_n] + \ldots + [\hat{r}_{1m} - c_n])\} d_2 +$$

$$f_{i1} D_{SD} + f_{i2} D_{CC} + \epsilon_{ist} \qquad (4.111')$$

and

$$\Delta a_{1it} = -\{(b_{i1} g_{11} + b_{i2} g_{12} + \ldots + b_{im} g_{1m})[\hat{r}_{11} - c_n] +$$

$$(b_{i1} g_{21} + b_{i2} g_{22} + \ldots + b_{im} g_{2m})[\hat{r}_{12} - c_n] + \ldots +$$

$$(b_{i1} g_{m1} + b_{i2} g_{m2} + \ldots + b_{im} g_{mm})[\hat{r}_{1m} - c_n]\} /$$

$$((g_{11} + g_{12} + \ldots + g_{1m})[\hat{r}_{11} - c_n] +$$

$$(g_{21} + g_{22} + \ldots + g_{2m})[\hat{r}_{12} - c_n] + \ldots + [\hat{r}_{1m} - c_n])\} d_2 -$$

$$b_{i1} a_{11t-1} - b_{i2} a_{12t-1} - \ldots b_{im} a_{1mt-1} +$$

$$F_{i1}\, D_{SD} + F_{i2}\, D_{CC} + \epsilon_{idt}. \qquad (4.112')$$

(4.111') can now be estimated directly by dropping the mth equation. Given that the own rate coefficients, m_{ii}, are the variances of the rates of returns which are positive, all the normalised own rate coefficients, g_{ii}, should have the same sign. All the static interest rate parameters in the first m-1 equations in (4.111') can now be identified. In addition, some of the parameter estimates for the mth equation can be derived from the symmetry restrictions. In the case where the symmetry restriction is not imposed none of the individual parameter estimates in the mth equation can be obtained although the estimated value of the mth asset can be derived from the balance sheet constraint. However, the numerator of each equation in (4.112') still contains the static interest rate parameters of the mth equation so that (4.112') still cannot be estimated as it stands. To facilitate estimation we reparametrise (4.112') as follows:

$$\begin{aligned}
\Delta a_{1it} = - \; & \{(b_{i1}\, g_{11} + b_{i2}\, g_{12} + \ldots + b_{im}\, g_{1m})\, [\hat{r}_{11} - c_n] + \\
& (b_{i1}\, g_{21} + b_{i2}\, g_{22} + \ldots + b_{im}\, g_{2m})\, [\hat{r}_{12} - c_n] + \ldots + \\
& g_i^{*}\, [\hat{r}_{1m} - c_n]\} \,/\, ((g_{11} + g_{12} + \ldots + g_{1m})\, [\hat{r}_{11} - c_n] + \\
& (g_{21} + g_{22} + \ldots + g_{2m})\, [\hat{r}_{12} - c_n] + \ldots + [\hat{r}_{1m} - c_n])\}\, d_2 - \\
& b_{i1}\, a_{11t-1} - b_{i2}\, a_{12t-1} - \ldots\, b_{im}\, a_{1mt-1} + \\
& F_{i1}\, D_{SD} + F_{i2}\, D_{CC} + \epsilon_{idt}. \qquad (4.112'')
\end{aligned}$$

Now (4.112'') can be estimated directly by dropping the mth equation. It is noteworthy that, unlike the linear dynamic models discussed in the last chapter, the normalised static interest rate parameters in (4.112''), except those for the mth equation can also be

identified so that the g_{ij}'s can be estimated directly together with the dynamic adjustment coefficients, b_{ij}'s. This implies that the symmetry restrictions can also be tested in the dynamic model.

As we mentioned in Chapter Three the Full Information Maiximum Likelihood estimates of the parameters are invariant to the choice of equation to be deleted from estimation. It is shown in Amemiya (1977) that the FIML estimator is consistent in nonlinear system of equations if ϵ is normally distributed but is generally inconsistent if ϵ is not normal. This result is contrary to that in the linear case where the FIML estimator derived from the normality assumption retains its consistency even if the true distribution is not normal. However, FIML is the only known efficient estimator for models that are nonlinear in their parameters [see for example, Amemiya (1977, 1983)], and it will be used for our estimation in the next section.

4.5 SOME SAFETY FIRST RESULTS FOR THE LONDON CLEARING BANKS: 1959 - 1983

In this section we present and discuss some empirical results of the Safety First models using balance sheet data of the London clearing banks for the period 1959 - 1983. There are five choice items in the pre-1971 period and seven in the post-1971 period. As in Chapter Three, the Government bond equation is dropped from estimation. We shall discuss the static and dynamic models separately.

4.5.1 Static Models

The system of equations in (4.111') are estimated subject to the restrictions in (4.109a'), (4.109b') and (4.109e') and with the Government bond equation deleted. Some of the parameter estimates for the Government bond equation can be derived from the restrictions. FIML results for the two periods are presented in Tables 4.1A and 4.1B. As no constant term is included in the estimation, neither the R-squared nor the Durbin-Watson statistic is reported in the tables.

The results in Table 4.1A are rather disappointing. As we mentioned earlier, all the normalised own rate coefficients in the numerators of the system of equations in (4.111) should have the same sign. From Table 4.1A, however, we observe that three of the five own rate coefficients are negatively signed. Moreover, only five of the fourteen independently estimated coefficients are statistically significant. The estimates in Table 4.1B are slightly more encouraging. All the seven own rate coefficients are correctly signed and more than half of the independently estimated interest rate coefficients are statistically different from zero. We shall not discuss these results any further, however, as residual plots indicate that first-order vector autocorrelation may be present.

Table 4.1A
FIML Estimates for the Safety First Static Model
With Symmetry Restrictions and Dummy Variables (1959:01 to 1971:06)

	CM	TB	CB	AD	GB
CMR	**-0.050**	0.222	0.187	0.122	0.209
	(0.315)	(1.175)	(2.518)	(1.973)	(3.466)
TBR	0.222	**-0.162**	0.174	0.497	-0.160
	(1.175)	**(0.439)**	(1.545)	(3.562)	(1.088)
CBR	0.187	0.174	**-0.007**	0.060	0.082
	(2.518)	(1.545)	**(0.108)**	(1.340)	(1.676)
ADR	0.122	0.497	0.060	**1.593**	0.759
	(1.973)	(3.562)	(1.340)	**(7.009)**	(4.295)
GBR	0.209	-0.160	0.082	0.759	
	(3.466)	(1.088)	(1.676)	(4.295)	
D_{SD}	65.272	12.048	-13.418	22.515	-86.417
	(0.760)	(0.069)	(0.250)	(0.123)	
D_{CC}	244.940	-501.680	201.210	457.090	-401.560
	(3.837)	(3.743)	(4.621)	(2.550)	
SER	160.808	328.358	109.712	338.834	250.307

Notes: (1) The interest rate coefficients refer to those in the numerator of each equation, i.e., they are the estimates for the g_{ij}'s in (4.111'); (2) Figures in parentheses are the ratios of estimated coefficients to asymptotic standard errors. Some t-ratios for the Government bond equation are not provided because this equation is not directly estimated. The ratios reported are provided by the symmetry restrictions.

Table 4.1B
FIML Estimates for the Safety First Static Model
With Symmetry Restrictions and Dummy Variables (1971:10 to 1983:06)

	CM	TB	CB	AD	LL	CD	GB
CMR	**0.326**	0.009	0.079	1.401	0.013	0.010	0.146
	(6.579)	(0.705)	(4.504)	(10.460)	(1.372)	(0.769)	(6.997)
TBR	0.009	**-0.015**	0.008	0.032	-0.014	0.015	0.006
	(0.705)	**(0.322)**	(0.219)	(1.104)	(0.645)	(0.503)	(0.300)
CBR	0.079	0.008	**0.089**	0.325	**0**.021	-0.055	0.021
	(4.504)	(0.219)	**(1.525)**	(8.539)	(0.869)	(1.427)	(1.021)
ADR	1.401	0.032	0.325	**6.227**	0.050	0.089	0.706
	(10.460)	(1.104)	(8.539)	**(11.756)**	(4.358)	(4.567)	(39.856)
LLR	0.013	-0.014	0.021	0.050	**0.022**	-0.021	0.007
	(1.372)	(0.645)	(0.869)	(4.358)	**(0.724)**	(0.627)	(0.746)
CDR	0.010	0.015	-0.055	0.089	-0.021	**0.053**	0.019
	(0.769)	(0.503)	(1.427)	(4.567)	(0.627)	**(1.057)**	(1.265)
GBR	0.146	0.006	0.021	0.706	0.007	0.019	
	(6.997)	(0.300)	(1.021)	(39.856)	(0.746)	(1.265)	
D_{SD}	287.300	317.130	-81.695	-1178.100	159.630	174.340	321.395
	(1.468)	(4.937)	(0.868)	(3.376)	(5.002)	(2.636)	
SER	613.559	199.214	202.226	869.516	113.160	185.934	476.969

Notes: (1) The interest rate coefficients refer to those in the numerator of each equation, i.e., they are the estimates for the g_{ij}'s in (4.111'); (2) Figures in parentheses are the ratios of estimated coefficients to asymptotic standard errors. Some t-ratios for the Government bond equation are not provided because this equation is not directly estimated. The ratios reported are provided by the symmetry restrictions.

To test whether first-order vector autocorrelation is present, we specify that the m × 1 disturbance vector ϵ_{st} in (4.111) follows the same autoregresive process as in (3.85). We shall test whether the matrix of autoregressive coefficients is diagonal or full. We shall also test the symmetry restrictions and the dummy variables both with and without assuming first-order vector autocorrelation among the error terms. The sample values of the maximised log-likelihood function for the various models are reported in Table 4.2, from which sample likelihood ratios for testing various hypotheses can be computed.

First-order Vector Autocorrelation

In Table 4.3 we present the sample likelihood ratio statistics for testing first-order vector autocorrelation in the safety first models. The null hypothesis of no first-order vector autocorrelation is tested against two alternatives of first-order vector autocorrelation: one with a diagonal matrix of autoregressive coefficients and the other with a full autoregressive matrix. Comparing the sample likelihood ratios in the first four columns of Table 4.3 with the Chi-square critical values in Table 3.7 we conclude that the null hypothesis of no first-order vector autocorrelation has to be rejected for both periods. The rejection is overwhelming and occurs in all the 24 cases: regardless of whether the autocorrelation matrix is assumed diagonal or full and regardless of whether the symmetry restrictions are imposed or not. When tested against the alternative of a diagonal matrix of autoregressive coefficients, the lowest sample likelihood ratio statistic is 1149.40 while with one degrees of freedom the 1% critical chi-square value is a mere 6.635. When the alternative is a full matrix of autoregressive coefficients, the lowest sample likelihood ratios are 1355.58 and 1222.78 in the pre-1971 period and post-1971 period respectively, compared with 1% critical chi-square values of 32.000 with sixteen degrees of freedom and of 58.619 with thirty-six degrees of freedom.

As for the specification of the matrix of first-order autoregressive coefficients, it is clear from the table that the null hypothesis of a diagonal matrix must be rejected for the pre-1971 period at all reasonable significance levels. The alternative of a full matrix

of autoregressive coefficients must be accepted in all cases with or without the symmetry restrictions and the dummy variables. For the post-1971 period, the null hypothesis of a diagonal matrix also has to be rejected in favour of a full matrix when symmetry restrictions are not imposed. However, when the symmetry restrictions are imposed we are unable to reject the null hypothesis of a diagonal matrix of autocorrelation coefficients at either the 5% or 1% significance levels.

Symmetry

Under the assumption of no first-order autocorrelation, the symmetry restrictions have to be rejected for both the pre-1971 period and the post-1971 period. When first-order autocorrelation is assumed, the symmetry restrictions can be accepted in all cases for the pre-1971 period at the 1% significance level, although not at the 5% level. For the post-1971 period, symmetry restrictions must be accepted at both the 5% and 1% significance levels with a diagonal matrix of autoregressive coefficients. However, when the matrix of autoregressive coefficients is specified as full, the symmetry restrictions have to be rejected. Since the diagonal matrix specification of the autoregressive coefficients has already been rejected in favour of the full matrix alternative when symmetry restrictions are imposed, we shall maintain the unrestricted models as empirically valid for the post-1971 period.

Dummy Variables

As with the symmetry restrictions, the test results for the dummy variables depend on whether first-order vector autocorrelation is assumed. With zero-order autocorrelation, the null hypothesis of no dummy variables, either separately or jointly, must be rejected at all reasonable significance levels for both periods. However, when first-order vector autocorrelation is specified, either with a diagonal or full matrix of autoregressive coefficients, the dummy variables are consistently rejected for both period, with or without the symmetry restrictions.

We therefore maintain within the Safety First framework the full matrix first-order vector autocorrelation model with symmetry restrictions as empirically valid for the pre-1971 period and the full matrix first-order vector autocorrelation model with no symmetry restrictions for the post-1971 period. FIML estimates of these two models are reported in Tables 4.6 and 4.7.

Table 4.2
Sample Values of the Maximised Log-likelihood Function
for the Static Safety First Models

	$\forall$ = full		$\forall$ = diagonal		$\forall$ = null	
	No Symm.	Symmetry	No Symm.	Symmetry	No Symm.	Symmetry
			1959:01 to 1971:06			
SSD Dummy CC Dummy	-2934.65 (44)	-2941.80 (38)	-2967.91 (29)	-2975.63 (23)	-3612.44 (28)	-3706.43 (22)
SSD Dummy No CC Dummy	-2937.74 (40)	-2945.60 (34)	-2971.77 (25)	-2978.92 (19)	-3621.60 (24)	-3738.76 (18)
CC Dummy No SSD Dummy	-2935.83 (40)	-2943.18 (34)	-2968.58 (25)	-2976.59 (19)	-3624.74 (24)	-3720.20 (18)
No SSD Dummy No CC Dummy	-2939.04 (36)	-2946.93 (30)	-2972.49 (21)	-2979.87 (15)	-3636.25 (20)	-3764.62 (14)
			1971:10 to 1983:06			
SSD Dummy	-5133.83 (84)	-5155.11 (69)	-5170.52 (49)	-5177.90 (34)	-5745.22 (48)	-5785.79 (33)
No SSD Dummy	-5134.97 (78)	-5157.15 (63)	-5171.19 (43)	-5178.40 (28)	-5821.15 (42)	-5868.02 (27)

Note: Figures in parentheses are the number of free parameters estimated.

Table 4.3
Sample Likelihood Ratios for Testing First-order Autocorrelation in the Static Safety First Models

	Ψ = full vs null		Ψ = diag vs null		Ψ = full vs diag	
	No Symm.	Symmetry	No Symm.	Symmetry	No Symm.	Symmetry
			1959:01 to 1971:06			
SSD Dummy CC Dummy	1355.58 (16)	1529.26 (16)	1289.06 (1)	1461.60 (1)	66.52 (15)	67.66 (15)
SSD Dummy No CC Dummy	1367.72 (16)	1586.32 (16)	1299.66 (1)	1519.68 (1)	68.06 (15)	66.64 (15)
CC Dummy No SSD Dummy	1377.82 (16)	1554.04 (16)	1312.32 (1)	1487.22 (1)	65.50 (15)	66.82 (15)
No SSD Dummy No CC Dummy	1394.42 (16)	1635.38 (16)	1327.52 (1)	1569.50 (1)	66.90 (15)	65.88 (15)
			1971:10 to 1983:06			
SSD Dummy	1222.78 (36)	1261.36 (36)	1149.40 (1)	1215.78 (1)	73.38 (35)	45.58 (35)
No SSD Dummy	1372.36 (36)	1421.74 (36)	1299.92 (1)	1379.24 (1)	72.44 (35)	42.50 (35)

Note: Figures in parentheses are the degrees of freedom under the null hypothesis.

Table 4.4
Sample Likelihood Ratios for Testing Symmetry in the Static Safety First Models

	$\forall$ = full	$\forall$ = diagonal	$\forall$ = null
	1959:01 to 1971:06		
SSD Dummy CC Dummy	14.30	15.44	181.98
SSD Dummy No CC Dummy	15.72	14.30	234.32
CC Dummy No SSD Dummy	14.70	16.02	190.90
No SSD Dummy No CC Dummy	15.78	14.76	256.74
	1971:10 to 1983:06		
SSD Dummy	42.56	14.76	81.14
No SSD Dummy	44.36	14.42	93.74

Note: There are six and fifteen degrees of freedom under the null hypothesis for the pre-1971 and post-1971 periods respectively.

Table 4.5
Sample Likelihood Ratios for Testing the Dummy Variables in the Static Safety First Models

	$\forall$ = full		$\forall$ = diagonal		$\forall$ = null	
	No Symm.	Symmetry	No Symm.	Symmetry	No Symm.	Symmetry
	1959:01 to 1971:06					
SSD Dummy No CC Dummy	6.18 (4)	7.60 (4)	7.72 (4)	6.58 (4)	18.32 (4)	64.66 (4)
CC Dummy No SSD Dummy	2.36 (4)	2.76 (4)	1.34 (4)	1.92 (4)	24.60 (4)	27.54 (4)
No SSD Dummy No CC Dummy	8.78 (8)	10.26 (8)	9.16 (8)	8.48 (8)	47.62 (8)	116.38 (8)
	1971:10 to 1983:06					
No SSD Dummy	2.28 (6)	4.08 (6)	1.34 (6)	1.00 (6)	151.86 (6)	164.46 (6)

Note: Figures in parentheses are the degrees of freedom under the null hypothesis.

Table 4.6
FIML Estimates for the Safety First Model with Symmetry Restrictions Assuming First-order Vector Autocorrelation ($\forall$ = full)
(1959:01 to 1971:06)

	CM	TB	CB	AD	GB
CMR	**0.879**	1.953	-0.022	-0.365	0.322
	(2.679)	(3.668)	(0.453)	(3.554)	(7.394)
TBR	1.953	**4.324**	-0.053	-0.803	0.707
	(3.668)	**(3.146)**	(0.470)	(3.150)	(11.804)
CBR	-0.022	-0.053	**-0.008**	0.017	-0.009
	(0.453)	(0.470)	**(1.438)**	(0.832)	(0.509)
ADR	-0.365	-0.803	0.017	**0.144**	-0.129
	(3.554)	(3.150)	(0.832)	**(2.606)**	(6.790)
GBR	0.322	0.707	-0.009	-0.129	
	(7.394)	(11.804)	(0.509)	(6.790)	
FIML estimates of autoregressive parameters in $\forall$					
	0.966	0.083	0.013	-0.069	
	(26.430)	(1.468)	(0.999)	(1.181)	
	0.042	**1.001**	-0.001	-0.088	
	(1.118)	**(21.387)**	(0.129)	(1.552)	
	0.107	-0.192	**0.981**	-0.013	
	(0.920)	(1.296)	**(31.962)**	(0.075)	
	0.019	0.037	0.005	**0.909**	
	(0.543)	(0.704)	(0.430)	**(14.676)**	
SER	53.910	68.718	14.715	75.902	27.816

Notes: (1) The interest rate coefficients refer to those in the numerator of each equation, i.e., they are the estimates for the g_{ij}'s in (4.111'); (2) Some coefficient estimates for the Government bond equation are not provided because this equation is not directly estimated and those given are derived from the symmetry restrictions; (3) Figures in parentheses are the ratios of estimated coefficients to asymptotic standard errors.

Table 4.7
FIML Estimates for the Unconstrained Safety First Model Assuming First-order Vector Autocorrelation (∀ = Diagonal)
(1971:10 to 1983:06)

	CM	TB	CB	AD	LL	CD	GB
CMR	**1.031**	0.098	0.159	0.706	-0.001	0.070	
	(**1.801**)	(0.769)	(1.072)	(2.603)	(0.001)	(0.751)	
TBR	0.102	**0.011**	0.016	0.062	-0.001	0.009	
	(0.700)	(**0.423**)	(0.557)	(0.691)	(0.088)	(0.611)	
CBR	0.150	0.013	**0.022**	0.128	0.001	0.005	
	(0.985)	(0.381)	(**0.463**)	(1.117)	(0.157)	(0.277)	
ADR	0.707	0.068	0.109	**0.480**	-0.001	0.049	
	(2.605)	(0.800)	(1.251)	(**2.155**)	(0.008)	(0.755)	
LLR	-0.003	-0.003	0.001	0.009	**0.001**	-0.003	
	(0.029)	(0.133)	(0.002)	(0.110)	(**0.053**)	(0.165)	
CDR	0.079	0.011	0.012	0.023	-0.001	**0.011**	
	(0.554)	(0.438)	(0.552)	(0.267)	(0.107)	(**0.522**)	
GBR	0.414	0.039	0.064	0.289	-0.001	0.027	
	(4.821)	(0.745)	(1.325)	(3.764)	(0.008)	(0.739)	
	FIML estimates of autoregressive parameters in ∀						
	0.892	0.004	0.018	0.005	-0.012	0.005	
	(**4.002**)	(0.060)	(0.288)	(0.017)	(0.479)	(0.105)	
	0.143	**0.897**	-0.028	-0.252	0.007	-0.001	
	(0.232)	(**9.138**)	(0.178)	(0.334)	(0.130)	(0.004)	
	-0.259	-0.022	**0.826**	0.433	-0.005	0.001	
	(0.426)	(0.179)	(**6.567**)	(0.490)	(0.076)	(0.009)	
	-0.069	-0.008	0.002	**1.006**	-0.005	-0.002	
	(0.343)	(0.145)	(0.041)	(**3.608**)	(0.249)	(0.059)	
	-0.567	0.044	-0.097	0.718	**0.938**	0.030	
	(0.710)	(0.195)	(0.331)	(0.542)	(**10.115**)	(0.161)	
	0.017	0.026	0.077	-0.120	0.024	**0.905**	
	(0.036)	(0.172)	(0.511)	(0.142)	(0.586)	(**11.956**)	
SER	386.883	97.330	88.316	484.696	40.439	82.979	177.835

Notes: (1) The interest rate coefficients refer to those in the numerator of each equation, i.e., they are the estimates for the g_{ij}'s in (4.111'); (2) Figures in parentheses are the ratios of estimated coefficients to asymptotic standard errors; (3) Estimates for the Government bond equation are not provided because this equation is not directly estimated.

From Table 4.3 we observe an interesting pattern of interest rate coefficients. While most of the interest rate coefficients are statistically different from zero, none of them is statistically significant in the Commercial bill equation. In fact, the own rate coefficient in the Commercial bill equation is negatively signed. Moreover, the Commercial bill rate coefficients in all the four equations are numerically small and statistically insignificant. As for the first-order autocorrelation coefficients, we note that all the elements on the leading diagonal of the matrix are significant and close to unity.

For the post-1971 period, we find from Table 4.4 that, once again, all the own interest rate coefficients have the same positive sign. Although fewer interest rate coefficients have significant t-ratios than in Table 4.1B, the standard errors of regression in Table 4.4 are much smaller than those in Table 4.1B, indicating better goodness of fit of the first-order vector autocorrelation model. We also note, that the own equation first-order autocorrelation coefficients are positive and statistically significant. Despite the fact that the full matrix specification of the autoregressive coefficients is accepted all the cross equation autoregressive coefficients are statistically insignificant. This once again suggests that in systems of equations one should not place too much emphasis on the statistical significance of the individual parameters, especially when some degree of multicollinearity is present.

4.5.2 Dynamic Models

In this subsection we discuss briefly some of the empirical results of the dynamic Safety First models. As we mentioned earlier, unlike the portfolio-theoretical dynamic models, the static interest rate coefficients can be estimated directly in the Safety First dynamic models so that the symmetry restrictions can also be tested. In order to test the symmetry restrictions as well as the validity of the dummy variables in the dynamic Safety First models we estimated the system of equations in (4.112') with various combinations of the restrictions and report in Table 4.8 the sample values of the maximised log-likelihood functions of these models.

In the last column of Table 4.8 we also give the sample likelihood ratio statistic for testing the symmetry restrictions in the dynamic models. For the pre-1971 period, the symmetry restrictions have to be accepted at the 1% significance level in all cases. At the 5% significance level we are also able to accept the symmetry restrictions in all but one cases. For the post-1971 period, the test results are unambiguous. The symmetry restrictions have to be accepted at all reasonable significance levels, either with or without the Special and Supplementary Deposits dummy variables.

In Table 4.9 we report the sample likelihood ratio statistics for testing the dummy variables. As with the dynamic models of Chapter 3 the credit ceilings dummy variable has to be rejected either on its own or jointly with the Special Deposits dummy variable; and the rejection occurs regardless whether the symmetry restrictions are imposed or not. In only one case where the symmetry restrictions and the credit ceiling dummy variable are present are we unable to reject the Special Deposits dummy variable at the 5% significance level. However, this is more due to the rejection of the credit ceilings dummy rather than the acceptance of the Special Deposits dummy variable. In any case, the model itself where this occurs has been rejected as empirically valid when tested against the alternative of no symmetry restrictions. For the post-1971 period, the SSD dummy variable also has to be rejected with or without the symmetry restrictions.

Thus, within the dynamic Safety First models we maintain, for both periods, the model with symmetry restrictions but no dummy variables as empirically valid. Their FIML estimates of the maintained models are presented in Tables 4.10 and 4.11.

However, the performance of the maintained models are rather disappointing for both periods. None of the individual interest rate coefficients is statistically different from zero (except a few of the composite interest rate coefficients) although some of coefficients of the lagged dependent variables are significant. This is in sharp contrast with the results of the dynamic models in Tables 3.23 and 3.24 where most of the interest rate coefficients are significant while none of the lagged dependent variable

coefficients is statistically different from zero. We note that the standard errors of regression in Table 3.23 are lower than in Table 4.10 for the pre-1971 period but for the post-1971 period they are generally higher in Table 3.24 than in Table 4.11, suggesting that the Safety First dynamic model may perform marginally better than the portfolio-theoretical dynamic model in the later period but slightly worse in the earlier period. The only encouraging thing in Tables 4.10 and 4.11 is that all the own interest rate coefficients, have the same sign. However, since these maintained models are to be further tested against non-nested alternatives in Chapter Six we shall not discuss the estimates in Tables 4.10 and 4.11 any further.

Table 4.8
Sample Values of the Maximised Log-likelihood Function and Likelihood Ratios for Testing Symmetry for the Dynamic Safety First Models

	No Symmetry	Symmetry	Likelihood Ratio
		1959:01 to 1971:06	
SSD Dummy CC Dummy	-2917.30 (52)	-2920.59 (46)	6.58 (6)
SSD Dummy No CC Dummy	-2921.49 (48)	-2922.34 (42)	1.70 (6)
CC Dummy No SSD Dummy	-2919.10 (48)	-2926.60 (42)	15.00 (6)
No SSD Dummy No CC Dummy	-2925.25 (44)	-2926.98 (38)	3.46 (6)
		1971:10 to 1983:06	
SSD Dummy	-5088.56 (96)	-5096.24 (81)	15.36 (15)
No SSD Dummy	-5092.26 (90)	-5101.75 (75)	18.98 (15)

Note: Figures in parentheses are the number of free parameters estimated.

Table 4.9
Sample Likelihood Ratios for the Testing the Dummy Variables in the Safety First Dynamic Models

	No Symmetry	Symmetry
	1959:01 to 1971:06	
SSD Dummy No CC Dummy	8.38 (4)	3.50 (4)
CC Dummy No SSD Dummy	3.60 (4)	12.02 (4)
No SSD Dummy No CC Dummy	15.90 (8)	12.78 (8)
	1971:10 to 1983:06	
No SSD Dummy	7.40 (6)	11.02 (8)

Note: Figures in parentheses are the degrees of freedom under the null hypothesis.

Table 4.10
FIML Estimates for the Dynamic Safety First Model
With Symmetry Restrictions (1959:01 to 1971:06)

	DCM	DTB	DCB	DAD	DGB
g_{i1}	**-0.466**	1.298	-0.531	0.030	0.410
	(0.322)	(0.411)	(0.358)	(0.177)	(0.927)
g_{i2}	1.298	**-3.742**	1.610	-0.204	-1.057
	(0.411)	**(0.596)**	(0.523)	(0.335)	(1.140)
g_{i3}	-0.531	1.610	**-0.716**	0.089	0.472
	(0.358)	(0.523)	**(0.473)**	(0.302)	(1.054)
g_{i4}	0.030	-0.204	0.089	**-0.099**	-0.089
	(0.177)	(0.335)	(0.302)	**(1.314)**	(0.748)
g_{i5}	0.410	-1.057	0.472	-0.089	
	(0.927)	(1.140)	(1.054)	(0.748)	
g_i^*	0.242	0.589	0.008	0.030	
	(5.378)	(7.174)	(0.374)	(0.369)	
b_{i1}	0.375	0.585	-0.017	-0.040	0.097
	(7.184)	(5.642)	(0.724)	(0.444)	
b_{i2}	0.252	0.691	0.014	-0.014	0.057
	(4.312)	(8.415)	(0.679)	(0.188)	
b_{i3}	0.141	0.822	0.045	-0.031	0.023
	(1.020)	(5.946)	(1.327)	(0.176)	
b_{i4}	0.227	0.558	0.007	0.061	0.147
	(4.512)	(6.112)	(0.302)	(0.644)	
b_{i5}	0.220	0.580	0.013	-0.002	0.189
	(4.345)	(6.554)	(0.522)	(0.016)	
SER	53.085	66.937	15.323	73.565	25.456

Notes: (1) The g_{ij}'s are the estimates for the long-run interest rate coefficients (1 = CMR, 2 = TBR, 3 = CBR, 4 = ADR and 5 = GBR) while the g_i^*'s are the estimates for the coefficient of Government bond rate in the numerator of each equation in (4.112''); (2) The b_{ij}'s are the adjustment coefficients (1 = CM_{t-1}, 2 = TB_{t-1}, 3 = CB_{t-1}, 4 = AD_{t-1}, 5 = GB_{t-1}); (3) Figures in parentheses are the ratios of estimated coefficients to asymptotic standard errors. Some t-ratios for the Government bond equation are not provided because this equation is not directly estimated. The ratios reported are provided by the symmetry restrictions.

Table 4.11
FIML Estimates for the Dynamic Safety First Model
With Symmetry Restrictions (1971:10 to 1983:06)

	DCM	DTB	DCB	DAD	DLL	DCD	DGB
g_{i1}	**-0.057**	-0.046	0.009	0.003	-0.031	-0.024	-0.103
	(0.455)	(0.599)	(0.183)	(0.019)	(0.589)	(0.582)	(0.582)
g_{i2}	-0.046	**-0.044**	0.019	0.017	-0.037	-0.019	-0.111
	(0.599)	**(0.409)**	(0.334)	(0.048)	(0.490)	(0.292)	(0.427)
g_{i3}	0.009	0.019	**-0.017**	-0.006	0.018	-0.001	0.065
	(0.183)	(0.334)	**(0.364)**	(0.039)	(0.372)	(0.036)	(0.355)
g_{i4}	0.003	0.017	-0.006	**-0.070**	0.010	0.008	0.033
	(0.019)	(0.048)	(0.039)	**(0.053)**	(0.043)	(0.047)	(0.050)
g_{i5}	-0.031	-0.037	0.018	0.010	**-0.037**	-0.002	-0.122
	(0.589)	(0.490)	(0.372)	(0.043)	**(0.481)**	(0.056)	(0.423)
g_{i6}	-0.024	-0.019	-0.001	0.008	-0.002	**-0.019**	-0.026
	(0.413)	(0.292)	(0.036)	(0.047)	(0.056)	**(0.334)**	(0.172)
g_{i7}	-0.103	-0.111	0.065	0.033	-0.122	-0.026	
	(0.582)	(0.427)	(0.355)	(0.050)	(0.423)	(0.172)	
g_{i}	0.328	0.007	0.045	0.558	-0.001	0.004	
	(2.791)	(0.171)	(0.873)	(3.942)	(0.066)	(0.151)	
b_{i1}	0.494	-0.009	0.030	0.447	-0.007	-0.003	0.048
	(3.442)	(0.166)	(0.487)	(2.657)	(0.379)	(0.085)	
b_{12}	0.286	0.141	0.062	0.504	-0.031	0.037	0.001
	(0.660)	(1.323)	(0.453)	(0.806)	(0.853)	(0.307)	
b_{i3}	0.607	0.031	0.214	0.068	-0.007	-0.002	0.089
	(1.390)	(0.293)	(2.341)	(0.156)	(0.159)	(0.017)	
b_{i4}	0.270	0.012	0.039	0.620	0.002	0.006	0.051
	(2.155)	(0.257)	(0.763)	(4.192)	(0.094)	(0.186)	
b_{i5}	-0.024	-0.057	0.172	0.995	0.128	-0.081	-0.133
	(0.030)	(0.267)	(0.664)	(0.853)	(1.690)	(0.457)	
b_{i6}	0.193	-0.062	0.061	0.825	-0.010	0.071	-0.078
	(0.419)	(0.538)	(0.486)	(1.305)	(0.180)	(0.967)	
b_{i7}	0.427	-0.005	0.048	0.399	-0.023	0.005	0.149
	(1.847)	(0.078)	(0.618)	(1.347)	(0.692)	(0.085)	
SER	344.484	94.415	85.197	420.541	37.749	85.485	164.478

Notes: (1) The g_{ij}'s are the estimates for the long-run interest rate coefficients (1 = CMR, 2 = TBR, 3 = CBR, 4 = ADR, 5 = LLR, 6 = CDR and 7 = GBR) while the g_i^*'s are the estimates for the coefficient of Government bond rate in the numerator of each equation in (4.112''); (2) The b_{ij}'s are the adjustment coefficients (1 = CM_{t-1}, 2 = TB_{t-1}, 3 = CB_{t-1}, 4 = AD_{t-1}, 5 = LL_{t-1}, 6 = CD_{t-1} and 7 = GB_{t-1}); (3) Figures in parentheses are the ratios of estimated coefficients to asymptotic standard errors. Some t-ratios for the Government bond equation are not provided because this equation is not directly estimated. The ratios reported are provided by the symmetry restrictions.

4.6 SUMMARY

In this chapter we have applied the Principle of Safety First to the modelling of bank portfolio behaviour. In terms of emphasis the models derived in this chapter are quite similar to those of Chapters Two and Three based on expected utility maximisation: both sets of models regard interest rate uncertainty as the only source of portfolio diversification. They also obtain similar symmetry results concerning the expected rates of return on the choice assets. Despite these similarities the Safety First portfolio models differ from those based on the expected utility maximisation paradigm in a number of important aspects.

First, unlike those models in the last two chapters, the properties of the Safety First models do not depend on any specific assumptions about the exact distribution of the actual return on a portfolio or the form of the utility function. The portfolio demand models developed in this chapter are therefore very general and robust in that they are consistent with a wide range of distributions of the rate of return on the portfolio. Indeed, one might argue that the Safety First models are distribution-free.

Second, the bank's optimal demand for the choice assets depends not only on the level of non-choice items but also on their expected rates of return although it is assumed that the rates of return on the choice and non-choice items are uncorrelated.

Third, the bank's optimal asset demand equations are highly nonlinear in the expected rates of return. The structure of the demand equations is such that even when the rates of return on the choice items are not correlated with each other, the demand for one choice asset will still depend upon its own expected rate of return as well as the expected rates of return on all other choice assets.

As we have seen in the last two sections the high degree of nonlinearity in the Safety First models has posed certain estimation problems. Not only do estimations take

longer to converge but parameter estimates are also more sensitive to small changes in data. The high degree of nonlinearity also meant that some of the parameter estimates are less accurate and this has resulted in a large number of interest rate coefficients being statistically insignificant, in both the static and dynamic models. On the other hand, the high degree of nonlinearity and the unique structure of the models also meant that, unlike the dynamic models developed and estimated in the last two chapters, the symmetry restrictions can also be tested in the dynamic Safety First models.

As in the portfolio-theoretical models examined in Chapter Three, the test results for the symmetry restrictions are not clear-cut. They are found to be consistent with the data in the dynamic models for both the pre-1971 period and the post-1971 period. However, in the static models the symmetry restrictions can only be accepted marginally by the pre-1971 data, but not by the post-1971 data. The test results for the dummy variables are unambiguous. As in the last chapter, the credit ceilings dummy is found to be inconsistent with the (pre-1971) data, either in the static models or in the dynamic models. The Special and Supplementary Deposit dummy variables are also rejected by the data from the two periods, once again in both the static and dynamic models. Like the portfolio-theoretical models in Chapter Three, the Safety First dynamic models perform rather badly in terms of goodness of fit. This similarity is not surprising, since the dynamic models in the two chapters are derived using the same general stock adjustment framework. The persistent poor performance of the dynamic models calls into question the adequacy of the general stock adjustment framework and/or the auxiliary assumptions regarding the structure of the transactions and disequilibrium costs. Further research in this area is obviously required.

APPENDIX TO CHAPTER FOUR: DERIVATION OF THE BANK'S OPTIMAL DEMAND FOR CHOICE ASSETS UNDER THE PRINCIPLE OF SAFETY FIRST

In this appendix we give the detailed derivation of the bank's optimal demand for choice assets. The procedures adpoted here are identical to those employed in Section 4.2 for the derivation of optimal demand for assets by an individual investor.

As shown in Figure 4.1, the bank will choose its portfolio at point P where the line RE is tangential to the efficient mean-variance frontier. To solve for the optimal value of μ_π we note that at the point of tangency μ_π in the mean-variance frontier must equal that in the linear function (4.23) with D replaced by R. Setting μ_π in (4.59) equal to the μ_π in (4.23) and expanding gives

$$(d\eta - k^2)\sigma_\pi^2 + 2k(d_1 - R - \delta d_2)\sigma_\pi - [R - (d_1 - \delta d_2)]^2 + (\delta^2 - \gamma)d^2 = 0$$

which can be written as a quadratic function of σ_π

$$a\sigma_\pi^2 + b\sigma_\pi + c = 0 \qquad \text{(A4.1)}$$

where

$$a = d\eta - k^2$$

$$b = 2k\,(d_1 - R - \delta d_2)$$

$$c = (\delta^2 - \gamma)d_2^2 - [R - (d_1 - \delta d_2)]^2.$$

As we have seen in Section 4.2, there will be in general two solutions for σ_π given by (4.26). But at the point of tangency only one root is allowed and this requires that

$$b^2 - 4ac = 0$$

or

$$4k^2 (d_1 - R - \delta d_2)^2 = 4 (d\eta - k^2)[(\delta^2 - \gamma)d_2^2 - (R - d_1 + \delta d_2)^2]$$

from which we obtain the optimal value for k^2 as

$$k^2 = d\eta[1 - \frac{(R - d_1 + \delta d_2)^2}{(\delta^2 - \gamma)d_2^2}]$$

or, given the definitions of d, γ, δ and η,

$$k^2 = \{e_1 - [(d_1 - R)/d_2]i_1\}' \Omega_{11}^{-1} \{e_1 - [(d_1 - R)/d_2]i_1\}. \qquad (A4.2)$$

The reciprocal of k^2 is, as we noted earlier, the upper bound of the probability that actual return at the end of the decision period will be less than or equal to the predetermined level, R, when the optimal portfolio is chosen. It should be noted that since k^2 is quadratic in $\{e_1 - [(d_1 - R)/d_2]i_1\}$ the upper bound of the probability, $1/k_2$, will be zero if and only if the determinant of Ω_{11}, $|\Omega_{11}|$, is zero. Since we have assumed that the rates of return on the choice items are not linearly dependent on each other, $|\Omega_{11}|$ will always be positive definite, so that at the optimum, the upper bound of the probability cannot be zero.

Now we are at a position to solve for the optimal asset holdings from the efficient mean-variance frontier by utilising the expression in (4.61) for k^2. At the point of tangency the slope of the linear function, k, must equal the slope of the efficient mean-variance frontier, which, from (4.59), is

$$\frac{\partial \mu_\pi}{\partial \sigma_\pi} = \frac{d\eta}{\mu_\pi - d_1 + \delta d_2} \sigma_\pi$$

$$= \frac{d\eta}{\mu_\pi - d_1 + \delta d_2} \left(\frac{\mu_\pi - R}{k} \right). \qquad \text{(A4.3)}$$

Setting (4.28) equal to (A4.3) gives the optimal value of μ_π:

$$\mu_\pi = \frac{d\eta R - k^2(d_1 - \delta d_2)}{d\eta - k^2}$$

which upon substituting for k^2 from (A4.2) becomes

$$\mu_\pi = \frac{(\delta^2 - \gamma)d_2^2}{(R - d_1 + \delta d_2)^2} + d_1 - \delta d_2. \qquad \text{(A4.4)}$$

We slot this value for μ_π into equations (4.55) and (4.56), so that

$$\lambda_1 = - \frac{d_2^2}{i_1' \, \Omega_{11}^{-1} \, i_1 \, (\delta d_2 - d_1 + R)}$$

$$= - \frac{d_2}{[e_1 - (d_1 - R)/d_2 \, i_1]' \, \Omega_{11}^{-1} \, i_1} \qquad \text{(A4.5)}$$

$$\lambda_2 = \frac{(R - d_1) \, d_2}{i_1' \, \Omega_{11}^{-1} \, i_1 \, (\delta d_2 - d_1 + R)}$$

$$= \frac{R - d_1}{[e_1 - (d_1 - R)/d_2 \, i_1]' \, \Omega_{11}^{-1} \, i_1}. \qquad \text{(A4.6)}$$

We note that (A4.5) and (A4.6) do not involve μ_π and σ_π^2. Substituting these values for λ_1 and λ_2 into equation (4.50) yields the bank's optimal asset holdings as

$$A_1 = - \frac{\Omega_{11}^{-1} \, [e_1 - (d_1 - R)/d_2 \, i_1]}{[e_1 - (d_1 - R)/d_2 \, i_1]' \, \Omega_{11}^{-1} \, i_1} \, d_2 \qquad \text{(A4.7)}$$

which is (4.60) in the main text.

Notes:

1. Samuelson (1970) shows it is a good approximation. See Paul A. Samuelson, "The Foundamental Approximation Theorem of Portfolio Analysis in Terms of Means, Variances and Higher Moments", Review of Economic Studies, vol. 37, 1970, pp. 537-542.

2. For an analysis of Professor Shackle's theory and its (albeit limited) application to portfolio selection see J.L. Ford, Choice, Expectation & Uncertainty, Martin Robertson, Oxford, 1983 and J.L Ford, Economic Choice Under Uncertainty — A Perspective Theory Approach, Edward Elgar, Hants, England, 1987.

3. This proof follows that of Ford (1983), which provides an extensive exposition of the one given by Roy (1952).

CHAPTER FIVE

RANDOM CASH REQUIREMENT, LIQUIDATION COSTS AND BANK PORTFOLIO BEHAVIOUR

5.1 INTRODUCTION

In the last chapter the Principle of Safety First was applied to the problem of bank portfolio selection. The asset demand equations derived from such a principle, though nonlinear in asset returns, can also be expressed in terms of the means and covariances of asset returns, similar to those obtained in Chapter Three from expected utility maximisation. This similarity between these two sets of models is not surprising since, although employing different objective functions, they are essentially speculative models in that both emphasise the importance of price (interest rate) uncertainty of assets. Indeed, both of them are capable of explaining bank portfolio diversification among those assets whose yields are subject to unforeseeable fluctuation.

Important price uncertainty of assets may be, but it is not, however, the only source of uncertainty the bank faces, as we have observed in Chapter One. Another area of uncertainty, which is equally, if not more, important and relevant to the bank's portfolio decision, is the uncertainty over cash flows during any period. As we have seen earlier, the main liabilities of a commercial bank are demand deposits and time deposits. As part of a national payment system banks are obliged to pay out their deposits without notice and on demand in the case of demand deposits and at short notice in the case of time deposits. On the other side of the balance sheet commercial loans and advances constitute a major part of a bank's assets. Not only does the bank not know the demand for loans during the decision period, but when a loan is granted, especially in the form of an overdraft, the bank is also not certain about the precise timing and pattern of its borrower's cash requirement which is to be met from the loan account.

During any decision period cash requirement is thus random and this poses a liquidity problem for the bank. On the one hand, in order to meet its customers' cash requirement the bank will have to hold in its portfolio adequate cash reserves. However, holding cash for all possible and remote contingencies requires a bank to forgo earnings since cash does not earn the bank any interest. On the other hand, if the bank's cash reserves held at the beginning of the period are insufficient to meet customers' requirement a reserve deficiency will occur, which makes certain costly adjustments necessary for the bank, e.g., the liquidation of other assets, or even emergency borrowings from the central bank.

The importance of uncertain cash requirement for the determination of a bank's optimal portfolio is first suggested by Edgeworth (1888). In this chapter we attempt, by applying the inventory-theoretical approach [see Arrow, Harris and Marschak (1951), Arrow, Karlin and Scarf (1958)] to the bank, to incorporate this aspect of uncertainty into a model of bank portfolio behaviour. The analysis is developed in terms of a choice model which regards random cash requirement as the only source of asset diversification. That is, it views asset diversification as providing alternative sources for meeting random cash requirement where each asset has a different rate of return and a different liquidation cost. As we have seen in Chapter One, there exist in the literature many theoretical models of bank portfolio behaviour based on the inventory-theoretical approach; but surprisingly, there have been little empirical work of commercial bank portfolio behaviour based on this approach. The model presented in this chapter is based on that developed by Gray and Parkin (1973) and subsequently applied to the analysis of the demand for financial assets by the private sector of the UK economy by Barrett, Gray and Parkin (1975).

This inventory-theoretical approach to bank portfolio behaviour might be criticised for ignoring the uncertainty over asset returns, which is the emphasis of models based on expected utility maximisation and the Principle of Safety First. It can be argued, however, since our concern here is with the bank's short-run portfolio behaviour it seems

realistic to assume that in the very short run, which in our case is a month, assets are not subject to yield fluctuation or only subject to very minor fluctuation.

In the models of bank portfolio behaviour derived in Chapter Three from expected utility maximisation, each asset demand equation is a linear function of the expected rates of return on all assets in the choice set. It is well known that interest rates on assets are highly correlated, thus making it difficult to obtain well-determined parameter estimates. Although the problem of multicollinearity can be partially mitigated by the imposition of symmetry and homogeneity restrictions, which reduces the number of parameters to be estimated, it cannot be resolved satisfactorily due to the presence of the expected rates of return on all assets in each and every equation. Moreover, in order to test the symmetry and homogeneity properties more general models without the symmetry and homogeneity restrictions have to be estimated. The main technical advantage of employing the inventory-theoretical approach in modelling bank portfolio behaviour is that, as we shall see below, it limits the number of interest rates to be included in each demand equation, thus alleviating the problem of multicollinearity considerably. As we shall see later, however, the inventory-theoretical model has its own empirical problems.

The optimal asset demand equations derived in this chapter are also functions of the mean and variance of the random cash requirement variable. As neither the mean nor the variance is directly observable they have to be estimated. In Barrett, Gray and Parkin (1975) both the mean and variance of the random cash variable are assumed to be a function of a certain exogenous variable and are then substituted into the asset demand equations so that they are estimated together with the parameters of the asset demand equations. As we shall discuss in more detail later, such an approach is inconsistent in that it uses information which is not available at the moment of decision. A major innovation of our model is the separate estimation using the Bayesian approach of the mean and variance of the cash requirement variable prior to the estimation of the parameters of the asset demand equations. This approach enables us to update the estimates of the mean and variance every time a new observation becomes available.

The estimates obtained for the mean and variance are then used as data in the estimation of the parameters of the bank's asset demand equations.

The rest of the chapter is organised as follows. Section 5.2 sketches the bank's optimal liquidation strategy when a random cash requirement occurs. The bank's optimal asset holdings are derived in Section 5.3, followed by a detailed examination of the comparative statics of the optimal asset demands in Section 5.4. In Section 5.5 we discuss the problems of empirical implementation of the theoretical bank portfolio selection model derived in Section 5.3 and the estimation of mean and variance of cash requirement. Some empirical results using UK commercial bank data for the period 1959-1971 are reported in Section 5.6. A summary of the chapter is given in the final section.

5.2 RANDOM CASH REQUIREMENT AND THE BANK'S OPTIMAL LIQUIDATION STRATEGY

Let A denote the n × 1 vector of assets and liabilities the bank can hold in its portfolio. As we discussed in the previous chapters, some of the items in the portfolio are determined exogenously, either by the monetary authorities or by the supply side, so that the bank is only able to choose the levels of a subset of A. We shall, as before, partition the bank's portfolio into choice and non-choice sets and treat liabilities as negative assets. Since all liabilities are regarded as non-choice items it will be assumed that the choice set contains assets only. During the decision period the bank faces random cash requirement which, if exceeding the bank's beginning-of-period holding of cash reserves, makes it necessary for the bank to liquidate its other portfolio items and entails liquidation costs.

It is postulated that the bank is able to estimate from past experience the probability density function of the random cash requirement. Let X denote net cash requirement during the decision period and f(X) its probability density function (pdf), then

$$\begin{aligned} \text{pdf of } X &= f(X) && \text{for } \alpha \leqslant X \leqslant \beta \\ &= 0 && \text{elsewhere} \end{aligned} \qquad (5.1)$$

where α and β are the lower and upper bounds of random cash requirement respectively. It is also assumed that the probability density function of random cash requirement is independent of the bank's non-choice items and independent of the bank's decision on the composition of its choice set portfolio. To allow for bankruptcy it is assumed that

$$\beta \geqslant A_1' \, i_1 + A_2' \, i_2 \qquad (5.2)$$

where A_1 is an m × 1 vector of choice assets, all of which are positive, and A_2 the

(n-m) × 1 vector of non-choice items. A typical element of A_1 is a_{1i} for all i = 1, 2, ..., m and that of A_2 is a_{2i} for all i = m+1, m+2, ..., n. By the definition of a pdf $f(X) \neq 0$ for $\alpha \leqslant X \leqslant \beta$. Furthermore, if the cumulative density function (cdf) of X is denoted by F(X), F(X) is monotonically increasing and

$$F(\beta) = \int_{\alpha}^{\beta} f(X)\, dX = 1. \qquad (5.3)$$

Since the bank does not have control over the levels of its non-choice items, in choosing its portfolio the bank is only concerned with the liquidation of the choice assets. The bank can neither alter the liquidation costs of nor the profit from its non-choice items. Let L_1 be the m-component vector of asset disposals, where by definition all the m components are non-positive, and C_1 the m-component vector of liquidation costs, a typical element of which is c_{1i}. In particular, it is assumed that c_{1i} is a linear homogeneous function of the amount liquidated of each asset and that all c_{1i}'s are different for i = 1, 2, ..., m. Under these assumptions the bank's profit function for the decision period can be written as

$$\prod = A_1' r_1 + A_2' r_2 + L_1' C_1 + q \qquad (5.4)$$

where r_1 is the m × 1 vector of rates of return on the choice assets, r_2 is the (n-m) × 1 vector of rates of return on the non-choice items and q is the total liquidation cost of the non-choice items, which is independent of the bank's decision regarding the choice set. All r_{1i}'s are also assumed different for i = 1, 2, ..., m. Since the cash requirement is stochastic the amounts of assets liquidated and hence profit will also be stochastic. It is thus postulated that the bank's objective is the maximisation of the expected value of $\prod$

$$E(\prod) = \mu_{\pi} = \hat{A}_1' r_1 + \hat{A}_2' r_2 + L_1^{*\prime} C_1 + \hat{q} \qquad (5.5)$$

subject to the balance sheet constraint

$$\hat{A}_1' \, i_1 + \hat{A}_2' \, i_2 = 0 \qquad (5.6)$$

where $\hat{A}_1$ is the vector of the expected (desired) values of A_1, a typical element of which is $\hat{a}_{1i}$, $\hat{A}_2$ is the vector of the expected values of A_2, a typical element of which is $\hat{a}_{2i}$, L_1^* is the m × 1 vector of expected values of choice asset liquidations and $\hat{q}$ is the total expected liquidation cost of the non-choice items. We may note that, since all liabilities are treated as negative assets and are included in the non-choice set, the term $-\hat{A}_2' \, i_2$ is the total expected disposable fund. This is a standard constrained maximisation problem with $\hat{A}_1$ being the vector of choice variables. However, before we proceed to solve for the optimal asset holdings we need to specify the expected asset liquidations, which in turn will depend upon the bank's liquidation strategy.

In order to obtain the expected liquidations of assets we first order the m components in $\hat{A}_1$ such that actual liquidation costs are increasing as we move down the vector, i.e., such that

$$c_{11} < c_{12} < c_{13} < \ldots < c_{1m}. \qquad (5.7)$$

Such an ordering is possible since we have assumed earlier that all c_i's are different. An asset i is said to be dominated by j if

$$r_{1i} < r_{1j} \quad \text{and} \quad c_{1i} > c_{1j}.$$

Given the objective function in (5.5) it is immediately clear that a zero holding of a dominated asset will always be optimal. Our earlier assumption that all components of $\hat{A}_1$ are positive hence ensures that dominated assets are excluded from the bank's choice set, and implies that, given the definition of a dominated asset, the vector r_1 is also ordered such that

$$r_{11} < r_{12} < r_{13} < \cdots < r_{1m}. \qquad (5.8)$$

Granted the orderings of assets defined by (5.7) and (5.8) it will always be optimal, in the event of a random cash requirement exceeding the bank's beginning-of-period cash holdings, to liquidate assets starting with the first and moving up the scale as far as necessary until the cash requirement is fully met [Gray and Parkin (1973)]. It is obvious that this strategy of liquidation will be optimal regardless of the levels of assets initially held by the bank. Given this optimal liquidation strategy the liquidation of each choice asset in the event of a certain cash requirement can now be specified as

$$\ell_i = \begin{cases} 0 & \text{for } \alpha \leqslant X \leqslant A_{1i-1} \qquad (5.9a) \\ -(X - A_{1i-1}) & \text{for } A_{1i-1} < X < A_{1i} \qquad (5.9b) \\ -\hat{a}_{1i} & \text{for } A_{1i} \leqslant X \leqslant \beta \qquad (5.9c) \end{cases}$$

where

$$A_{1i} = \sum_{j=1}^{i} \hat{a}_{1j}.$$

(5.9a) states that the ith asset is not to be liquidated if the random cash requirement is less than or equal to the sum of the first (i-1) choice assets; if the random cash requirement is greater than the sum of the first (i-1) choice assets but still less than the sum of the first i choice assets, then by (5.9b) the ith asset will be partially liquidated and the amount liquidated will be the difference between the total cash requirement and the sum of the first (i-1) choice assets; should, however, the total cash requirement exceed the sum of the first i choice assets, (5.9c) requires that the ith choice asset be entirely liquidated.

Using (5.9) together with the pdf of X in (5.1) the expected liquidation of the ith asset is then

$$\varrho_i^* = -\int_{A_{1i-1}}^{A_{1i}} X\, f(X)\, dX + \int_{A_{1i-1}}^{A_{1i}} A_{1i-1}\, f(X)\, dX - \int_{A_{1i}}^{\beta} \hat{a}_{1i}\, f(X)\, dX. \qquad (5.10)$$

Multiplying the expected liquidation ϱ_i^* by its unit liquidation cost ς_i and summing over all m choice assets we obtain the total expected liquidation costs as

$$C_1' L_1^* = \sum_{i=1}^{m} c_{1i} \{ -\int_{A_{1i-1}}^{A_{1i}} X\, f(X)\, dX + \int_{A_{1i-1}}^{A_{1i}} A_{1i-1}\, f(X)\, dX - \int_{A_{1i}}^{\beta} \hat{a}_{1i}\, f(X)\, dX \} \qquad (5.11)$$

which expresses total expected liquidation costs in terms of the choice vector $\hat{A}_1$ and thus enables us to solve for the optimal asset holdings by the bank. We do this in the next section.

5.3 THE BANK'S OPTIMAL ASSET HOLDINGS

We write

$$C(\hat{A}_1) = C_1' \; L_1^*$$

so that the bank's problem can be restated as choosing A_1 to maximise

$$\mu_\pi = \hat{A}_1' \; r_1 + \hat{A}_2' \; r_2 + C(\hat{A}_1) + \hat{q}$$

subject to

$$\hat{A}_1' \; i_1 + \hat{A}_2' \; i_2 = 0.$$

Forming the Lagrangean

$$L = \hat{A}_1' \; r_1 + \hat{A}_2' \; r_2 + C(\hat{A}_1) + \hat{q} + \lambda \; (\hat{A}_1' \; i_1 + \hat{A}_2' \; i_2)$$

and differentiating it with respect to the choice vector $\hat{A}_1$ and the Lagrangean multiplier λ yield the following first-order conditions:

$$\frac{\partial L}{\partial \hat{A}_1} = r_1 + C'(\hat{A}_1) + \lambda \; i_1 = 0 \qquad (5.12a)$$

$$\frac{\partial L}{\partial \lambda} = \hat{A}_1' \; i_1 + \hat{A}_2' \; i_2 = 0 \qquad (5.12b)$$

where $C'(\hat{A}_1)$ is the vector of partial derivatives of $C(\hat{A}_1)$ with respect to the components of $\hat{A}_1$. Despite the daunting appearance of the expected liquidation cost function in (5.11) the partial derivatives of this function with respect to $\hat{A}_1$ turn out to be neat and

simple. Differentiating $C(A_1)$ with respect to a particular element of A_1, say a_k, gives a typical element of $C'(A_1)$ as

$$C_{1k}'(A_1) = - c_{1k} \int_{A_{1k}}^{\beta} f(X)\, dX + \sum_{i=1}^{m} c_{1i} \int_{A_{1i-1}}^{A_{1i}} f(X)\, dX$$

which upon utilising the definition of a pdf and (5.3) can be rewritten as

$$C_{1k}'(A_1) = - c_{1k} \left[\int_{\alpha}^{\beta} f(X)\, dX - \int_{\alpha}^{A_{1k}} f(X)\, dX\right] +$$

$$\sum_{i=k+1}^{m} c_{1i} \left[\int_{\alpha}^{A_{1i}} f(X)\, dX - \int_{\alpha}^{A_{1i-1}} f(X)\, dX\right]$$

$$= - c_{1k} [1 - F(A_{1k})] + \sum_{i=k+1}^{m} c_{1i} [F(A_{1i}) - F(A_{1i-1})] \qquad (5.13)$$

where $F(A_{1i})$ is the cumulative probability function of X evaluated at A_{1i}. There will be m such equations in the system. Substitution of (5.13) into (5.12a) for each $C_{1i}'(A_1)$ thus enables us to rewrite the (m + 1) first-order conditions as

$$r_{11} - c_{11}[1 - F(A_{11})] + \sum_{i=2}^{m} c_{1i}[F(A_{1i}) - F(A_{1i-1})] + \lambda = 0$$

$$r_{12} - c_{12}[1 - F(A_{12})] + \sum_{i=3}^{m} c_{1i}[F(A_{1i}) - F(A_{1i-1})] + \lambda = 0$$

$$\vdots \qquad (5.14)$$

$$r_{1m-1} - c_{1m-1}[1 - F(A_{1m-1})] + c_{1m}[F(A_{1m}) - F(A_{1m-1})] + \lambda = 0$$

$$r_{1m} - c_{1m}[1 - F(A_{1m})] + \lambda = 0$$

$$A_1' i_1 + A_2' i_2 = 0.$$

We note that the summation term in each of the first (m - 1) equations differs by only one component from that in the next equation, suggesting that the system of equations may be solved by the conventional subtraction method. Before we proceed in this direction, however, it is necessary to specify the exact functional form of the distribution of the random cash requirement variable. We assume that the random cash requirement variable, X, is normally distributed with mean μ_x and variance σ_x^2. The normality assumption is made here mainly on the grounds that X is the sum of a large number of independent changes in different deposit and loan accounts.

The one disadvantage of the normal distribution is that it is not possible to derive an exact formula for the cumulative density function, since, as it is well known, the normal density function cannot be analytically integrated. It is fortunate, however, that the problem can be solved by once again resorting to the Tchebycheff's inequality, which, as noted in the previous chapter, becomes an equality for a normally distributed variable. That is, if X is normally distributed with mean μ_x and variance σ_x^2, then for any real number $\tau > 0$

$$\Pr(|X - \mu_X| \geq \tau) = \frac{\sigma_X^2}{\tau^2}.$$

Since X, being a normal variable, is symmetric, it immediately follows

$$\Pr(X - \mu_X \geq \tau) = \frac{\sigma_X^2}{2\tau^2}.$$

Let $\tau = A_i - \mu_x$, we then have

$$\Pr(X \geq A_{1i}) = \frac{\sigma_X^2}{2(A_{1i} - \mu_X)^2} \qquad (5.15)$$

so that from the definition of a cumulative density function

$$F(A_{1i}) = Pr(X \leqslant A_{1i}) = 1 - Pr(X \geqslant A_{1i})$$

$$= 1 - \frac{\sigma_x^2}{2(A_{1i} - \mu_x)^2}. \qquad (5.16)$$

Now substitution of (5.16) into (5.14) allows us to express the first-order conditions directly in terms of A_i:

$$r_{11} - \frac{c_{11}\,\sigma_x^2}{2(A_{11} - \mu_x)^2} + \frac{\sigma_x^2}{2} \sum_{i=2}^{m} c_{1i} \left[\frac{1}{(A_{1i-1} - \mu_x)^2} - \frac{1}{(A_{1i} - \mu_x)^2} \right] + \lambda = 0$$

$$r_{12} - \frac{c_{12}\,\sigma_x^2}{2(A_{12} - \mu_x)^2} + \frac{\sigma_x^2}{2} \sum_{i=3}^{m} c_{1i} \left[\frac{1}{(A_{1i-1} - \mu_x)^2} - \frac{1}{(A_{1i} - \mu_x)^2} \right] + \lambda = 0$$

$$\vdots \qquad (5.17)$$

$$r_{1m-1} - \frac{c_{1m-1}\sigma_x^2}{2(A_{1m-1} - \mu_x)^2} + \frac{c_{1m}\,\sigma_x^2}{2} \left[\frac{1}{(A_{1i-1} - \mu_x)^2} - \frac{1}{(A_{1i} - \mu_x)^2} \right] + \lambda = 0$$

$$r_{1m} - \frac{c_{1m}\,\sigma_x^2}{2(A_{1m} - \mu_x)^2} + \lambda = 0$$

$$A_1' \, i_1 + A_2' \, i_2 = 0.$$

Subtracting the second equation from the first in (5.17) gives

$$r_{11} - r_{12} - \frac{\sigma_x^2 (c_{11} - c_2)}{2(A_{11} - \mu_x)^2} = 0.$$

Recalling that $A_{11} = \hat{a}_{11}$ and rearranging we obtain

$$\hat{a}_{11} = \left(\frac{\sigma_x^2 (c_{12} - c_{11})}{2(r_{12} - r_{11})} \right)^{1/2} + \mu_x. \qquad (5.18)$$

Similarly, subtract the third equation from the second in (5.17) to obtain

$$r_{12} - r_{13} - \frac{\sigma_x^2 (c_{12} - c_{13})}{2(A_{12} - \mu_x)^2} = 0$$

which given that $A_{12} = \hat{a}_{11} + \hat{a}_{12}$ yields

$$\hat{a}_{12} = \left(\frac{\sigma_x^2 (c_{13} - c_{12})}{2(r_{13} - r_{12})}\right)^{1/2} - \hat{a}_{11} + \mu_x$$

$$= \left(\frac{\sigma_x^2 (c_{13} - c_{12})}{2(r_{13} - r_{12})}\right)^{1/2} - \left(\frac{\sigma_x^2 (c_{12} - c_{11})}{2(r_{12} - r_{11})}\right)^{1/2}. \qquad (5.19)$$

In the same fashion, for k = 3, 4, ..., m-1, we have

$$\hat{a}_{1k} = \left(\frac{\sigma_x^2 (c_{1k+1} - c_{1k})}{2(r_{1k+1} - r_{1k})}\right)^{1/2} - \left(\frac{\sigma_x^2 (c_{1k} - c_{1k-1})}{2(r_{1k} - r_{1k-1})}\right)^{1/2}. \qquad (5.20)$$

The solution for the mth asset can then be derived through the balance sheet constraint. Summing the solutions for assets 1 to m-1 and noting the cancelling of terms give

$$A_{1m-1} = \left(\frac{\sigma_x^2 (c_{1m} - c_{1m-1})}{2(r_{1m} - r_{1m-1})}\right)^{1/2} + \mu_x.$$

But from the balance sheet constraint (5.12b)

$$\hat{a}_{1m} = - \hat{A}_2' \, i_2 - A_{1m-1}$$

so that

$$\hat{a}_{1m} = - \hat{A}_2' \, i_2 - \left(\frac{\sigma_x^2 (c_{1m} - c_{1m-1})}{2(r_{1m} - r_{1m-1})}\right)^{1/2} - \mu_x. \qquad (5.21)$$

Equations (5.18), (5.20) and (5.21) thus form the optimal solution system for the choice asset vector $\hat{A}_1$. The bank's optimal holdings for the first and the last choice

assets are given by equations (5.18) and (5.21) while the demand for all the interior choice assets is given by equation (5.20). To recapitulate, the bank's optimal choice asset holdings are

$$\hat{a}_{11} = \left(\frac{\sigma_x^2 (c_{12} - c_{11})}{2(r_{12} - r_{11})}\right)^{1/2} + \mu_x \tag{5.22a}$$

$$\hat{a}_{1k} = \left(\frac{\sigma_x^2 (c_{1k+1} - c_{1k})}{2(r_{1k+1} - r_{1k})}\right)^{1/2} - \left(\frac{\sigma_x^2 (c_{1k} - c_{1k-1})}{2(r_{1k} - r_{1k-1})}\right)^{1/2}$$

$$\text{for } k = 2, 3, \ldots, m - 1 \tag{5.22b}$$

$$\hat{a}_{1m} = - A_2' \, i_2 - \left(\frac{\sigma_x^2 (c_{1m} - c_{1m-1})}{2(r_{1m} - r_{1m-1})}\right)^{1/2} - \mu_x. \tag{5.22c}$$

As we anticipated in the introduction to this chapter only those rates of return and liquidation costs of the adjacent assets enter each choice asset demand equation. It is obvious from (5.42) that the optimal asset demand equations are not defined when the interest rates of any two adjacent choice assets are equal. While this possibility is precluded by our assumption that all interest rates are different, as we shall see later, such an assumption may not always be satisfied in reality. Also, it is very interesting to note that the demand for the first m-1 choice assets are independent of total expected disposable funds (the non-choice items) so that a *ceteris paribus* increase in total expected disposable funds (the non-choice items) all goes into the last choice asset. This result follows of course from the unique ordering of the choice assets and depends on the assumption that changes in total expected disposable funds (the non-choice items) do not affect the parameters of the probability distribution of the random cash requirement variable. As we can see from (5.22) the bank's demand for all the choice assets depend upon the variance of the random cash requirement variable while its demand for the first and last choice assets also depend upon the mean of the cash requirement variable. Therefore, if the variance of random cash requirement is made a

function of total expected disposable funds (the non-choice items), the bank's optimal demand for all the m choice assets will then be dependent upon total expected disposable funds (the non-choice items). In the next section, we examine in detail the properties of these asset demand equations.

5.4 COMPARATIVE STATICS AND THE NONNEGATIVITY CONDITION

In this section we first investigate the effects on choice asset demand of changes in the rates of return, the liquidation costs, the parameters of the distribution of random cash requirement and total expected disposable funds. We shall also establish under what conditions the optimal holdings of all assets in the choice set are positive. Given the equations for optimal choice asset holdings in (5.22) the comparative statics are easily derived.

5.4.1 Response of Choice Asset Demand to Changes in the Rates of Return

As the choice asset demand equations take different forms we shall examine the first, the last and the interior assets separately.

For the first choice asset, differentiating (5.22a) with respect to r_{11} and r_{12} gives

$$\frac{\partial \hat{a}_{11}}{\partial r_{11}} = \frac{\sigma_x \, (c_{12} - c_{11})^{1/2}}{2^{3/2} \, (r_{12} - r_{11})^{3/2}} > 0 \tag{5.23}$$

$$\frac{\partial \hat{a}_{11}}{\partial r_{12}} = - \frac{\sigma_x \, (c_{12} - c_{11})^{1/2}}{2^{3/2} \, (r_{12} - r_{11})^{3/2}} < 0. \tag{5.24}$$

For the last choice asset, we differentiate (5.22c) with respect to r_{1m} and r_{1m-1}:

$$\frac{\partial \hat{a}_{1m}}{\partial r_{1m}} = \frac{\sigma_x \, (c_{1m} - c_{1m-1})^{1/2}}{2^{3/2} \, (r_{1m} - r_{1m-1})^{3/2}} > 0 \tag{5.25}$$

$$\frac{\partial \hat{a}_{1m}}{\partial r_{1m-1}} = - \frac{\sigma_x \, (c_{1m} - c_{1m-1})^{1/2}}{2^{3/2} \, (r_{1m} - r_{1m-1})^{3/2}} < 0. \tag{5.26}$$

For the interior choice assets, we obtain by differentiating (5.22b) with respect to the rates of return:

$$\frac{\partial \hat{a}_{1k}}{\partial r_{1k}} = \frac{\sigma_x}{2^{3/2}} \left[\frac{(c_{1k+1} - c_{1k})^{1/2}}{(r_{1k+1} - r_{1k})^{3/2}} + \frac{(c_{1k} - c_{1k-1})^{1/2}}{(r_{1k} - r_{1k-1})^{3/2}}\right] > 0 \qquad (5.27)$$

$$\frac{\partial \hat{a}_{1k}}{\partial r_{1k-1}} = \frac{\partial \hat{a}_{1k-1}}{\partial r_{1k}} = - \frac{\sigma_x (c_{1k} - c_{1k-1})^{1/2}}{2^{3/2} (r_{1k} - r_{1k-1})^{3/2}} < 0 \qquad (5.28)$$

$$\frac{\partial \hat{a}_{1k}}{\partial r_{1k+1}} = \frac{\partial \hat{a}_{1k+1}}{\partial r_{1k}} = - \frac{\sigma_x (c_{1k+1} - c_{1k})^{1/2}}{2^{3/2} (r_{1k+1} - r_{1k})^{3/2}} < 0. \qquad (5.29)$$

The implications of the above results are obvious. Given the original ordering of the choice assets all the own rate responses are positive while the adjacent rate responses are negative and symmetric, implying that all choice assets are perfect substitutes for their neighbours. In fact, for the first and the last choice assets, the own rate response is equal in magnitude to the adjacent rate response, while for the interior choice assets, the own rate response equals in magnitude the sum of the two adjacent rate responses. As a corollary of these properties we observe that the familiar Cournot condition also holds for the inventory-theoretical model since

$$\sum_{i=1}^{2} \frac{\partial \hat{a}_{1i}}{\partial r_{11}} = \sum_{i=m-1}^{m} \frac{\partial \hat{a}_{1i}}{\partial r_{1m}} = \sum_{i=2}^{m-1} \frac{\partial \hat{a}_{1i}}{\partial r_{1k}} = 0 \qquad \text{for } k = 2, 3, \ldots, m-1.$$

5.4.2 Response of Choice Asset Demand to Changes in the Liquidation Costs

We now turn to the examination of the response of choice asset demand to changes in the liquidation costs of the choice assets. Differentiating equations (5.22a), (5.22b) and (5.22c) with respect to the relevant liquidation costs yields:
for the first choice asset,

$$\frac{\partial \hat{a}_{11}}{\partial c_{11}} = - \frac{\sigma_x}{2^{3/2} [(r_{12} - r_{11})(c_{12} - c_{11})]^{1/2}} < 0 \qquad (5.30)$$

$$\frac{\partial \hat{a}_{11}}{\partial c_{12}} = \frac{\sigma_x}{2^{3/2}\,[(r_{12} - r_{11})(c_{12} - c_{11})]^{1/2}} > 0; \qquad (5.31)$$

for the last choice asset,

$$\frac{\partial \hat{a}_{1m}}{\partial c_{1m}} = -\frac{\sigma_x}{2^{3/2}\,[(r_{1m} - r_{1m-1})(c_{1m} - c_{1m-1})]^{1/2}} < 0 \qquad (5.32)$$

$$\frac{\partial \hat{a}_{1m}}{\partial c_{1m-1}} = \frac{\sigma_x}{2^{3/2}\,[(r_{1m} - r_{1m-1})(c_{1m} - c_{1m-1})]^{1/2}} > 0; \qquad (5.33)$$

and for the interior choice assets,

$$\frac{\partial \hat{a}_{1k}}{\partial c_{1k}} = -\frac{\sigma_x}{2^{3/2}\,[(r_{1k+1} - r_{1k})(c_{1k+1} - c_{1k})]^{1/2}} - \frac{\sigma_x}{2^{3/2}\,[(r_{1k} - r_{1k-1})(c_{1k} - c_{1k-1})]^{1/2}} < 0 \qquad (5.34)$$

$$\frac{\partial \hat{a}_{1k}}{\partial c_{1k-1}} = \frac{\partial \hat{a}_{1k-1}}{\partial c_{1k}} = \frac{\sigma_x}{2^{3/2}\,[(r_{1k} - r_{1k-1})(c_{1k} - c_{1k-1})]^{1/2}} > 0 \qquad (5.33)$$

$$\frac{\partial \hat{a}_{1k}}{\partial c_{1k+1}} = \frac{\partial \hat{a}_{1k+1}}{\partial c_{1k}} = \frac{\sigma_x}{2^{3/2}\,[(r_{1k+1} - r_{1k})(c_{1k+1} - c_{1k})]^{1/2}} > 0. \qquad (5.36)$$

These results are very similar to those derived in the last subsection. The only difference is that the own liquidation cost responses are negative and the adjacent asset liquidation cost responses are positive and symmetric. Again, we observe:

$$\sum_{i=1}^{2} \frac{\partial \hat{a}_{1i}}{\partial c_{11}} = \sum_{i=m-1}^{m} \frac{\partial \hat{a}_{1i}}{\partial c_{1m}} = \sum_{i=2}^{m-1} \frac{\partial \hat{a}_{1i}}{\partial c_{1k}} = 0 \qquad \text{for } k = 2, 3, \ldots, m-1.$$

5.4.3 Response of Choice Asset Demand to Changes in the Parameters of the Distribution of Random Cash Requirement and Total Expected Disposable Funds

As it is the random cash requirement which provides the source of asset diversification in the inventory-theoretical model it is very natural to ask how changes in the parameters of the distribution of random cash requirement affect the bank's demand for choice assets. However, before we probe into this question we consider briefly the effect on choice asset demand of a change in total expected disposable funds or the non-choice items, $\hat{A}_2$' i_2.

(i) Change in Total Expected Disposable Funds

As we can see from equations (5.22a) and (5.22b), the bank's optimal demand for the first (m - 1) choice assets is independent of the total expected disposable funds, $\hat{A}_2$' i_2. Therefore, a change in total disposable funds can only affect the bank's optimal demand for the last choice asset. Indeed, as we can easily appreciate from (5.22c),

$$\frac{\partial \hat{a}_{1m}}{\partial \hat{a}_{2i}} = 1 \qquad (5.37)$$

for all i = m+1, m+2, ... n. This is, then, a special case of the Engel condition, implying that if only the bank's total expected disposable funds (non-choice items) increases, the total increase will go into the highest yielding choice asset.

(ii) Change in the Mean of Random Cash Requirement, μ_x

The mean of the random cash requirement variable enters only the first and the last equations of the bank's choice asset demand. As we can see from equations (5.22a) and (5.22c) a rise in μ_x will increase the holding of the first choice asset and decrease that of the last choice asset by the same amount.

(iii) Change in the Variance of Random Cash Requirement, σ_x^2

Differentiating the choice asset demand equations in (5.22) with respect to the standard deviation of the random cash requirement variable, σ_x, we have for the first choice asset,

$$\frac{\partial \hat{a}_{11}}{\partial \sigma_x} = \left[\frac{(c_{12} - c_{11})}{2(r_{12} - r_{11})}\right]^{1/2} > 0, \tag{5.38}$$

for the interior choice assets,

$$\frac{\partial \hat{a}_{1k}}{\partial \sigma_x} = \left(\frac{1}{2}\right)^{1/2} \left[\left(\frac{c_{1k+1} - c_{1k}}{r_{1k+1} - r_{1k}}\right)^{1/2} - \left(\frac{c_{1k} - c_{1k-1}}{r_{1k} - r_{1k-1}}\right)^{1/2}\right] = \frac{\hat{a}_{1k}}{\sigma_x} > 0 \tag{5.39}$$

since by assumption $\hat{a}_{1k} > 0$, and for the last choice asset,

$$\frac{\partial \hat{a}_{1m}}{\partial \sigma_x} = -\left(\frac{(c_{1m} - c_{1m-1})}{2(r_{1m} - r_{1m-1})}\right)^{1/2} < 0. \tag{5.40}$$

Therefore, any increase in the variability of cash requirement will increase the bank's demand for the first m - 1 choice assets and reduce the demand for the last choice asset. It is also clear from (5.38) - (5.40) that

$$\sum_{i=1}^{m} \frac{\partial \hat{a}_{1i}}{\partial \sigma_x} = 0.$$

5.4.4 The Nonnegativity Condition

It is assumed earlier that all the m components of A_1 are positive. Under what conditions, then, will this assumption be satisfied? From (5.22a), for the holding of the

first choice asset to be positive, i.e., for $a_{11} > 0$, we require that

$$\sigma_x \left(\frac{c_{12} - c_{11}}{2(r_{12} - r_{11})}\right)^{1/2} > - \mu_x$$

or

$$\left(\frac{c_{12} - c_{11}}{r_{12} - r_{11}}\right) > - 2^{1/2} \frac{\mu_x}{\sigma_x} . \tag{5.41}$$

For the holding of any of the interior choice assets to be positive, i.e., for $a_{1k} > 0$, $k = 2, 3, ..., m-1$, it is necessary that

$$\sigma_x \left(\frac{c_{1k+1} - c_{1k}}{2(r_{1k+1} - r_{1k})}\right)^{1/2} > \sigma_x \left(\frac{c_{1k} - c_{1k-1}}{2(r_{1k} - r_{1k-1})}\right)^{1/2}$$

or

$$\left(\frac{c_{1k+1} - c_{1k}}{r_{1k+1} - r_{1k}}\right)^{1/2} > \left(\frac{c_{1k} - c_{1k-1}}{r_{1k} - r_{1k-1}}\right)^{1/2}. \tag{5.42}$$

Similarly, for $\hat{a}_{1m} > 0$ we require

$$- \hat{A}_2' \, i_2 - \mu_x > \sigma_x \left(\frac{c_{1m} - c_{1m-1}}{2(r_{1m} - r_{1m-1})}\right)^{1/2}$$

or

$$2^{1/2} \frac{-\hat{A}_2' \, i_2 - \mu_x}{\sigma_x} > \left(\frac{c_{1m} - c_{1m-1}}{r_{1m} - r_{1m-1}}\right)^{1/2}. \tag{5.43}$$

Given the initial orderings of the m choice assets (5.41) - (5.43) imply that

$$2^{1/2}\,\frac{-\hat{A}_2'\,i_2 - \mu_x}{\sigma_x} > \left(\frac{c_{1m} - c_{1m-1}}{r_{1m} - r_{1m-1}}\right)^{1/2} > \left(\frac{c_{1m-1} - c_{1m-2}}{r_{1m-1} - r_{1m-2}}\right)^{1/2} > \ldots >$$

$$\left(\frac{c_{12} - c_{11}}{r_{12} - r_{11}}\right)^{1/2} > -\frac{2^{1/2}\,\mu_x}{\sigma_x}$$

or

$$-\frac{\sigma_x}{2^{1/2}\,\mu_x} > \left(\frac{r_{12} - r_{11}}{c_{12} - c_{11}}\right)^{1/2} > \ldots > \left(\frac{r_{1m-1} - r_{1m-2}}{c_{1m-1} - c_{1m-2}}\right)^{1/2} >$$

$$\left(\frac{r_{1m} - r_{1m-1}}{c_{1m} - c_{1m-1}}\right)^{1/2} > \frac{\sigma_x}{2^{1/2}(-\hat{A}_2'\,i_2 - \mu_x)}. \tag{5.44}$$

Under these conditions all m choice asset holdings will be positive. From the first-order conditions in (5.14) we deduce

$$\frac{r_{1i+1} - r_{1i}}{c_{1i+1} - c_{1i}} = 1 - F(A_{1i}). \tag{5.45}$$

Further, recalling that

$$\hat{A}_{1m} = \hat{A}_1'\,i_1 = -\hat{A}_2'\,i_2$$

we then obtain from (5.16)

$$\frac{\sigma_x}{2^{1/2}(-\hat{A}_2'\,i_2 - \mu_x)} = [1 - F(A_{1m})]^{1/2} \tag{5.46}$$

and

$$\frac{\sigma_x}{2^{1/2}(0 - \mu_x)} = [1 - F(0)]^{1/2}. \tag{5.47}$$

Using (5.45) - (5.47) we are able to restate (5.44) as

$$1 - F(0) > 1 - F(A_{11}) > \ldots > 1 - F(A_{1m-1}) > 1 - F(A_{1m})$$

or

$$F(A_{1m}) > F(A_{1m-1}) > \ldots > F(A_{11}) > F(0). \qquad (5.48)$$

Since F(.) is the cumulative density function of random cash requirement (5.48) is always satisfied. Therefore, the bank's optimal portfolio will indeed ensure positive holdings for all the m assets in the choice set.

We shall not consider the short-run adjustment of the equilibrium model developed in this chapter; this is because in deriving the equilibrium model we have already taken into account of the liquidation costs of assets which are part of the adjustment costs.

5.5 PROBLEMS OF AGGREGATION AND ESTIMATION

Before we can estimate and test the model in (5.22) a number of issues need to be resolved. First, the model in (5.22) depicts the optimal portfolio behaviour of an individual bank, we need therefore to make some assumptions regarding aggregation over banks. Second, it is necessary to order the choice assets. This ordering of choice assets is crucial since it determines which asset is adjacent to another, and as we can see from (5.22), only the own interest rate and liquidation cost and those of the adjacent assets enter the choice asset demand functions. Third, both the mean and variance of the cash requirement variable are not observable. We can either treat them as parameters or variables in the choice asset demand equations. For reasons to be discussed below they will be regarded as variables and thus need to be estimated prior to the estimation of the choice asset demand equations. We deal with each of these issues in turn in the following three subsections.

5.5.1 Aggregation

We shall employ the same definitions of A_1 and A_2 as in Chapter Three so that there are five choice assets in the pre-1971 period and seven choice assets in the post-1971 period. In order to aggregate over banks we assume that

(1) all banks face the same decision period which is one calender month and maximise the same objective function of the form in (5.5);

(2) the expected values of the non-choice items are given by their actual levels;

(3) interest rate and liquidation cost for the same asset are the same for all banks;

(4) the level of random cash requirement differs from one bank to another but the underlying generating process is the same for all banks; in addition, the random cash requirements of all the banks are independently distributed from each other.

The first two assumptions are the same as in the previous two chapters and require no

further discussion. As for the third assumption it is a fact that the London clearing banks face more or less the same interest rates and given the similarities in their scale and operations it seems reasonable to assume that they also face the same liquidation costs. While banks may differ in the level of their actual cash requirement, because the nature of their operations are very similar it seems also justified to assume that the cash requirements of all the London clearing banks follow the same type of distribution.

Given the above assumptions we can write the aggregate optimal choice asset demand equations of all the banks under consideration as

$$\hat{a}_{11} = \left(\frac{\sigma_x^2 \, (c_{12} - c_{11})}{2(r_{12} - r_{11})}\right)^{1/2} + \mu_x \qquad (5.49a)$$

$$\hat{a}_{1k} = \left(\frac{\sigma_x^2 \, (c_{1k+1} - c_{1k})}{2(r_{1k+1} - r_{1k})}\right)^{1/2} - \left(\frac{\sigma_x^2 \, (c_{1k} - c_{1k-1})}{2(r_{1k} - r_{1k-1})}\right)^{1/2}$$

$$\text{for } k = 2, 3, \ldots, m - 1 \qquad (5.49b)$$

$$\hat{a}_{1m} = - A_2' \, i_2 - \left(\frac{\sigma_x^2 \, (c_{1m} - c_{1m-1})}{2(r_{1m} - r_{1m-1})}\right)^{1/2} - \mu_x \qquad (5.49c)$$

where $\mathbf{A_2}$ is as defined in Chapter Three, $\hat{a}_{11}$, $\hat{a}_{1k}$, $\hat{a}_{1m}$, μ_x and σ_x^2 are the sums of $\hat{a}_{11}$, $\hat{a}_{1k}$, $\hat{a}_{1m}$, μ_x and σ_x^2 over all banks respectively. In order to estimate the system in (5.49) we need to specify which quantities are variables and which are parameters. Since we do not have information on the c_{1i}'s it seems natural to treat the quantities, $(c_{1i} - c_{1i-1})$ for all i = 2, 3, ..., m, as paramters. This implicitly assumes that the $(c_{1i} - c_{1i-1})$ are constant over time. However, as we shall see shortly such an assumption poses a difficult problem in the empirical implementation of the choice asset demand equations.

5.5.2 Ordering the Assets

The problem arises from the unique ordering of the choice assets. But how do we order the choice assets? From (5.7) and (5.8) we see that the ordering of the liquidation costs must be the same as the ordering of the interest rates in each decision period otherwise the nonnegativity conditions for the complete set of choice assets could not be possibly satisfied. Since we do have information on the interest rates this should enable us to order the choice assets.

The simplest way is to order the assets on the basis of the sample means of the interest rates. Table 5.1 presents the sample statistics of the interest rates in ascending order of the mean rates for the two sample periods. While the sample means of the interest rates do rank the choice assets uniquely for both the pre-1971 and post-1971 periods, it is clear from Figures 5.1 and 5.2 that interest rate orderings changed over time. Indeed, there are five different orderings of the interest rates out of 150 obervations in the pre-1971 period and sixty distinctive orderings out of a total of 141 observations in the post-1971 period. The nonnegativity conditions imply that the ordering of liquidation costs also changed the same number of times and this creates a difficult problem. Whenever the ordering of the choice assets changes the structure of the asset demand equations in (5.49) has to change, too, so that the independent variables appearing in each equation will be different. However, in order that the system of equations in (5.49) can be estimated meaningfully the ordering of the liquidation costs has to be the same in a sufficiently large number of decision periods. To be more precise, the number of decision periods which have the same interest rate ordering should be at least as large as the number of independent parameters corresponding to that particular ordering in the system of choice asset demand equations.

Table 5.1
Sample Statistics of Interest Rates: 1959:01 1983:06

	Mean	Minimum	Maximum	Standard deviation
		1959:01 to 1971:06		
CMR	4.90	2.69	7.13	1.33
TBR	5.52	3.13	7.88	1.41
GBR	6.39	4.38	9.32	1.28
CBR	7.11	4.63	9.63	1.54
ADR	7.93	6.00	10.00	1.32
		1971:10 TO 1983:06		
CMR	9.50	2.88	16.75	3.33
TBR	10.46	4.27	16.28	3.05
CDR	11.39	4.56	18.06	3.28
LLR	11.50	4.56	18.81	3.34
GBR	11.61	5.38	17.10	2.37
CBR	11.70	5.06	17.81	3.10
ADR	13.04	7.00	19.00	3.12

Table 5.2
Choice Asset Interest Rate Ordering in the Pre-1971 Period

Ordering	No. of Observations
CMR < TBR < GBR < CBR < ADR	125
CMR = TBR < GBR < CBR < ADR	1
CMR < TBR < CBR < GBR < ADR	8
CMR < GBR < TBR < CBR < ADR	9
TBR < CMR < GBR < CBR < ADR	7

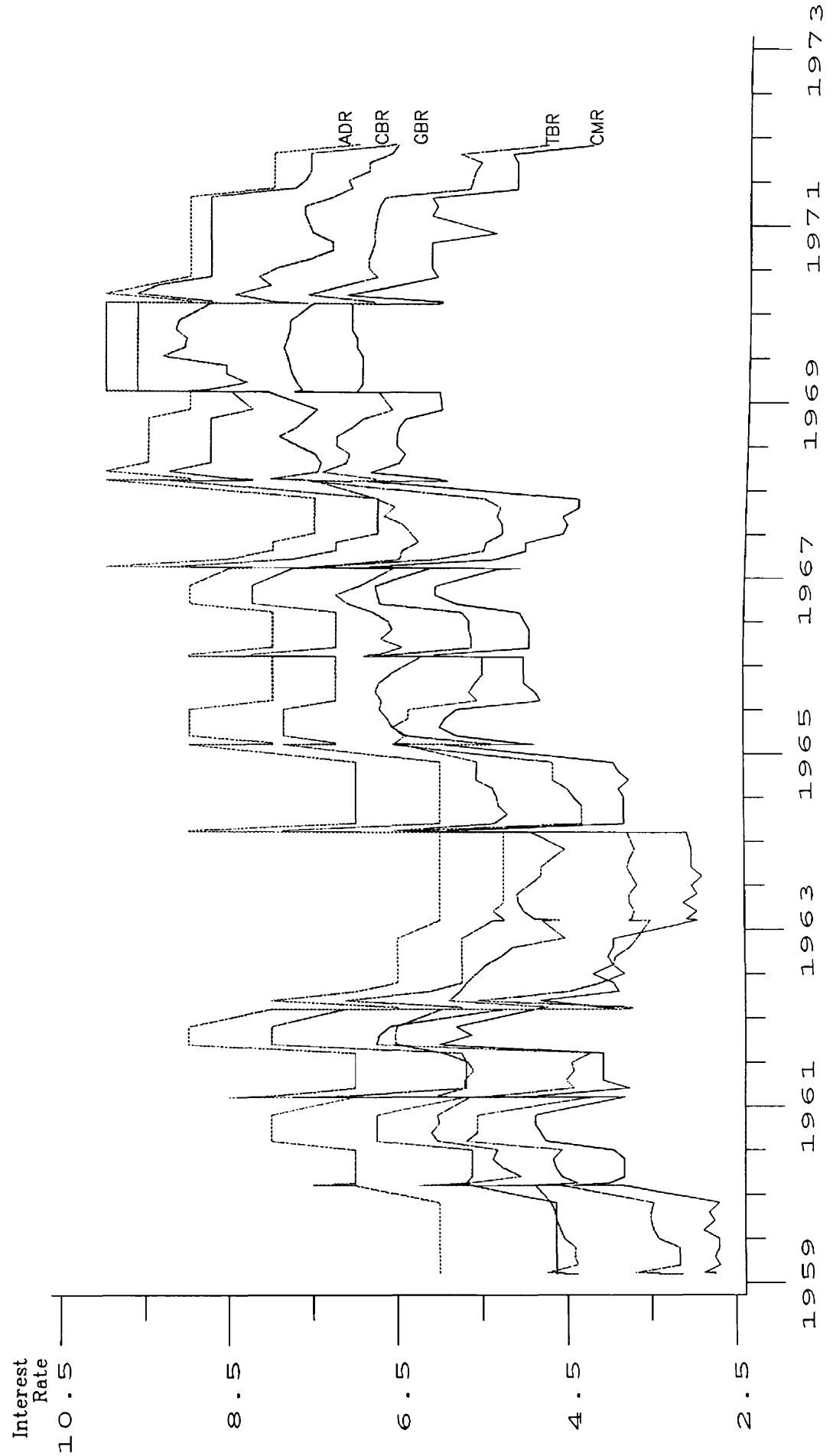


Figure 5.1
Interest Rates From 1959:01 to 1971:09

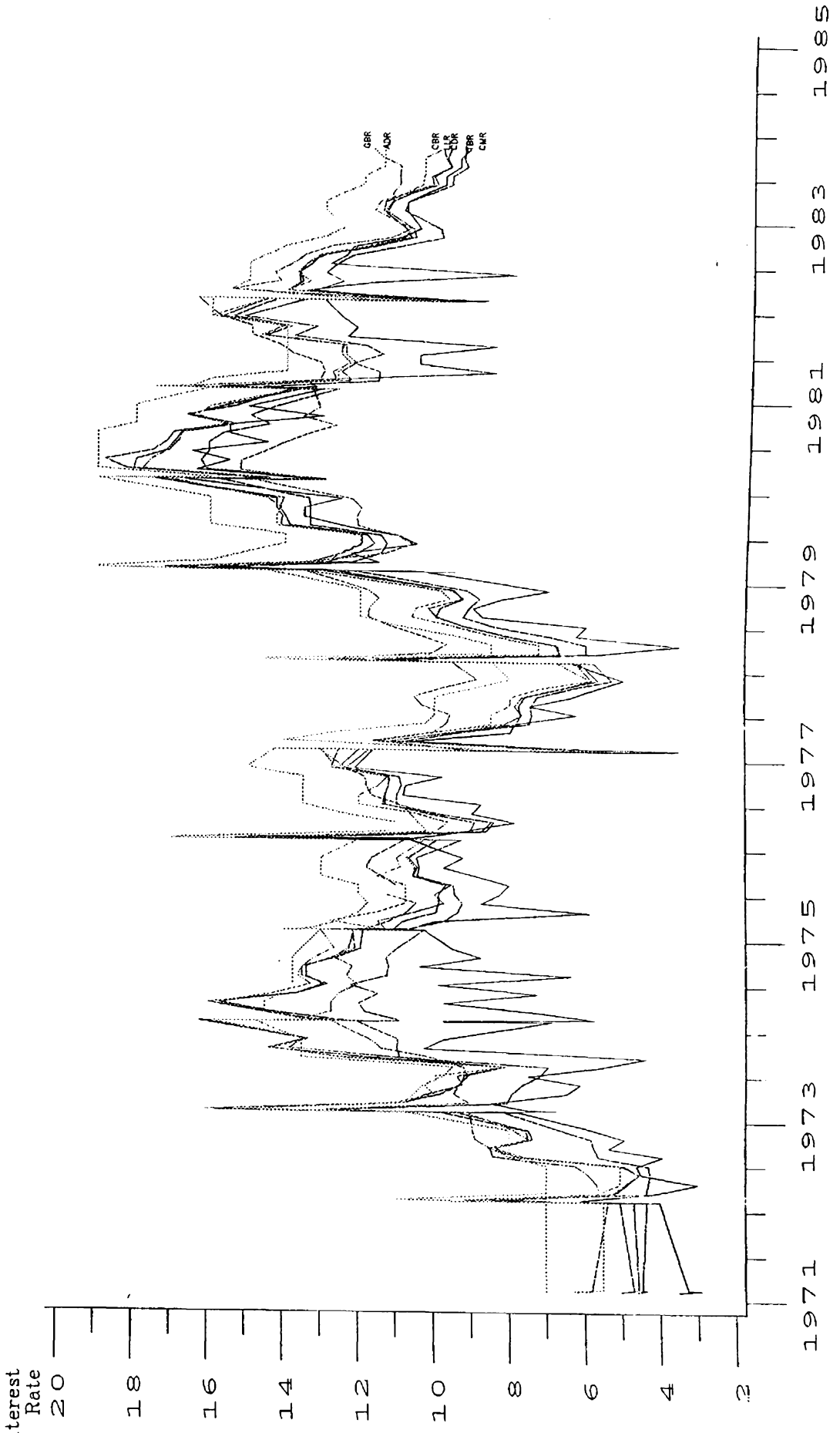


Figure 5.2
Interest Rates From 1971:10 to 1983:08

The five different orderings in the pre-1971 period are given in Table 5.2 and they consist of 125, 1, 8, 9, and 7 observations respectively. Of the sixty distinctive orderings in the post-1971 period only three have more than six observations each. There are five choice assets in the pre-1971 period and seven choice assets in the post-1971 period. As we shall see later that there will be at least, respectively, four and six independent parameters corresponding to each unique asset ordering for the pre- and post-1971 periods. This means that the theoretical model of asset demand in (5.49) cannot be meaningfully estimated for the post-1971 period and we may take this as a refutation of the inventory-theoretical model by the post-1971 data. We are, luckily, able to estimate the theoretical model in (5.49) for the pre-1971 period except for one particular ordering which has only one observation (1962:6). In fact, for this particular ordering our choice asset demand model in (5.49) cannot be defined since the interest rates of the first two choice assets, Call money and Treasury bills, are equal. For reasons of consistency we shall exclude this particular observation from our estimation. Unfortunately, when we are to compare in the next chapter the inventory-theoretical model with some of the models developed in the preceding chapters it will be necessary to reestimate those models without the above mentioned observation. Alternatively, one could employ a dummy variable to account for this particular observation, but as it turns out, the presence of such a dummy variable introduces data singularity into the model.

One could, of course, aggregate the choice assets into fewer groups so as to reduce the number of possible different orderings. However, there are two basic problems with such an alternative. First, as each of the aggregated asset groups contains more than one type of assets, it will be difficult to find a suitable interest rate as the rate of return on the aggregated asset. Second, and more important for our purposes, it will not be possible to test the precautionary models using the aggregated assets against those models developed in the previous chapters, since the number of equations in the models will be different. For these reasons, we shall not consider here further aggreagting the choice assets.

The system of equations in (5.49) describe the optimal aggregate portfolio behaviour of banks. Actual behaviour may deviate from the optimal behaviour randomly and, as before, it is postulated that the vector of random errors enter the demand equations additively. In addition, we shall introduce into the asset demand equations the Special Deposit and credit ceilings dummy variables which are also presumed to be additive. Thus, actual aggregate choice asset demand equations of the banks, corresponding to one particular asset ordering can be written as

$$a_{11} = \left(\frac{\sigma_x^2 (c_{12} - c_{11})}{2(r_{12} - r_{11})}\right)^{1/2} + \mu_x + f_{11} D_{SD} + f_{12} D_{CC} + \epsilon_{1t} \qquad (5.50a)$$

$$a_{1k} = \left(\frac{\sigma_x^2 (c_{1k+1} - c_{1k})}{2(r_{1k+1} - r_{1k})}\right)^{1/2} - \left(\frac{\sigma_x^2 (c_{1k} - c_{1k-1})}{2(r_{1k} - r_{1k-1})}\right)^{1/2}$$

$$+ f_{k1} D_{SD} + f_{k2} D_{CC} + \epsilon_{kt}$$

$$\text{for } k = 2, 3, \ldots, m - 1 \qquad (5.50b)$$

$$a_{1m} = - A_2' i_2 - \left(\frac{\sigma_x^2 (c_{1m} - c_{1m-1})}{2(r_{1m} - r_{1m-1})}\right)^{1/2} - \mu_x$$

$$+ f_{m1} D_{SD} + f_{m2} D_{CC} + \epsilon_{mt} \qquad (5.50c)$$

where D_{SD} and D_{CC} are the Special Deposit and credit ceilings dummies and f_{i1} and f_{i2} their respective coefficients. The ϵ_{it}'s for i = 1, 2, ..., m, are random errors which are assumed to be normally distributed with means zero and covariance matrix Σ. It is important to note that when there are more than one orderings of the choice assets during the sample period there will be other terms in the right-hand side of equations, corresponding to different orderings of the choice assets. In order to estimate the system of equations in (5.50) for the whole sample period it will be necessary to set up the independent variables (except the dummies) in such a way that the variables corresponding to one particular ordering will have zero values when a different ordering is in effect. This means that there will be more than one interest rate difference terms

in the first and last equations and more than two such terms in the interior equations. It should also be noted that we will not be able to use our empirical model to make forecasts or predictions with observations which have orderings of choice assets different from those occurred in the sample period.

5.5.3 Estimating the Mean and Variance of Cash Requirement

Before we can estimate the aggregate system of choice asset demand equations in (5.50) some further assumptions concerning the mean and variance of the cash requirement variable are required. As we can see from the individual bank's system of choice asset demand equations in (5.22) the parameters of the density function of the random variable enter the demand equations in two ways: the mean of the cash requirement, μ_x, enters additively, but only in the first and last choice asset equations, while the variance of the random variable, σ_x^2, enters each and every choice asset equation multiplicatively. However, both the mean and the variance of the cash requirement variable are unknown to the banks. In their study on the demand for financial assets by the personal sector of the UK economy, Barrett, Gray and Parkin (1975) postulate that both the mean and the variance of the cash requirement are linear functions of some exogenous variables, i.e.,

$$\mu_X = \alpha_1 T \qquad \text{and} \qquad \sigma_X^2 = \alpha_2 T$$

where T is the sum of consumption and tax payments [see Barrett, Gray and Parkin (1975), p. 508]. The above relations are then substituted for μ_x and σ_x^2 into the asset demand equations and α_1 and α_2 are estimated together with other parameters in the choice asset demand equations. Such an approach is very *ad hoc* and indeed inconsistent in that in estimating the mean and variance of cash requirement at time t the decision-maker is also using information which only becomes available after time t. In this section we apply the Bayesian approach to the bank's expectations formation regarding future cash requirement. The mean and variance of cash requirement are first

estimated and their estimates are then used as data in the estimation of the bank's choice asset demand equations.

The approach adopted here is quite straightforward. It is postulated that the bank believes actual cash requirement is influenced by some underlying factors. The precise relationship between actual cash requirement and these underlying factors is not known, but the bank holds *prior* beliefs about the parameters of this relationship represented by the joint *prior* probability distribution of the parameters. As more information becomes available the bank revises its *prior* beliefs regarding the parameters according to Bayes' rule and uses the revised, *posterior* probability distribution as the *prior* probability density for the next period. Each time when new data about actual cash requirement and the underlying variables becomes available this process of updating is repeated [for an introduction to the Bayesian approach see, for example, Zellner (1971)]. The model presented below is a simple one, but it can be extended to accommodate for more elaborate specifications of the generating process of actual cash reuqirement without any difficulty.

Suppose that the bank sees actual cash requirement at time t as generated by the following normal process:

$$X_t = Z_t' b + \epsilon_t \qquad \epsilon_t \sim N(0, \sigma_\epsilon^2) \qquad (5.51)$$

where X_t is actual cash requirement at time t, Z_t is a k × 1 vector of exogenous variables, b is a k × 1 vector of unknown parameters and ϵ_t is the error term normally distributed with mean zero and unknown variance σ_ϵ^2. If the parameters b and σ_ϵ^2 were known the bank's expectations of cash requirement during the decision period would be given by

$$\mu_X = E(X_t) = Z_t' b \qquad (5.52)$$

and

$$\sigma_X^2 = E[X_t - E(X_t)]^2 = \sigma_\epsilon^2. \qquad (5.53)$$

However, the parameters b and σ_e^2 are unknown, but the bank holds *prior* beliefs about them represented by the joint *prior* distribution of b and σ_e^2, denoted by $P(b, \sigma_e^2)$. Specifically, we assume that the *prior* is a normal-gamma density of the following form

$$P(b, \sigma_\epsilon^2) = P_1(b|\sigma_\epsilon^2)\ P_2(\sigma_\epsilon^2) \qquad (5.54)$$

where

$$P_1(b|\sigma_\epsilon^2) \propto \sigma_\epsilon^{-k} \exp\{-(1/2\sigma_\epsilon^2)\ (b - \beta)'\ \Omega\ (b - \beta)\} \qquad (5.55)$$

is the conditional *prior* density of b given σ_e^2, which is normal with mean vector β and known dispersion matrix Ω^{-1} of order $k \times k$, and

$$P_2(\sigma_\epsilon^2) \propto \sigma_\epsilon^{-2(\alpha-1)} \exp\{-\gamma/\sigma_\epsilon^2\} \qquad (5.56)$$

is the marginal *prior* density of σ_e^2, which is gamma with parameters $\alpha > 0$ and $\gamma > 0$. We may note in passing that the mean and variance of the gamma distribution is [see, for example, Broemeling (1985), p. 442]

$$E(\sigma_\epsilon^2) = \gamma/(\alpha - 1), \qquad \alpha > 0 \qquad (5.57)$$

and

$$Var(\sigma_\epsilon^2) = \gamma^2/(\alpha - 1)^2\ (\alpha - 2), \qquad \alpha > 2. \qquad (5.58)$$

As more information becomes available the bank revises its *prior* beliefs regarding

the parameters b and σ_ϵ^2 according to Bayes' rule. This gives its *posterior* density of the parameters b and σ_ϵ^2:

$$P^*(b, \sigma_\epsilon^2 | X, Z) \propto P(b, \sigma_\epsilon^2)\, \ell(X | b, \sigma_\epsilon^2, Z) \tag{5.59}$$

where $\ell(X | b, \sigma_e^2, Z)$ is the likelihood function of observing X given b, σ_e^2 and Z, i.e., it is the conditional density of the sample random variables given b and σ_e^2.

Following a set of T observations on X_t and Z_t we can write (5.51) as

$$X = Z\, b + \epsilon \tag{5.60}$$

where X is a T × 1 vector of observations on X_t, Z is a T × k matrix of observations on the k independent variables, b is a k × 1 vector of unknown parameters as defined before, and ϵ is a T × 1 vector of error terms normally and independently distributed with mean vector zero and common variance σ_e^2. Since $\epsilon \sim N(0, \sigma_e^2 I)$, the random vector X is also a normal process and thus the likelihood function is

$$\ell(X | b, \sigma_\epsilon^2, Z) \propto \sigma_\epsilon^{-T} \exp\{ -(1/2\sigma_\epsilon^2)\, (X - Zb)'\, (X - Zb)\}. \tag{5.61}$$

Noting that

$$(X - Zb)'\, (X - Zb) = [X - Z\hat{b} - Z(b - \hat{b})]'[X - X\hat{b} - Z(b - \hat{b})]$$

$$= (X - Z\hat{b})'(X - Z\hat{b}) + (b - \hat{b})'\, Z'Z\, (b - \hat{b}) \tag{5.62}$$

we can rewrite the likelihood function as follows:

$$\ell(X | b, \sigma_\epsilon^2, Z) \propto \sigma_\epsilon^{-T} \exp\{ -(1/2\sigma_\epsilon^2)\, [S + (b - \hat{b})'\, Z'Z\, (b - \hat{b})]\} \tag{5.61'}$$

where

$$\hat{b} = (Z'Z)^{-1} Z'X \tag{5.63}$$

is the least squares estimator of b and

$$S = (X - Z\hat{b})' (X - Z\hat{b}) \tag{5.64}$$

is the least squares sample residuals.

Combining the *prior* density in (5.54) with the likelihood function in (5.61) yields the *posterior* density of the parameters b and σ_e^2:

$$P^*(b, \sigma_\epsilon^2 | X, Z) \propto \sigma_\epsilon^{-(T+k+2\alpha-2)} \times$$

$$\exp\{ -(1/2\sigma_\epsilon^2) [(b - \beta)'\Omega(b - \beta) + (X - Zb)'(X - Zb) + 2\gamma]\} \tag{5.65}$$

which, upon completing the square on b and utilising (5.61'), (5.63) and (5.64), can be restated as

$$P^*(b, \sigma_\epsilon^2 | X, Z) \propto \sigma_\epsilon^{-k} \exp\{ -(1/2\sigma_\epsilon^2) (b - \beta^*)' \Omega^* (b - \beta^*)\} \times$$

$$\sigma_\epsilon^{-(T+2\alpha-2)} \exp\{ -\gamma^*/\sigma_\epsilon^2\} \tag{5.66}$$

where

$$\beta^* = (Z'Z + \Omega)^{-1} (Z'X + \Omega\beta) \tag{5.67a}$$

$$\Omega^* = Z'Z + \Omega \tag{5.67b}$$

$$\gamma^* = \gamma + (1/2)[X'X - (Z'X + \Omega\beta)' \beta^*]. \qquad (5.67c)$$

Therefore, the joint *posterior* density is also normal-gamma. It is clear from (5.66) that the marginal *posterior* moments of σ_e^2 are

$$E(\sigma_\epsilon^2 | X, Z) = [2\gamma + X'X - (Z'X + \Omega\beta)' \beta^*](T + 2\alpha - 2)^{-1} \qquad (5.68)$$

and

$$Var(\sigma_\epsilon^2 | X, Z) = E(\sigma_\epsilon^2 | X, Z)^2 (T + 2\alpha - 4)^{-1}. \qquad (5.69)$$

The marginal *posterior* density of b is derived by integrating (5.66) with respect to σ_e^2:

$$P^*(b | X, Z) \propto \{ 2\gamma + X'X - (Z'X + \Omega\beta) \beta^* + (b - \beta^*)' \Omega^* (b - \beta^*)\}^{-(T+2\alpha+k)/2} \qquad (5.70)$$

which is a k-dimensional t density with $(T + 2\alpha)$ degrees of freedom, location vector

$$\beta^* = (Z'Z + \Omega)^{-1} (Z'X + \Omega\beta) \qquad (5.71)$$

and dispersion matrix

$$Var(b | X, Z) = \Omega^{*-1} [2\gamma + X'X - (Z'X + \Omega\beta)' \beta^*](T + 2\alpha - 2)^{-1}$$

$$= \Omega^{*-1} E(\sigma_\epsilon^2 | X, Z). \qquad (5.72)$$

From the *posterior* density of b and σ_e^2 we can then derive the predictive mean and the variance of random cash requirement. Replacing the unknown values of b and

σ_e^2 by their *posterior* moments we have at time t the predictive mean of random cash requirement

$$\mu_X = E(X_t) = Z_t' \beta^* \qquad (5.73)$$

and its variance

$$\sigma_X^2 = E(\sigma_\epsilon^2 | X, Z) \qquad (5.74)$$

where β^* and $E(\sigma_e^2 | X, Z)$ are given by (5.71) and (5.68) respectively. However, both (5.68) and (5.71) involve the parameters of the *prior* distributions of b and σ_ϵ^2 which are unknown for the first decision period of our sample. One could either assign arbitrary values to these parameters or estimate them from equation (5.51) using data prior to the sample period. Since data is available prior to the sample period we shall opt for the latter. Once the parameter values of the *priors* of b and σ_ϵ^2 are obtained for the first decision period, the predictive mean and variance of the cash requirement variable can then be derived directly from (5.73) and (5.74). Moreover, it can be shown that, provided the parameters of the *prior* distributions are estimated from equation (5.51) by ordinary least squares using data before the first decision period of the sample, (5.71) is equivalent to the Recursive Least Squares formula for the parameter vector b [see, for example, Harvey (1981), p. 54].

The random cash requirement variable, X_t, is, however, not directly observable. We shall assume that a bank's actual holding of cash represents its actual cash requirement. In addition, we assume that for an individual bank (5.51) takes the following form:

$$CA_t = b_1 TD_{t-1} + b_2 AD_{t-1} + \epsilon_t \qquad \epsilon_t \sim N(0, \sigma_X^2) \qquad (5.75)$$

where CA_t is the bank's actual cash holdings, TD_{t-1} and AD_{t-1} are its actual total deposits and advances at time $t-1$. Since we have assumed that the random cash

requirements of the banks are independent from each other the aggregate mean, μ_x, and variance, σ_x^2 are simply the sums of all μ_x's and σ_x^2's. Thus

$$CA_t = b_1\ TD_{t-1} + b_2\ AD_{t-1} + \epsilon_t \qquad \epsilon_t \sim N\ (0,\ \sigma_x^2) \tag{5.76}$$

where CA_t is the sum of CA_t over all banks and TD_{t-1} and AD_{t-1} are the aggregate total deposits and advances at time t-1 respectively.

The parameters of the *prior* distributions used for the first decision period (1959:01) are based on least squares estimates of (5.76) for the period 1958:01 - 1958:12. Each time we estimate (5.76) when an additional observation is made available we obtain the predicted value of CA_t as the mean of the cash requirement variable and the sample estimate of σ_x^2 as its variance. These estimates are then used in the estimation of the aggregate choice asset demand equations in (5.50).

Letting

$$b_{ij} = (c_{1i} - c_{1j})^{1/2} \tag{5.77}$$

$$R_{ij} = \left(\frac{\sigma_x^2}{2(r_{1i} - r_{1j})} \right)^{1/2}, \tag{5.78}$$

we can write the banks' actual aggregate choice asset demand equations corresponding to one particular ordering of the assets, as

$$a_{11} = b_{21}\ R_{21} + \mu_x + f_{11}\ D_{SD} + f_{12}\ D_{CC} + \epsilon_{1t} \tag{5.79a}$$

$$a_{1k} = b_{k+1,k}\ R_{k+1,k} - b_{k,k-1}\ R_{k,k-1} + f_{k1}\ D_{SD} + f_{k2}\ D_{CC} + \epsilon_{kt}$$

$$\text{for } k = 2,\ 3,\ \ldots,\ m - 1 \tag{5.79b}$$

$$a_{1m} = -\ A_2'\ i_2 - b_{m,m-1}\ R_{m,m-1} - \mu_x + f_{m1}\ D_{SD} + f_{m2}\ D_{CC} + \epsilon_{mt} \tag{5.79c}$$

(5.79) is a system of linear equations and can now be directly estimated by FIML.

Unfortunately, the transformation of the variables in (5.57) and (5.58) also means that we will not be able to distinguish in empirical estimation between the effects on choice asset demand of σ_x^2 and the interest rate term $(r_{1i} - r_{1j})$, and thus makes the interpretation of empirical results difficult. From (5.78) we see that all R_{ij}'s have the same numerator so that, for a particular period, differences in the R_{ij}s simply reflect differences in the interest rate terms, $(r_{1i} - r_{1j})$. Given the definition in (5.77) all b_{ij}'s in (5.79) should still be positive. However, over time, the σ_x^2 also changes. Thus empirically, there is no guarantee that all b_{ij}'s will be positive. We discuss briefly some of the empirical results of the above model for the London clearing banks in the next section.

5.6 SOME EMPIRICAL RESULTS FOR THE LONDON CLEARING BANKS: 1959-1971

As we mentioned earlier, due to the large number of distinctive orderings of the interest rates of the choice assets the model in (5.79) cannot be applied to the post-1971 data. Thus, the system of equations in (5.79) is estimated using data for the London clearing banks during the period 1959:01 to 1971:06 only. To circumvent the singularity problem the Government bond equation is once again dropped from estimation. And as in previous chapters the system is estimated both with and without first-order vector autocorrelation. We shall also test whether the matrix of first-order autoregressive parameters is full or diagonal. The sample values of the maximised log-likelihood function for the different models are reported in Table 5.3. Based on these information various hypotheses can be tested using the likelihood ratio statistic. We discuss below a few of these test results.

Table 5.3
Sample Values of the Maximised Log-likelihood Function
for the Precautionary Models of Bank Portfolio Behaviour
1959:01 to 1971:06

	Ψ = full	Ψ = diagonal	Ψ = null
SSD Dummy CC Dummy	-3054.88 (34)	-3091.29 (22)	-3979.53 (18)
SSD Dummy No CC Dummy	-3061.51 (30)	-3096.63 (18)	-4007.03 (14)
CC Dummy No SSD Dummy	-3060.89 (30)	-3096.34 (18)	-4013.57 (14)
No SSD Dummy No CC Dummy	-3068.35 (26)	-3102.29 (14)	-4114.04 (10)

Note: Figures in parentheses are the number of free coefficients estimated.

Table 5.4
Sample Likelihood Ratios for Testing First-order Vector Autocorrelation in the Precautionary Models of Bank Portfolio Behaviour
1959:01 to 1971:06

	$\forall$ = full vs null	$\forall$ = diag. vs null	$\forall$ = full vs diag.
SSD Dummy CC Dummy	1849.30 (16)	1776.48 (1)	72.86 (15)
SSD Dummy No CC Dummy	1891.04 (16)	1820.80 (1)	70.24 (15)
CC Dummy No SSD Dummy	1905.36 (16)	1834.46 (1)	70.90 (15)
No SSD Dummy No CC Dummy	2091.38 (16)	2023.50 (1)	67.88 (15)

Note: Figures in parentheses are the degrees of freedom under the null hypothesis.

Table 5.5
Sample Likelihood Ratios for Testing the Dummy Variables in the Precautionary Models of Bank Portfolio Behaviour
1959:01 to 1971:06

	$\forall$ = full	$\forall$ = diagonal	$\forall$ = null
SSD Dummy No CC Dummy	13.26 (4)	10.68 (4)	55.00 (4)
CC Dummy No SSD Dummy	12.02 (4)	10.10 (4)	68.08 (4)
No SSD Dummy No CC Dummy	26.94 (8)	22.00 (8)	269.02 (8)

Note: Figures in parentheses are the degrees of freedom under the null hypothesis.

First-order Autocorrelation

It is clear from Table 5.4 that the null hypothesis of no first-order autocorrelation must be rejected in all cases and regardless of whether the matrix of autoregressive parameters is assumed to be diagonal or full. The rejection is overwhelming: the lowest sample likelihood ratio statistic for testing the null hypothesis against the alternatives of full autoregressive matrix is 1849.30 while, with sixteen degrees of freedom, the critical chi-square value is only 13.28% at the 1% significance level. The rejection of no first-order vector autocorrelation is equally decisive when tested against the alternative of a diagonal matrix of autoregressive coefficients: the lowest sample likelihood ratio statistic is 1776.48, compared with a 1% critical chi-square value of 6.635 with one degrees of freedom. We also have to reject the null of a diagonal autoregressive matrix in favour of the full matrix alternative in all the 4 cases, with or without the dummy variables. The acceptance of the full matrix specification of the autoregressive coefficients is consistent with the test results in the last two chapters.

Dummy Variables

Unlike the models in the previous chapters, however, we are unable to accept the null hypothesis of no Special Deposit and credit ceilings dummies jointly at both the 5% and 1% significance levels. As we can see from Table 5.5, the null must be rejected in all 3 cases, with or without first-order vector autocorrelation and regardless of whether the matrix of autoregressive coefficients is assumed full or diagonal. In the case of $\forall$ = null, the test result is conclusive: the sample likelihood ratio is 269.02, compared with a 1% chi-square critical value of 20.09. This test result is less decisive, though, when first-order autocorrelation is assumed, for the likelihood ratio test statistics of 26.94 ($\forall$ = full) and 22.00 ($\forall$ = diagonal) are only marginally higher than the 1% chi-square critical value. The dummy variables may also be accepted separately in all cases at the 5% significance level, although the null of no Special Deposit dummy or no credit ceilings dummy may be accepted at the 1% significance level when the models are assumed to

follow the first-order vector autoregressive process.

Given the above test results we shall regard the model with the dummy variables and a full matrix of first-order autoregressive coefficients as the maintained hypothesis within the precautionary framework of bank portfolio behaviour. FIML estimates of the maintained model are reported in Table 5.6. The performance of the model is in general very poor.

First, as we mentioned earlier, all the coefficients of R_{ij}'s, b_{ij}'s, should be positive. However, as we can see from Table 5.6, three of the ten coefficients of R_{ij}'s are negatively signed. Interestingly, all the three coefficients involved the Government bond equation, which was dropped from direct estimation because of the singularity problem. In fact, only two of the ten coefficients of R_{ij}'s are statistically different from zero. This poor performance is probably due to the small number of observations corresponding to some of the orderings of choice assets. Second, despite the fact that the Special Deposit and credit ceilings dummies have to be accepted none of the t-ratios of the dummy coefficients is statistically significant. More disturbingly, the coefficient of credit ceilings dummy variable in the Advance equation is positively signed. Not surprisingly, the standard errors of regression in Table 5.6 are in general higher than those of comparable models for the same period reported in the previous two chapters. On the positive side, all the four own equation autoregressive coefficients are correctly signed and statistically significant. In fact, all the four autoregressive coefficients of the Advance equation are statistically different from zero. As the maintained model in Table 5.6 is to be tested against other non-nested alternatives in the next chapter, no policy implications will be discussed here.

Table 5.6
FIML Estimates of the Precautionary Model with SSD and CC Dummy Variables Assuming First-order Vector Autocorrelation (∀ = Full)
1959:01 to 1971:06

	CM	TB	GB	CB	AD
R_{12}	20.306	-20.306			
	(8.761)	(8.761)			
R_{21}	-4.967	4.967			
	(2.331)	(2.331)			
R_{23}		0.242	-0.242		
		(0.477)	(0.477)		
R_{31}	0.716*		-0.716*		
	(0.439)		(0.439)		
R_{32}		-0.013	0.013		
		(0.021)	(0.021)		
R_{34}			0.091	-0.091	
			(0.422)	(0.422)	
R_{42}		-0.088		0.088	
		(0.069)		(0.069)	
R_{43}			0.026*	-0.026*	
			(0.137)	(0.137)	
R_{53}			2.107*		-2.107*
			(0.862)		(0.862)
R_{54}				-0.830	0.830
				(0.674)	(0.674)
D_{SD}	122.070	-105.480	1.684	-13.291	-4.983
	(1.661)	(0.295)		(0.092)	(0.001)
D_{CC}	89.109	-104.100	-28.883	13.143	30.731
	(1.710)	(1.113)		(1.295)	(0.357)
	FIML estimates of autoregressive parameters in ∀				
	0.602	-0.221		0.021	0.494
	(6.918)	(1.450)		(0.927)	(3.767)
	-0.172	**0.771**		0.008	0.290
	(1.789)	**(6.221)**		(0.381)	(2.503)
	0.151	-0.045		**1.006**	-0.199
	(1.442)	(0.380)		**(50.199)**	(1.497)
	-0.051	-0.083		0.003	**1.079**
	(1.182)	(1.625)		(0.275)	**(20.296)**
SER	78.699	104.756	28.004	15.610	123.950

Note: (1) Figures in parentheses are the ratios of estimated coefficients to asymptotic standard errors. Some t-ratios for the Government bond equation are not reported because this equation is not directly estimated. The ratios reported are provided by the symmetry restrictions. (2) The R_{ij}'s are defined as follows:

$R_{12} = (\sigma_x^2/2(CMR - TBR)$ $R_{21} = (\sigma_x^2/2(TBR - CMR)$
$R_{23} = (\sigma_x^2/2(TBR - GBR)$ $R_{31} = (\sigma_x^2/2(GBR - CMR)$
$R_{32} = (\sigma_x^2/2(GBR - TBR)$ $R_{34} = (\sigma_x^2/2(GBR - CBR)$
$R_{42} = (\sigma_x^2/2(CBR - TBR)$ $R_{43} = (\sigma_x^2/2(CBR - GBR)$
$R_{53} = (\sigma_x^2/2(ADR - GBR)$ $R_{54} = (\sigma_x^2/2(ADR - CBR)$.

(3) The starred estimates are of the wrong sign.

5.7 CONCLUDING REMARKS

In this chapter we developed and estimated a precautionary model of bank portfolio behaviour based on the inventory-theoretical framework. Unlike the models explored in the last two chapters, the precautionary model regards random cash requirements and thus their associated liquidation costs as the sole source of bank portfolio diversification and ignores uncertainty over rates of return on assets. In the precautionary model assets can only be distinguished by their rates of return and their degree of liquidity, which is measured by their liquidation cost. Two assets which have the same rate of return and the same liquidation cost are thus viewed as the same asset. Indeed, the logical validity of the precautionary model depends on the crucial assumption that the ordering of assets by their rates of return is the same as that by their liquidation cost. It is this unique ordering of assets which ensures that the bank's optimal demand for an asset is a function of only its own rate of return and liquidation cost and those of its adjacent assets. This feature of the precautionary model helps to alleviate the empirical problem of multicollinearity among interest rates found in the models of the previous chapters. However, this obvious empirical advantage is offset, and indeed, outweighed, by other implementational difficulties which resulted from the restrictions imposed by the unique ordering of assets. In order to estimate the precautionary model it is necessary for a particular ordering of assets to remain unchanged for a reasonably large number of sample points. In reality, however, the ordering of assets changed frequently. In fact, there were so many distinctive orderings of assets in the post-1971 period it was not possible to estimate the precautionary model for that period. Even when it was possible to estimate the model for the pre-1971 period, there were very few observations corresponding to some orderings of assets and it was difficult to obtain accurate estimates for the parameters. Largely due to these difficulties, the performance of the precautionary model is rather unsatisfactory.

CHAPTER SIX

NON-NESTED TESTS OF THE BANK PORTFOLIO BEHAVIOUR MODELS

6.1 INTRODUCTION

In the last three chapters we developed and estimated three different sets of models of bank portfolio behaviour. The natural question one is to ask now is which, if any, of the models is consistent with the portfolio behaviour of the London clearing banks. Before we proceed to compare the different models, however, the fundamental question of what constitutes a "good" model must first be addressed. Although there is no uniform criteria in economics for choosing among rival theories or models, at the abstract level two factors are usually considered [Desai (1981)]. The first of these is the logical consistency and completeness of a theory relative to its rival, or generality. It is often the case that each theory claims to be more general than its rivals and attempts to cast the rival theory as a special case of itself. As the title The General Theory of Employment, Interest and Money made it clear, generality was the criterion used by Keynes to bolster the superiority of his theory over the neoclassicals. Keynes claimed that the neoclassical full employment equilibrium was a special case of his own theory which predicted multiple equilibria one of which was the full employment equilibrium [Keynes (1936)]. To answer this claim Hicks reformulated the neoclassical and the Kaynesian models so that the latter became a special case, the so-called "depression economics" variant of the neoclassical model [Hicks (1937)].

A second weapon often evoked at the abstract level by economists is simplicity or parsimony. In the words of Milton Friedman, "A hypothesis is important if it 'explains' much by little" [Friedman (1953), p. 14]. Explanation and prediction are the goals of economics as well as most other sciences. However, the mechanisms by which economic variables are generated are invariably complex. When a theory or a model is

constructed, it is not intended to be an acurate or comprehensive description of the world, but rather, it is intended to simplify the underlying process in such a way that only the essential features are brought out. Thus if a theory can be encapsulated in a few key relationships and if the theory meets the other criterion of generality and consistency, then this should reinforce the claims of a theory. "Simple statements ... are to be prized more highly than less simple ones because they tell us more; because their empirical content is greater; and because they are better testable" [Popper (1959), p. 142].

Generality and simplicity are thus two important criteria on which rival theories can be evaluated relative to each other at the abstract level of discourse. The concept of simplicity is especially important and relevant at the empirical level. From the econometric point of view the most important advantage of a simple model is that it contains a small number of variables and parameters. Given the fact that not all economic variables are observable and recorded it is essential that only those important variables are incorporated in a model if the model is to be estimated and tested against data. Simplicity is also important for the identifiability of a model. Broadly speaking, a model is identifiable if only one set of parameter values is consistent with the data. The identifiability of a model is important for two reasons. Firstly, if a model is not identified it is reasonable to ask whether the model actually makes sense in the way it has been specified. Secondly and relatedly, it is important to examine what meaning, if any, can be given to the estimates computed from a model which is not identified. The main point to note here is that if a model is not identifiable, the estimates cannot in general be interpreted in any meaningful way. Although identifiability is precisely defined in econometrics it is closely related to the question of simplicity or parsimony. In general, the more parsimonious a model is, the less likely it is to suffer from problems of identifiability.

As we indicated earlier, one of the objectives of building a model is to explain systematically the movements in the dependent variables. If the model is at all

successful, the predictions based on the explanatory variables should be reasonably close to the observed values of the dependent variables at all points in the sample. Thus, another criterion frequently used at the empirical level to compare rival models is their "goodness of fit". The goodness of fit of a regression model may be characterised by a measure such as the coefficient of multiple correlation, R^2 or $\bar{R}^2$, the coefficient of multiple correlation adjusted for the degrees of freedom. More will be discussed on this measure later.

"The only relevant test of the validity of a hypothesis is comparison of its predictions with experience" [Friedman (1953), p. 7]. Thus another often cited criterion in comparing models is the accuracy of their predictions, predictive power. The predictive power of a model should not be confused with the goodness of fit criterion. A model is specified and estimated on the basis of a given sample. The accuracy of predictions of the model within the sample is captured by measures such as R^2. Another objective of model building is to predict future movements of the dependent variables. An important question is, therefore, whether the estimated model can predict the dependent variables outside the sample, given the relevant information on the explanatory variables. Success in this respect is much more convincing than the attainment of good fit within the sample, since the preferred specification is being validated against a set of fresh observations.

Given these considerations, how do the three approaches to bank portfolio behaviour compare with each other?

Both the portfolio-theoretical and Safety First models regard uncertainty over rates of return on assets as the source of asset diversification, while the precautionary model ignores interest rate uncertainty and considers asset diversification as a means of meeting random cash requirements. Although the portfolio-theoretical and Safety First models both emphasise the importance of interest rate uncertainty they differ in their assumptions regarding the bank's objective function. The portfolio-theoretical model, analogous to

consumer behaviour, considers the bank an expected utility maximiser. In the Safety First model, the bank is assumed to minimise the probability that the return from its chosen portfolio falls short of a predetermined disaster level. On the other hand, the precautionary model, more in line with firm behaviour, regards the bank as an expected profit maximiser. As a result of the different emphasis and objectives, the asset demand equations derived from the three approaches also differ in their functional forms. Thus, at the theoretical level, none of the models is more general than the other and nor can any of them be cast as a special case of the other. Indeed, none of the models can be nested within the other.

While both the portfolio-theoretical and Safety First models express asset demand in terms of expected rates of return and their covariances, the Safety First models are highly nonlinear so that they are much more difficult to estimate than the linear portfolio-theoretical models. At the empirical level, the portfolio-theoretical and Safety First models both suffer from multicolinearity problems because the expected rates of return on all choice assets, which are usually highly correlated, appear in each and every asset demand equation. In the precautionary model, the bank's optimal asset demand equations are functions of the rates of return and liquidation costs as well as the mean and variance of the random cash requirement variable. As a result of the unique ordering of choice assets, only the own rate of return and those of the adjacent assets appear in each asset demand equation of the precautionary models, thus alleviating the problem of multicolinearity among interest rates. However, as we have seen in Chapter Five, the precautionary models suffer from other implementational problems which also result from the unique ordering of the choice assets. Thus, in terms of ease of empirical implementation, none of the models is superior to the other. Also in terms of "goodness of fit", the three sets of models are quite similar, although the static models perform universally better than the dynamic models. Which of the models should be accepted as empirically valid, if any at all, can only be determined by formal econometric tests.

The rest of this chapter is devoted to this issue. In Section 6.2, we review briefly a number of tests developed recently in the literature for testing non-nested alternatives. Although the origins of some of these tests go back to the 1960s and 1970s they have rarely been employed in applied econometrics. Our purpose here, however, is not to undertake a comprehensive review and assessment of the existing non-nested tests, but to provide a brief introduction to the main tests and to examine which is more suitable for our task of comparing the models developed in the previous three chapters. In Section 6.4, we first restate briefly the models to be tested and then report and discuss the non-nested test results on these models. The final section provides some concluding remarks and suggestions for future research.

6.2 MODEL SELECTION AND NON-NESTED TESTS

We are concerned with two or more non-nested multivariate models, which may be either sets of restricted reduced forms of linear simultaneous equations or nonlinear, seemingly unrelated equations models. By non-nested models we simply mean that neither model can be obtained from the other by the imposition of appropriate parametric restrictions. To allow both linear and nonlinear forms the two competing models may be written as

$$H_0: \quad A_{it} = f_{it}(X_t, B) + \epsilon_{0it} \tag{6.1}$$

$$H_1: \quad A_{it} = g_{it}(Z_t, \Upsilon) + \epsilon_{1it} \tag{6.2}$$

where i (=1 to m) indexes equations and t (=1 to T) indexes observations. Thus A_{it} is the tth observation on the ith dependent variable. f_{it} and g_t are continuous and second-order differentiable functions with respect to all their arguments: vectors of exogenous variables X_t and Z_t and unknown parameter vectors B ($k_1 \times 1$) and Υ ($k_2 \times 1$). It is assumed that for given t the ϵ_{jit} (j=0 or 1) are serially independent and multivariate normal with means zero and unknown nonsingular covariance matrix Ω_j (j = 0 or 1) and independent of X_t and Z_t, provided that the data are actually generated by H_j.

To conform with our previous notations the systems (6.1) and (6.2) may be written in vector form as

$$H_0: \quad A = f(X, B) + \epsilon_0 \tag{6.3}$$

$$H_1: \quad A = g(Z, \Upsilon) + \epsilon_1 \tag{6.4}$$

where A is the mT $\times$ 1 vector of observations on all the n dependent variables. f(.)

and g(.) are the corresponding mT × 1 vectors of predictions, **X** and **Z** are matrices of predetermined variables, B and Υ are k_1 × 1 and k_2 × 1 vectors of unknown parameters, and ϵ_0 and ϵ_1 are mT × 1 disturbance vectors with means zero and nonsingular covariance matrices $\Omega_0 \otimes I$ and $\Omega_1 \otimes I$ respectively. The vector on the ith dependent variable will be denoted by $f_i(.)$ and $g_i(.)$. Note that individual parameters may appear in several different equations. We denote the complete parameter sets of models (6.3) and (6.4) by the vectors θ_0 and θ_1 so that

$$\theta_0' = \{B', st\ (\Omega_0)'\} \qquad \text{and} \qquad \theta_1' = \{\Upsilon', st\ (\Omega_1)'\}$$

where st (Ω_0) and st (Ω_1) are the vectors formed by stacking the matrices Ω_0 and Ω_1 by columns.

Until recently, the usual practice among applied econometricians when faced with choosing among non-nested models like those in (6.3) and (6.4) was to rely on the "goodness of fit" criterion and to select that model which has the least residual variance, or equivalently, the one which has the highest adjusted multiple correlation coefficient, R^2. The rationale for such "goodness of fit" procedure is that the use of residual variance as a choice criterion gives rise "on average" to the correct choice provided that (1) one of the alternative models considered is the "true model" in the sense that all the classical assumptions for that model are satisfied, and (2) the explanatory variables entering the specification of the models should all be fixed in repeated samples [Theil (1961)]. One serious problem with the "goodness of fit" procedure is the notion of a "true model". The assumptions underlying the classical models are made not because they are optimal from the viewpoint of economic theory but because they are extremely convenient for estimation and hypothesis testing purposes. Thus, there is no theoretical justification for expecting a correctly specified model to possess all the characteristics of the classical regression models.

More importantly, using the "goodness of fit" criterion in selecting among

non-nested models will always lead to the selection of one model regardless of whether or not it can predict the consequences of separate alternatives [Pesaran (1981)]. That is to say, the model is evaluated on the basis of its own performance. This may be justified in the case of nested models, but is inappropriate for testing non-nested models. We are faced with a body of data and a set of alternative hypotheses. Since the latter are non-nested by assumption, we cannot rank them by level of generality as can be done when the models are nested. There is thus no maintained hypothesis; each model is on an equal footing with every other model. It is a fact that, in econometrics, we never have a maintained hypothesis which we believe with certainty; it is therefore essential that we always use the models we possess to organise the evidence in different ways and ask whether the patterns which result are consistent with views we currently hold. In considering whether an alternative hypothesis, together with the data, contains sufficient information to reject the currently maintained hypothesis, the question of whether that alternative "fits" well or badly, even if meaningful, is certainly irrelevant. Indeed, one might argue that "a hypothesis, which one would not wish to consider seriously on its own right, can be a perfectly effective tool for disproving an alternative, even if that alternative may in some aspects seem more promising" [Pesaran and Deaton (1978), p. 678]. It is thus important that tests between non-nested models should encompass the possibility of rejecting all the alternatives considered. The use of the "goodness of fit" procedure obviously does not satisfy such a requirement.

Another approach to the problem of comparing non-nested models is to nest the separate models in a general artificial model and then to apply the likelihood ratio to test the competing models as special cases. More specifically, if we denote the likelihood functions of the two models in (6.3) and (6.4) by $L_0(\theta_0)$ and $L_1(\theta_1)$ respectively, a general comprehensive model can be written as

$$L(\theta_0, \theta_1, \theta) = F(L_0, L_1, \theta) \qquad (6.5)$$

where F can take many different forms as long as $\int F dA = 1$. The adoption of such an

approach in the case of normally distributed regression models will result in a comprehensive model which includes all the regressors as its explanatory variables and then one would try to apply the usual standard regression tests. However, such a comprehensive test procedure suffers potentially from two serious drawbacks. First, since F in (6.5) can take many different forms there are many different ways of constructing a comprehensive model out of any two alternatives and this, in general, will introduce a certain degree of arbitrariness into the analysis [Pesaran (1974)]. More importantly, the inclusion of all regressors into the comprehensive model will usually result in the high degree of multicollinearity among the explanatory variables and thus make it difficult, if not impossible, to estimate the resultant comprehesive model. This problem is especially serious in the case of competing models which have the same explanatory variables and differ only in their functional forms. When a large number of non-nested hypotheses is under consideration, a comprehensive model is likely to be so general as to be useless in practice.

To circumvent the potential problem of multicolinearity a number of procedures based on this principle have been proposed recently to test non-nested models. The first of these, referred to as the N test, is derived by Pesaran (1974) and Pesaran and Deaton (1978). The N test is based on a statistic proposed by Cox (1961, 1962) and tests whether one econometric model, H_0, could possibly have generated the data, by seeing whether the performance of a non-nested alternative model, H_1, is consistent with the truth of H_0. Let ℓ_{10} be the log of the maximum likelihood ratio, then if we maintain H_0 against H_1, the Cox statistic is defined as

$$T_0 = \ell_{10} - T\,[\underset{T\to\infty}{\text{plim}}_0\ (\ell_{10}/T)]_{\theta_0=\hat{\theta}_0} \qquad (6.6)$$

where plim_0 denotes the probability limit when H_0 is true, $\ell_{10} = L_0(\hat{\theta}_0) - L_1(\hat{\theta}_1)$, and $\hat{\theta}_0$ and $\hat{\theta}_1$ are the maximum likelihood estimators of θ_0 and θ_1 under H_0 and H_1 respectively. Cox (1961) shows that, given that H_0 is true, T_0 is asymptotically normally distributed with mean zero and variance

$$V(T_0) = V_0(L_{10}) - (1/T)\eta'\Phi^{-1}\eta \tag{6.7}$$

where $L_{10} = L_0(\theta_0) - L_1(\theta_{10})$ with $\theta_{10} = \text{plim}_0\theta_1$,

$$\Phi = -\,\text{plim}_0 \frac{1}{T} \frac{\partial^2 L_0}{\partial\theta_0\partial\theta_0'}$$

is the asymptotic information matrix of H_0, and

$$\eta = T \frac{\partial[\text{plim}_0(\ell_{10}/T)]}{\partial\theta_0}.$$

If we maintain H_0 against H_1 the N test is defined as

$$N_0 = T_0 \,/\, V_0(T_0)^{1/2} \tag{6.8}$$

where $V_0(T_0)$ is the maximum likelihood estimate of $V_0(T_0)$ in (6.7). Pesaran and Deaton (1978) show that T_0 is given by

$$T_0 = (T/2)\,\log\{|\Omega_1|/|\Omega_{10}\} \tag{6.9}$$

where $\Omega_{10} = \text{plim}_0\Omega_1$ is the maximum likelihood estimator of Ω_{10}. The ijth element of the maximum likelihood estimator of Ω is shown to be

$$\hat{\omega}_{ij10} = \hat{\omega}_{ij0} + (1/T)\{f_i(B_0) - g_i(\Upsilon_{10})\}'\{f_j(B_0 - g_j(\Upsilon_{10})\} \tag{6.10}$$

where $\hat{\omega}_{ij0}$ is the ijth element of the maximum likelihood estimate of Ω_0 and Υ_{10} is given by

$$\{\frac{\partial g(\theta_{10})}{\partial\Upsilon_{10}}\}'\,(\Omega_{10}^{-1} \otimes I)\{f(B_0 - g(\Upsilon_{10})\} = 0. \tag{6.11}$$

The maximum likelihood estimator of $V_0(T_0)$ is

$$V_0(T_0) = d' \Psi 0^{-1} d \tag{6.12}$$

where

$$d = \{I - F (F' \Psi 0^{-1} F)^{-1} F' \Psi_0^{-1}\} h \tag{6.13}$$

$$F = \left(\frac{\partial f(B)}{\partial B}\right)_{\beta=\beta} \tag{6.14}$$

$$h = (\Omega_0 \Omega_{10}^{-1} \otimes I)\{f(B) - g(\Upsilon_{10})\} \tag{6.15}$$

$$\Psi_0 = \Omega_0 \otimes I. \tag{6.16}$$

Note that F is evaluated at B = B and g at $\Upsilon = \Upsilon_{10}$ where Υ_{10} is the maximum likelihood estimator of Υ when H_0 is true. We also note that h is the residual vector of the regression of f(B) on H_1, transformed by premultiplication by $(\Omega_0 \Omega_{10}^{-1} \otimes I)$. Given that T_0 is asymptotically normally distributed with mean zero under H_0, it follows that N_0 is asymptotically normally distributed with mean zero and variance one, i.e. $N_0 \sim N(0, 1)$. If N_0 is significantly less than zero we may conclude that the null model H_0 is rejected in a direction towards the alternative model H_1, and if N_0 is significantly greater than zero we may conclude that H_0 is rejected in a direction away from H_1.

The procedure proposed by Pesaran and Deaton (1978) for computing N_0 is as follows:

(1) First estimate both H_0 and H_1 by full information maximum likelihood (FIML) and obtain estimates for B, Υ, Ω_0 and Ω_1.

(2) Then take the predicted values from H_0, f(B), and use these as dependent

variables in a second FIML regression of H_1. The estimated variance-covariance matrix of this regression is then added to Ω_0 to derive Ω_{10} using (6.10) and T_0 is calculated using (6.9).

(3) Using the residual vector from the regression of f(B) on H_1 and the formula in (6.15) we obtain the vector h.

(4) After evaluating F and Ψ_0 according to (6.14) and (6.16) we compute a generalised least squares regression to obtain d. $V_0(T_0)$ is then calculated using (6.12).

Thus, the computation of N_0 test involves performing three nonlinear regressions and one linear regression. It should be noted that the procedure for computing T_0 as set out above is incorrect in one important aspect. The equations which jointly determine Υ_{10} and Ω_{10} are (6.10) and (6.11). From these equations it is obvious that the covariance matrix which must be used when estimating Υ_{10} is Ω_{10}. However, in step 2 Υ_{10} is computed by a FIML regression of f(B) on H_1. The covariance matrix which is implicitly used by this regression is the second term on the right-hand side of (5.10), which is clearly not proportional to Ω_{10}. Since H_1 is not the true model, as pointed out by Davidson and MacKinnon (1983), there is no reason for the estimate of Υ which result from step 2 above to be consistent for Υ_{10}. The correct way of estimating Υ_{10} is to use equations (6.10) and (6.11) and this would usually require a special-purpose computer programme.

It should be stressed that the N_0 provides a test of H_0 against the evidence provided by H_1 only and tells us nothing whatsoever about the validity of H_1. The possible outcomes of such a test is that H_0 is not rejected, that H_0 is rejected in a direction towards H_1 and that H_0 is rejected in a direction away from H_1. But one cannot conclude from N_0 that H_0 is rejected in favour of H_1, as Pesaran (1974) has misleadingly asserted [p. 158]. If we wish to test the validity of H_1, we must reverse

the role of the two models and compute the test statistic N_1. This means that the computations from stage (2) onwards will have to be performed again. Moreover, the use of the N_0 statistic enables one to test the null hypothesis against only one alternative at a time. This could be a disadvantageous when there exist more than one alternative hypotheses under consideration because then the above procedure will have to be performed for each alternative. Added to that, the implementation of the N_0 test by Pesaran and Deaton (1978), as described above, is unnecessarily complicated. It should be noted that the N test does not measure the relative fitness of a model; rather it is a test of whether a given hypothesis can or cannot explain the performance of an alternative hypothesis against the data. Indeed, it is not even necessary for the alternative to yield a satisfactory explanation of the phenomena under examination [Pesaran and Deaton (1978)].

One particular form of making a comprehensive model out of (6.3) and (6.4) is to combine the likelihood functions L_0 and L_1 (for H_0 and H_1) exponentially with weights $(1 - \lambda)$ and λ, respectively, and then normalise so that the resultant likelihood function integrates to one. This is the form suggested by Cox (1961, 1962) and later used by Atkinson (1970) and Pesaran (1982). Pesaran (1982) shows that this procedure yields the following comprehensive model:

$$A = (1 - \lambda)\ (\Omega\ \Omega_0^{-1} \otimes I)\ f(B) + \lambda\ (\Omega\ \Omega_1^{-1} \otimes I)\ g(Y) + \epsilon \qquad (6.17)$$

where Ω is the covariance matrix of the ϵ and is defined by

$$\Omega^{-1} = (1 - \lambda)\ \Omega_0^{-1} + \lambda\ \Omega_1^{-1}. \qquad (6.18)$$

Thus, the covariance matrix of ϵ in (6.17) is the inverse of a convex combination of the inverse matrices under H_0 and H_1.

As it stands, the comprehensive model in (6.17) is not very useful, since it is not

possible to estimate all the parameters simultaneously; indeed, many of the parameters will not be identified. However, remember that the estimate of λ itself is not important; our aim is to test whether or not λ is zero. To achieve this we may simply, following Durbin (1970), linearise the test regression about $B = B$, $\Omega_0 = \Omega_0$ and $\lambda = 0$. The test statistic is then the usual t ratio on the regressor corresponding to λ, which is asymptotically normally distributed with mean zero and variance one if H_0 is true [see Davidson and MacKinnon (1983)]. This is the test proposed by Davidson and MacKinnon (1983) and it is called the P_1 test, since it is similar to the P test for the univariate case derived in Davidson and MacKinnon (1981). The P_1 test is based on the following artificial regression:

$$A - f(B) = f_\beta(B)b + \lambda (\Omega_0 \Omega_1^{-1} \otimes I) \{\hat{g}(Y) - f(B)\} + \epsilon \qquad (6.19)$$

where $f(B)$ and $\hat{g}(Y)$ denote the fitted values of $f(X, B)$ and $g(Z, Y)$ based on the FIML estimates of H_0 and H_1, respectively, and $f_\beta(B)$ denotes the matrix of partial derivatives of $f(B)$ with respect to the vector of parameters evaluated at $B = B$. We note that, under H_0, ϵ is distributed as $N(0, \Omega_0)$, so that (6.19) must be estimated by GLS using a covariance matrix proportional to Ω_0.

Thus, in the case of two competing models the steps required to calculate the P_1 test as described above are as follows [Davidson and MacKinnon (1983)]:

1) The models H_0 and H_1 in (6.3) and (6.4) are estimated by FIML to obtain $f(B)$, $\hat{g}(Y)$, Ω_0 and Ω_1. Differentiate $f(B)$ with respect to B and evaluate $f_\beta(B)$ at $B = B$ so as to derive $f_\beta(B)$.

2) Compute the $m \times m$ triangular matrix Ω_G such that $\Omega_G' \Omega_G = \Omega_0^{-1}$.

3) Premultiply the $mT \times 1$ vector $\hat{g}(Y) - f(B)$ by the $m \times m$ matrix $\Omega_0 \Omega_1^{-1}$ to form the vector P.

4) Run a GLS regression of A - f(B) on $f_\beta(B)B$ and P. Namely, premultiply the regressand and all the regressors by $(I \otimes \Omega_\sigma)$, and then perform an OLS regression. The t ratio on P is then the P_1 test.

If H_0 is true, then the true value of λ in (6.19) is zero. Thus, if the estimate of λ is not significantly different from zero, we accept H_0. Obviously, if H_1 is true, the estimate of λ will converge asymptotically to one. Therefore, when λ is significantly close to unity we reject H_0 in a direction towards H_1; if, on the other hand, λ is significantly different from unity we reject H_0 in a direction away from H_1. However, the rejection of H_0 does not imply the truth of H_1. This is because the t statistic on λ from (6.19) is conditional on the truth of H_0, not on the truth of H_1. As with the N test, the P_1 statistic as described in the above steps provides a test of H_0 against the evidence provided by H_1 only and does not reveal anything about the truth of H_1. If H_1 is to be tested against evidence provided by H_0, the role of the two competing models must be reversed and the above steps repeated. Once again, there are three possible outcomes. Thus, for each pairwise test there are altogether six possible outcomes.

Notice that if λ is restricted to zero, the estimates of b should be identically zero since, by the first-order conditions for B, $f_\beta(B) = 0$. This is a convenient way to check the calculations. When there are more than one alternative models at a time there would be as many regressors like P (the second term on the right-hand side of (6.19)) as there are alternative models. In such a case, the appropriate P_1 test is the asymptotic F test of the hypothesis that all of the coefficients, λ_1's, are zero. It should be noted that the P_1 test is asymptotic and therefore, some care should be taken in interpreting the test statistic in small samples.

Little is known about the relative performance of the N and the P_1 tests. The fact that the N test is based on the maximum likelihood principle suggests that it should

have good power [MacKinnon (1983)]. Davidson and MacKinnon (1983) show that, provided the process actually generated the data is "close" to the null hypothesis, the P_1 statistic tends to minus the same random variable as the N statistic. Thus, on asymptotic grounds there is no reason to prefer one test to another. However, as we can see from the foregoing discussion, the P_1 test is much simpler to calculate than the N test which requires not only a special-purpose programme to calculate Υ and Ω_{10} but also an elaborate procedure to compute the variance of the test statistic. In addition, the N test can only be performed to test the null hypothesis against one alternative model at one time. Of course, whether one *should* test a model against several alternative hypotheses simultaneously will depend on whether the true model is among the alternatives. If one of the alternatives is true, it is best to test a model against that alternative alone. If, on the other hand, the true model is not among the alternative hypotheses, then testing against several of them jointly may have higher power than testing against each of them individually [MacKinnon (1983)]. In practice, we usually do not know *a priori* whether one of the alternatives is true or not. But being able to test more than one alternative models is clearly an advantage. It is for these reasons that we shall employ the P_1 statistic in testing our models in the following section.

6.3 THE MODELS AND NON-NESTED TEST RESULTS

The models we are to test in this section are those maintained in the previous three chapters. There are all together eleven maintained models: four static and three dynamic models for the pre-1971 period, and two static and two dynamic models for the post-1971 period. For convenience, we first restate briefly all the eleven models. FIML estimates of these models have already been reported and examined in the last three chapters and will not be repeated here. In the second half of this section we discuss the non-nested test results on these models.

6.3.1 The Models

The letter H is used with subscripts to label the hypotheses; the first letter subscript indicates whether the model is static (S) or dynamic (D) while the first number subscript denotes if the model is for the pre-1971 period (1) or for the post-1971 period (2). The models can only be tested against each other within the same category (static or dynamic) and for the same period. We note that all the maintained static models follow the first-order vector autocorrelation process with a full matrix of autoregressive coefficients. Models for the pre-1971 period have five equations while those for the post-1971 period have seven equations. We shall use the same notations for the equations as before and no notational distinctions will be made for the two different periods. All models are estimated by FIML with the Government bond equation deleted. For comparability with H_{S14}, H_{S11}, H_{S12} and H_{S13} are reestimated without the observation for 1962:06 (see Section 5.5.2 in Chapter Five).

The first of the maintained models is the unconstrained static portfolio-theoretical model with independent non-choice variables, intercepts and a full matrix of autoregressive coefficients for the pre-1971 period. The model is without the liquid asset ratio constraints and is also unconstrained by the symmetry and the homogeneity restrictions.

H_{S11}:

$$A_1 = \alpha + G\ \hat{r}_1 + H\ A_2 + \epsilon_{st} \qquad (6.20)$$

with

$$\epsilon_{st} = \forall\ \epsilon_{st-1} + \omega_{st}$$

$$i'\ \alpha = 0$$

$$i'\ G = 0$$

$$i'\ H = -i$$

where A_2 = (CN SD EA DD TD), and $\forall$ is a 5 × 5 full matrix. FIML estimates of this model are reported in Table 3.13.

The second maintained model for the pre-1971 period is the portfolio-theoretical model with intercepts, credit ceilings dummy variables and the liquid asset ratio constraints. The model is also constrained by the theoretical restrictions of symmetry and homogeneity.

H_{S12}:

$$A_1 = \alpha + G_1\ \hat{r}_1 + H_1\ A_2^* + f\ D_{CC} + \epsilon_{st} \qquad (6.21)$$

with

$$\epsilon_{st} = \forall\ \epsilon_{st-1} + \omega_{st}$$

$$G_1 = \begin{bmatrix} g_{11} & g_{12} & g_{13} & 0 & 0 \\ g_{12} & g_{22} & g_{23} & 0 & 0 \\ g_{13} & g_{23} & g_{33} & 0 & 0 \\ 0 & 0 & 0 & g_{44} & g_{45} \\ 0 & 0 & 0 & g_{45} & g_{55} \end{bmatrix} \qquad H_1 = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{31} & h_{32} \\ 0 & h_{42} \\ 0 & h_{52} \end{bmatrix}$$

$G_1 \; i = 0$

$i' \; G_1 = 0'$

$i' \; H_1 = - \; i'$

$\alpha_1 + \alpha_2 + \alpha_3 = \alpha_4 + \alpha_5 = 0$

$i' \; f = 0'$

where $A_2^* = (R_r \; W^*)'$. FIML estimates of this model are presented in Table 3.19.

The third maintained static model for the pre-1971 period is the Safety First model with symmetry restrictions but no dummy variables.

H_{S13}:

$$a_{1it} = - \{(g_{i1}[\hat{r}_{11} - c_n] + g_{i2}[\hat{r}_{12} - c_n] + \ldots + g_{i5}[\hat{r}_{15} - c_n]) \; /$$

$$((g_{11} + g_{12} + \ldots + g_{15}) \; [\hat{r}_{11} - c_n] +$$

$$(g_{12} + g_{22} + \ldots + g_{25}) \; [\hat{r}_{12} - c_n] + \ldots + [\hat{r}_{15} - c_n])\} \; d_2 + \epsilon_{ist} \qquad (6.22)$$

with

$$\epsilon_{st} = \forall\ \epsilon_{st-1} + \omega_{st}$$

and i = 1, 2, ..., 5. Table 4.6 contains the FIML estimates of this model.

The last maintained static model for the pre-1971 period is the precautionary model developed in Chapter Five. The model has both Special Deposit and credit ceilings dummy variables.

H_{S14}:

$$a_{11} = b_{21}\ R_{21} + \mu_x + f_{11}\ D_{SD} + f_{12}\ D_{CC} + \epsilon_{1t}$$

$$a_{1k} = b_{k+1,k}\ R_{k+1,k} - b_{k,k-1}\ R_{k,k-1} + f_{k1}\ D_{SD} + f_{k2}\ D_{CC} + \epsilon_{kt}$$

$$\text{for } k = 2,\ 3,\ 4 \qquad (6.23)$$

$$a_{15} = -\ A_2'\ i_2 - b_{54}\ R_{54} - \mu_x + f_{51}\ D_{SD} + f_{52}\ D_{CC} + \epsilon_{5t}$$

with

$$\epsilon_{st} = \forall\ \epsilon_{st-1} + \omega_{st}.$$

FIML estimates of the model are given in Table 5.6.

There are three dynamic models for the pre-1971 period. The first of these is the unconstrained portfolio-theoretical model with intercepts, Special Deposit dummy variables and non-choice items as independent variables. The model is not constrained by the homogeneity restrictions.

H_{D11}:

$$\Delta A_{1t} = K + \Phi \hat{r}_1 + \Lambda A_{2t} - B A_{1t-1} + F D_{SD} + \epsilon_{dt} \qquad (6.24)$$

with

$$i' K = 0$$

$$i' \Phi = 0$$

$$i' \Lambda = -i$$

$$i' B = i$$

$$i' F = 0$$

where A_2 is as defined in H_{S11} above and F is the 5 × 1 vector of dummy coefficients. FIML estimates of this model are in Table 3.23.

The second maintained dynamic model is also a portfolio-theoretical model with intercepts and Special Deposit dummy variables but constrained by the liquid asset ratio restrictions. The model is not constrained by the homogeneity restrictions.

H_{D12}:

$$\Delta A_{1t} = K + \Phi_1 r_1 + \Lambda_1 A^*_{2t} - B A_{1t-1} + F D_{SD} + \epsilon_{dt} \qquad (6.25)$$

with

$$\Phi_1 = \begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} & 0 & 0 \\ \varphi_{21} & \varphi_{22} & \varphi_{23} & 0 & 0 \\ \varphi_{31} & \varphi_{32} & \varphi_{33} & 0 & 0 \\ 0 & 0 & 0 & \varphi_{44} & \varphi_{45} \\ 0 & 0 & 0 & \varphi_{54} & \varphi_{55} \end{bmatrix} \qquad \Lambda_1 = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \\ \lambda_{31} & \lambda_{32} \\ 0 & \lambda_{42} \\ 0 & \lambda_{52} \end{bmatrix}$$

$$i' \Phi_1 = 0'$$

$$i' \Lambda_1 = - i'$$

$$K_1 + K_2 + K_3 = K_4 + K_5 = 0$$

$$i' F = 0$$

$$i' B = i'$$

where A_2^* is as defined in H_{S12} above. Table 3.26 gives FIML estimates for this model.

The third maintained dynamic model is the Safety First model with symmetry restrictions and no dummy variables.

H_{D13}:

$$\Delta a_{1it} = - \{(b_{i1} g_{11} + b_{i2} g_{12} + \ldots + b_{i5} g_{15}) [\hat{r}_{11} - c_n] +$$

$$(b_{i1} g_{12} + b_{i2} g_{22} + \ldots + b_{i5} g_{25}) [\hat{r}_{12} - c_n] + \ldots +$$

$$(b_{i1} g_{15} + b_{i2} g_{25} + \ldots + b_{i5} g_{55}) [\hat{r}_{15} - c_n]\} /$$

$$((g_{11} + g_{12} + \ldots + g_{15})\,[\hat{r}_{11} - c_n] +$$

$$(g_{12} + g_{22} + \ldots + g_{25})\,[\hat{r}_{12} - c_n] + \ldots + [\hat{r}_{15} - c_n])\}\,d_2 -$$

$$b_{i1}\,a_{11t-1} - b_{i2}\,a_{12t-1} - \ldots\, b_{i5}\,a_{15t-1} + \epsilon_{idt} \qquad (6.26)$$

with

$$\sum_{i=1}^{5} b_{ij} = 1$$

for all j = 1, 2, ..., 5. FIML estimates for this model are presented in Table 4.10.

There are only four maintained models for the post-1971 period: two static and two dynamic. These models have seven equations. The first maintained static model for the post-1971 period is the portfolio-theoretical model with a single wealth variable, intercepts and a full matrix of autoregressive coefficients. The model is constrained by both symmetry and homogeneity restrictions.

H_{S21}:

$$A_1 = \alpha + G\,\hat{r}_1 + H\,w + \epsilon_{st} \qquad (6.27)$$

with

$$\epsilon_{st} = \forall\,\epsilon_{st-1} + \omega_{st}$$

$$i'\,\alpha = 0$$

$$G = G'$$

$G' \; i = 0$

$i' \; G = 0$

$i' \; H = -1$

where $w = i' \; A_2$. FIML estimates of this model are reported in Table 3.14.

The second maintained static model for the post-1971 period is the Safety First model with symmetry restrictions and no dummy variables. This model has the same structure as H_{S13} but has two more equations.

H_{S22}:

$$a_{1it} = -\{(g_{i1}[\hat{r}_{11} - c_n] + g_{i2}[\hat{r}_{12} - c_n] + \ldots + g_{i7}[\hat{r}_{17} - c_n]) /$$

$$((g_{11} + g_{12} + \ldots + g_{17})[\hat{r}_{11} - c_n] +$$

$$(g_{12} + g_{22} + \ldots + g_{27})[\hat{r}_{12} - c_n] + \ldots + [\hat{r}_{17} - c_n])\} d_2 + \epsilon_{ist} \qquad (6.28)$$

with

$$\epsilon_{st} = \forall \epsilon_{st-1} + \omega_{st}$$

and i = 1, 2, ..., 7. Table 4.11 gives the FIML estimates for this model.

The first maintained dynamic model for the post-1971 period is the portfolio-theoretical model with intercepts, Special Deposit dummy variables and

non-choice items as a single wealth variable. The model is constrained by homogeneity restrictions.

H_{D21}:

$$\Delta A_{1t} = K + \Phi \hat{r}_1 + \Lambda w - B A_{1t-1} + F D_{SD} + \epsilon_{dt} \qquad b \ (6.29)$$

with

$i' K = 0$

$i' \Phi = 0$

$\Phi' i = 0$

$i' w = -1$

$i' B = i$

$i' F = 0$

where w is as defined in H_{S21}. FIML estimates for this model are contained in Table 3.24.

The other maintained dynamic model for the post-1971 period is the Safety First model with symmetry restrictions and no dummy variables. This model has the same structure as that of H_{D13} for the pre-1971 period.

H_{D21}:

$$\Delta a_{1it} = - \{(b_{i1}\, g_{11} + b_{i2}\, g_{12} + \ldots + b_{i7}\, g_{17})\, [\hat{r}_{11} - c_n] +$$

$$(b_{i1}\, g_{12} + b_{i2}\, g_{22} + \ldots + b_{i7}\, g_{27})\, [\hat{r}_{12} - c_n] + \ldots +$$

$$(b_{i1}\, g_{17} + b_{i2}\, g_{27} + \ldots + b_{i7}\, g_{77})\, [\hat{r}_{17} - c_n]\} \;/$$

$$((g_{11} + g_{12} + \ldots + g_{17})\, [\hat{r}_{11} - c_n] +$$

$$(g_{12} + g_{22} + \ldots + g_{27})\, [\hat{r}_{12} - c_n] + \ldots + [\hat{r}_{17} - c_n])\}\, d_2 -$$

$$b_{i1}\, a_{11t-1} - b_{i2}\, a_{12t-1} - \ldots\, b_{i7}\, a_{17t-1} + \epsilon_{idt} \qquad (6.30)$$

with

$$\sum_{i=1}^{7} b_{ij} = 1$$

for all j = 1, 2, ..., 7. Table 4.11 gives the FIML estimates for this model.

6.3.2 Non-nested Test Results

As we indicated earlier, the P_1 test can be used to test H_0 against more than one alternative model at a time. However, since we do not have any *a priori* information about the truth of the alternatives, we shall conduct only pairwise tests. Each of the models stated above will in turn be used as the maintained hypothesis. In Tables 6.1 - 6.3 we report the P_1 test statistics for all the eleven models. In each cell we give the λ value and its associated asymptotic t-ratio. The t-ratio informs us whether the null hypothesis should be rejected while the value of λ indicates whether the null hypothesis is rejected in a direction towards or away from the alternative hypothesis. Given these test statistics our discussion can be very brief.

Table 6.1
P_1 Test Statistics for H_{S11}, H_{S12}, H_{S13} and H_{S14}

Alternative Hypothesis:		H_{S11}	H_{S12}	H_{S13}	H_{S14}
Maintained Hypothesis:	H_{S11}		0.985 (3.118)	1.106 (4.143)	0.708 (4.148)
	H_{S12}	0.800 (6.439)		1.023 (5.268)	0.610 (4.876)
	H_{S13}	0.757 (6.597)	0.863 (3.670)		0.490 (5.311)
	H_{S14}	0.338 (17.108)	0.365 (17.747)	0.358 (18.546)	

Note: The first number in each cell is the estimate for λ *and the figure in the parentheses is the asymptotic t-ratio.*

Table 6.2
P_1 Test Statistics for H_{D11}, H_{D12}, and H_{D13}

Alternative Hypothesis:		H_{D11}	H_{D12}	H_{D13}
Maintained Hypothesis:	H_{D11}		1.199 (1.826)	0.864 (1.840)
	H_{D12}	0.807 (5.398)		0.712 (2.998)
	H_{D13}	0.779 (8.919)	0.819 (5.337)	

Note: The first number in each cell is the estimate for λ *and the figure in the parentheses is the asymptotic t-ratio.*

Table 6.3
P_1 Test Statistics for H_{S21}, H_{S22}, H_{D21} and H_{D22}

Alternative Hypothesis:		H_{S21}	H_{S22}	H_{D21}	H_{D22}
	H_{S21}		0.796 (2.964)		
	H_{S22}	0.671 (2.794)			
Maintained Hypothesis:					
	H_{D21}				0.906 (1.658)
	H_{D22}			0.740 (2.848)	

Note: The first number in each cell is the estimate for λ *and the figure in the parentheses is the asymptotic t-ratio.*

Let us first examine the P_1 test statistics on the four static models for the pre-1971 period. It is clear from Table 6.1 that all the four static models for the pre-1971 period must be rejected. All the asymptotic t-ratios are well above the critical t values at all reasonable significance levels. As we have seen in Chapters Three, the models H_{S11} and H_{S12} are derived from the same principle and very similar in structure. This close resemblance between H_{S11} and H_{S12} is also reflected in the test results. When H_{S11} is taken to be the maintained hypothesis, H_{S11} is rejected in the direction towards H_{S12} and H_{S13}. Similarly, H_{S12} is also rejected in the direction towards H_{S13}. When H_{S13} is taken to be the maintained hypothesis, the λ value against H_{S11} is very close to the λ value against H_{S12} as the alternative. We also note from Table 6.1 that testing H_{S14} against H_{S11}, H_{S12} and H_{S13} give very similar results: H_{S14} is rejected in the direction away from all the three alternatives and the λ values against H_{S11}, H_{S12} and H_{S13} are almost identical. This similarity is also discernible when the role of H_{S14} is reversed.

Table 6.2 gives the P_1 test statistics on the three dynamic models for the pre-1971 period. Clearly, both H_{D12} and H_{D13} must be rejected. The rejection occurs regardless of whether the alternative is each other or H_{D11}. The λ values also indicate that both

H_{D12} and H_{D13} are rejected in the direction torwards H_{D11}. In fact, when H_{D11} is the maintained hypothesis the estimated λ values against H_{D12} and H_{D13} are both statistically insignificant. Thus, H_{D11} cannot be rejected against the evidence provided by H_{D12} and H_{D13}, and we therefore maintain H_{D11} as the empirically consistent model for the pre-1971 period.

How do the post-1971 models fair in the non-nested test? As we can see from Table 6.3 the test results are not dissimilar to those for the pre-1971 period. Both the static models, H_{S21} and H_{S22}, must be rejected. With H_{S21} as the maintained hypothesis the asymptotic t-ratio is 2.964, well above the critical t values at all reasonable significance levels. When H_{S22} is taken as the maintained hypothesis, the asymptotic t-ratio is similarly at 2.794. For the two dynamic models, H_{D22} has to be rejected against the evidence provided by H_{D21}. However, we are unable to reject H_{D21} against the evidence provided by H_{D22}. Thus, for the post-1971 period, we maintain H_{D21} as the empirically valid model.

FIML estimates of the two maintained models are reported in Tables 3.23 and 3.24. As we can see from the tables, both models perform badly in terms of the traditional "goodness of fit" measure, the R^2. Although we have omitted a number of observations in our estimation for the purpose of forecasting, because of the poor performance of the two models we shall not conduct any post-sample forecasting. The above test results also highlight the point that the P_1 test, and indeed non-nested tests in general, are quite different from comparisons of R^2 statistics or likelihoods. On the one hand, all the static models, which fit universally better than any of the dynamic models, have to be rejected. On the other hand, two of the dynamic models, which are clearly less satisfactory than their static counterparts in terms of fit, cannot be rejected by the evidence. The P_1 test is thus not a measure of relative fit; it is a measure of whether a given hypothesis can or cannot explain the performance of alternative hypotheses against the evidence.

6.4 CONCLUDING REMARKS AND SOME SUGGESTIONS FOR FUTURE RESEARCH

Since the publication of the seminal paper by Parkin *et al* in 1970 the empirical literature on the portfolio behaviour of commercial banks in general, and of UK commercial banks in particular, has been dominated by one particular type of models — the mean-variance models based on expected utility maximisation under the assumption of risk aversion. The major attraction of this type of models is that they yield clearly defined demand equations and prior information concerning the parameters to be estimated. Most of these models are static models. They are linear in both their parameters and variables and are therefore also easy to estimate. In general, this type of models perform well in terms of the traditional "goodness of fit" measures and many authors have drawn important policy conclusions from them. Although the properties of this type of models such as homogeneity and symmetry have been tested by many studies the validity of these models have never been tested against other alternative models.

In this study we attempted to extend the traditional portfolio-theoretical models and at the same time to apply two alternative approaches to model the portfolio behaviour of commercial banks. These are the Safety First and precautionary approaches. We built and estimated both static and dynamic models. Although these alternative approaches are themselves unsatisfactory since each of them, like the portfolio-theoretical approach, emphasises one particular aspect of the bank's operations, they provide us with alternative models which can be used to test the validity of the existing models. As these various sets of models are non-nested within each other, a recently developed non-nested test statistic is employed to test them. To those who are eager to look for policy implications from econometric models our test results are very disappointing. We are left without a satisfactory model: every static model we have considered must be rejected against one or more alternatives and although we are unable to reject two of the dynamic models their performance in terms of "goodness of fit" is extremely poor. Disappointing this situation may be, we do not, however, consider this to be undesirable.

It is our belief that in the existing empirical literature some hypotheises are accepted too readily. One useful message emerging from this study is that models which perform well in terms of "goodness of fit" measures are not necessarily empirically consistent and one should not make important policy conclusions based on these models alone.

Although we are left without a satisfactory model our test results indicate that more efforts should be directed towards improving the dynamic models. A number of factors may be responsible for the poor performance of the dynamic models. We suggest a number of areas where future research might be fruitful. The list is based on some of the major shortcomings of the existing models and it is by no means exhaustive.

1) The structure of the disequilibrium and transactions costs. In deriving the dynamic models we assumed in Chapter Three that both the disequilibrium and transactions cost functions take the quadratic form. As we already indicated this assumption may be inadequate and other functional forms of the costs should be tested.

2) The two-stage optimisation procedure. In both Chapters Two and Three, we adopted a two-stage optimisation procedure. First, the bank chooses a portfolio which is optimal in the absence of transactions costs. Once such an optimal portfolio has been derived the bank then decides whether to adjust its portfolio by taking into account of the transactions costs as well as of the costs which would incur if it does not move into such an optimal portfolio. This two-stage procedure is in general sub-optimal. One possible way of improving this situation would be to specify transactions or adjustment costs directly in the bank's objective function and then carry out the optimisation process.

3) The bank's expectations formation process. The asset demand equations derived in both Chapter Three and Chapter Four depend upon the expected rates of return on assets and their variances and covariances, which are unobservable and unknown

to the bank. In deciding its optimal portfolio the bank will have to form expectations about them. One possible direction would be to model the bank's expectations formation process regarding interest rates explicitly. In other words, the bank would first estimate the expected rates of return on assets and their variances and covariances and then use these estimates as data in the estimation of its optimal demand equations. One could, for example, employ the Bayesian approach to model the bank's expectations concerning interest rates in the same way as the bank's expectations about future cash requirement are constructed in Chapter Five.

4) Combining the different approaches. Each of the three sets of models emphasises one important aspect of the bank's operations. Another potentially useful direction would be to combine aspects of the different approaches. As we have indicated in Chapter One, it will be difficult in a general n-asset model to combine features of the different approaches. However, one feasible option would be to partition the choice assets into sub-groups according to their liquidity and risk characteristics, and then to apply different approaches to these sub-groups. For example, one might partition the assets into a liquid set and an illiquid set. As the liquid assets are useful for meeting random cash demand one could then apply the precautionary approach to model the bank's demand for these assets. On the other hand, illiquid assets such as loans and advances have higher risks as well as higher rates of return and one could apply either the portfolio-theoretical approach or the Safety First approach to this subset of assets. The difficulty with such an option lies in choosing an appropriate method of combining the various sub-portfolios, which would be optimal.

APPENDIX: DATA SOURCES AND VARIABLE DEFINITIONS

In this appendix, we give the data sources and definitions of variables.

1. Sources

Balance sheet data of the London clearing banks for the period 1959:01 - 1969:12 are from Bank of England, Statistical Abstracts, no. 1, Table 9, pp. 35-39. Balance sheet data for the period 1970:01 - 1983:08 are from Bank of England Quarterly Bulletin, statistical annex, vols. 11-23. Interest rate data for the period 1959:01 - 1969:12 are from Bank of England, Statistical Abstracts, no. 1, Tables 29 and 30, pp. 170-178. Interest rate data for the period 1970:01 - 1983:08 are from Bank of England Quarterly Bulletin, statistical annex, vols. 11-23.

2. Variable Definitions

Interest rates:

ADR = the Bank rate (1959:01 - 1972:09), or minimum lending rate (1972:10 - 1981:07), or base rate (1981:08 - 1983:08) plus two percentage points;

CBR = the average of discount market's buying rates for trade bills (three months);

CDR = the sterling certificate of deposit three-month average rate;

CMR = the average of call money rates for the London clearing banks;

DDR = the London clearing banks' deposit rate (seven days' notice);

GBR = the redemption yields for short-dated British government stocks;

LLR = the average of three-month rates for local authorities' temporary loans;

TBR = the discount market's tender rate for Treasury bills;

Assets and Liabilities:

AD = Advances and loans other than to nationalised industries (1959:01-1971:09); sterling advances (1971:10 - 1975:04); sterling advances to UK, sterling advances to overseas and other sterling loans (1975:05 - 1983:08);

CA = Cash, balances with the Bank of England and balancing items;

CB = Commercial bills and other bills (1959:01 - 1971:09); commercial bills, other bills and acceptances (1971:10 - 1975:04); commercial bills, sterling other bills and acceptances (1975:05 - 1983:08);

CD = Sterling certificates of deposits;

CM = Call money (1959:01 - 1971:09); money at call and short notice to discount houses and other borrowers and sterling balance with other banks (1971:10 - 1975:04); money at call and short notice, and sterling loans to banks and discount houses other than reserve assets (1975:05 - 1983:08);

DD = Current accounts (demand deposits) (1959:01 - 1971:09);

EA = Advances to nationalised industries (1959:01 - 1971:09);

GB = British government stocks and other investment (1959:01 - 1971:09); British government stocks up to five years to maturity, and British government stocks over five years and undated (1971:10 - 1975:04); British government stocks up to one year to maturity, British government stocks over one year and undated, and sterling other investment (1975:05 - 1983:08);

LL = Sterling loans to local authorities (1971:10 - 1983:08);

NP = Net foreign currency position - total foreign currency liabilities minus total foreign currency assets (1971:10 - 1983:08);

SD = Special Deposits with Bank of England (1959:01 - 1971:09); Special and Supplementary deposits with Bank of England (1971:10 - 1980:07);

SL = Total sterling liabilities (1971:10 - 1983:08);

TD = Deposit and other accounts (1959:01 - 1971:09).

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